

THE PARABOLIC ANDERSON MODEL: THE ASYMPTOTICS OF THE STATISTICAL MOMENTS AND LIFSHITZ TAILS REVISITED

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ABSTRACT. The Anderson Hamiltonian $H = -\Delta + V$ on $l^2(\mathbb{Z}^d)$ is an object of intense research in probability theory as well as in mathematical physics. Here Δ is the discrete Laplacian and $V = \{V(x) : x \in \mathbb{Z}^d\}$ is an i.i.d. random field taking values in \mathbb{R} .

In probability theory one considers the parabolic Anderson model (PAM) $\partial_t u = -Hu$ on $\mathbb{R}_+ \times \mathbb{Z}^d$ with initial condition $u(0, x) = 1$. The large-time behaviour of (PAM) is strongly linked to the occurrence of high peaks in the fields V and $u(t, x)$. These intermittency peaks give the main contribution to the formation of the statistical moments and determine the correlation structure of $u(t, x)$. The latter quantities are extensively studied in the stochastics literature using path integral methods and the theory of large deviations.

The parabolic Anderson model is also closely connected to the theory of Anderson localization. The large-time behaviour of the statistical moments and the correlation structure correspond to Lifshitz tails, i.e. to the asymptotics of the integrated density of states and to properties of localized eigenstates.

Our aim is to study the relation between the probabilistic and the spectral theoretic point of view combining spectral arguments and the Laplace method. In particular we want to present a unified approach to control the asymptotics of the statistical moments interpolating between the quantum and the classical regime.

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