A Novel Facility Design Approach to Improve the Revenue of Public-Storage Warehouses

Yeming (Yale) Gong †, René de Koster ‡, J.B.G.(Hans) Frenk §, and Adriana F. Gabor †

† EM LYON Business school, France. ‡ Erasmus University, the Netherlands. § Sabaci University, Turkey.

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This research is based on our visits of 53 warehouse facilities in Europe (e.g. Rotterdam, Bonn, Lyon), America (e.g. Chicago, Philadelphia), and Asia (e.g. Shanghai, Hong Kong) from December 2007 to July 2009.

- Shurgard (EU, Rotterdam, left 1), MiniCo (China, Hong Kong, left 2), Public Storage (USA, Philly, left 3), and Big Orange (Singapore, left 4).
A operation mode where the revenue not the cost matters

- A large warehouse (left). **But...** few workers (right).
- So the revenue, not the cost (in traditional warehouse literature) is their major concern.
The internal view of a PS warehouse

Heterogeneous storage units in Rotterdam Shurgard warehouse.
What is the major business problem?

Storage unit type ↔ Market segment

- Storage types do not fit to the corresponding market segments.

<table>
<thead>
<tr>
<th>demand type</th>
<th>demand quan.</th>
<th>supply type</th>
<th>supply quan.</th>
</tr>
</thead>
<tbody>
<tr>
<td>7 m²</td>
<td>10</td>
<td>7 m²</td>
<td>2</td>
</tr>
<tr>
<td>5 m²</td>
<td>2</td>
<td>5 m²</td>
<td>10</td>
</tr>
</tbody>
</table>
Could we find a new facility design approach to be able to provide a layout design to improve the revenue management of public storage warehouses?

Storage layout design in a Shurgard warehouse.
Literature

- Literature on warehousing operations.
  *Many*: Focus on cost management.
  *Few*: Emphasize the revenue management.

- Literature on revenue management.
  *Many*: Airline management, hotel management.
  *Few*: Warehouses.
A case to introduce the basic problem

- Visited a *Public Storage* warehouse near Sears tower, Chicago.
- Environment: Downtown → high demand → customers are lost when a storage unit type is fully occupied.
Upon arrival of a customer for storage type $i$, if no units of this type are available, this customer is lost.

The equilibrium probability that an arriving type-$i$ customer is lost is given by the $M/G/x_i/x_i$ Erlangs B-formula

$$B(x_i, \lambda_i, \beta_i) = \frac{(\lambda_i \beta_i)^{x_i}}{x_i!} \left[ \sum_{j=0}^{x_i} \frac{(\lambda_i \beta_i)^{j}}{j!} \right]^{-1}.$$
Formulation

- **Objective**: To maximize the expected revenue.
- **Constraints**: Space capacity with a total area $C$.
- **Decision variables**: $x_i$, the amount of storage units of type $i$ with storage area $c_i$.

**Model-1**: 

$$\max \sum_{i=1}^{K} r_i \beta_i \lambda_i (1 - B(x_i, \lambda_i, \beta_i))$$

$$s.t. \sum_{i=1}^{K} c_i x_i \leq C, x_i \in \mathbb{Z}^+, 1 \leq i \leq K$$
An optimal DP algorithm

(I) Evaluate $R_m(y)$ for feasible $y \in \{0, 1, \ldots, C\}$. Where $R_m(y)$: revenue to go, given $y$ storage space left and we are now at storage type $m$.

(II) Set $k \leftarrow K - 1$.

*Repeat.*

(II.1) Evaluate $R_k(y) = \max_{x_k \in \mathcal{L}_k(y)} \{f_k(x_k) + R_{k+1}(y - g_k(x_k))\}$.

Where $f(x_i) := r_i \beta_i \lambda_i v(x_i)$, and $g(x_i) := c_i x_i$.

(II.2) $k \leftarrow k - 1$.

*Until* $k = 1$.

(III) Output the optimal objective value $R_1(C)$ and construct the optimal solution $(x_1^*, \ldots, x_m^*)$. 

Revenue Management and Public Storage Warehouses
Further exploration: Service level

Let $s_i$ denote the upper bound of acceptable loss probability for customer class $i$, we have service level constraint $B(x_i) \leq s_i$.

**Model-1-S:**

$$\text{max} \sum_{i=1}^{K} \beta_i r_i (1 - B(x_i))$$

$$\text{s.t.} \sum_{i=1}^{K} c_i x_i \leq C, B(x_i) \leq s_i, x_i \in \mathbb{Z}_+, 1 \leq i \leq K$$
A case to introduce the upgrade problem

- Requested a smaller unit (left) and was upgraded to a larger unit (right).
- By the term upgrade, we refer to an offer to a customer a storage unit with higher service class.
Units are reserved apriori for upgraded customers.

A type $k$ customer who finds all the $x_k$ units busy may choose to be upgraded and use one of the $y_k$ reserved units, if one is available.

If all $y_k$ units are busy, the customer is lost.
**Formulation**

- \( r_k m(x_k, \rho_k) \) is the revenue obtained from \( x_k \) units by PASTA.
- \( r_{k+1} \eta_{k+1}(x_k) \beta_k \left( 1 - P_{\text{rej}}(x_k, y_k) \right) \) is the revenue obtained from upgrade flows.

The optimization problem can now be written as:

\[
\max \sum_{k=1}^{K} \left[ r_k m(x_k, \rho_k) + r_{k+1} \eta_{k+1}(x_k) \beta_k \left( 1 - P_{\text{rej}}(x_k, y_k) \right) \right] \\
\sum_{k=1}^{K-1} (c_k x_k + c_{k+1} y_k) + c_K x_K \leq C \\
x_k, y_k \in \mathbb{Z}_+
\]

The above maximization problem can be solved via dynamic programming.

But, it is tough to get \( P_{\text{rej}}(x_k, y_k) \) the rejection probability that an upgraded type \( k \) customer will find the reserved \( y_k \) units busy.
Protocol for upgrade operations without apriori reservation

- One does not reserve capacity for upgraded customers in advance. A customer of type $k$ who finds all the units of type $k$ busy, may get any available unit of type $k + 1$ (if an available unit exists).
Table: Apply the basic model to design W Chicago Public Storage warehouse

<table>
<thead>
<tr>
<th>Items</th>
<th>class 1</th>
<th>class 2</th>
<th>class 3</th>
<th>class 4</th>
<th>class 5</th>
<th>class 6</th>
<th>class 7</th>
<th>revenue</th>
</tr>
</thead>
<tbody>
<tr>
<td>Type (ft²)</td>
<td>5 × 5</td>
<td>5 × 10</td>
<td>7.5 × 10</td>
<td>10 × 10</td>
<td>10 × 15</td>
<td>10 × 20</td>
<td>10 × 25</td>
<td></td>
</tr>
<tr>
<td>Prices</td>
<td>81</td>
<td>93</td>
<td>170</td>
<td>170</td>
<td>305</td>
<td>348</td>
<td>397</td>
<td></td>
</tr>
<tr>
<td>Ave. demand</td>
<td>10</td>
<td>15</td>
<td>35</td>
<td>45</td>
<td>20</td>
<td>5</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>Ave. storage time</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>3</td>
<td>3</td>
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<td></td>
</tr>
<tr>
<td>Old design</td>
<td>21</td>
<td>30</td>
<td>43</td>
<td>120</td>
<td>30</td>
<td>20</td>
<td>15</td>
<td>42109</td>
</tr>
<tr>
<td>New opt. design</td>
<td>25</td>
<td>34</td>
<td>79</td>
<td>96</td>
<td>45</td>
<td>17</td>
<td>6</td>
<td>48233 (+14.54%)</td>
</tr>
</tbody>
</table>

- **Performance**: The average revenue is improved by +14.54%.
- **Management insight**: Too many units with 10 × 10. In 2008, managers just redesigned the warehouses. They found previously they had too few units with 10 × 10. We show they have overplayed it. While we admit 10 × 10 is the most popular type, we should increase its neighbor types 7.5 × 10 and 10 × 15.
Design warehouses without upgrade operations

Table: Apply the basic model to design van Buren Public Storage Warehouse (Chicago)

<table>
<thead>
<tr>
<th>Items</th>
<th>class 1</th>
<th>class 2</th>
<th>class 3</th>
<th>class 4</th>
<th>class 5</th>
<th>class 6</th>
<th>class 7</th>
<th>class 8</th>
<th>revenue</th>
</tr>
</thead>
<tbody>
<tr>
<td>Type</td>
<td>5 × 5</td>
<td>5 × 10</td>
<td>7.5 × 10</td>
<td>10 × 10</td>
<td>10 × 10</td>
<td>10 × 10</td>
<td>10 × 10</td>
<td>10 × 10</td>
<td></td>
</tr>
<tr>
<td>Prices</td>
<td>66</td>
<td>78</td>
<td>102</td>
<td>139</td>
<td>281</td>
<td>348</td>
<td>398</td>
<td>480</td>
<td></td>
</tr>
<tr>
<td>Ave. demand</td>
<td>19</td>
<td>61</td>
<td>48</td>
<td>80</td>
<td>11</td>
<td>10</td>
<td>8</td>
<td>9</td>
<td></td>
</tr>
<tr>
<td>Ave. storage</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>Old design</td>
<td>41</td>
<td>63</td>
<td>52</td>
<td>126</td>
<td>43</td>
<td>41</td>
<td>0</td>
<td>0</td>
<td>45649</td>
</tr>
<tr>
<td>New design</td>
<td>40</td>
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<td>23</td>
<td>101</td>
<td>19</td>
<td>24</td>
<td>16</td>
<td>19</td>
<td>52693   (+14.68%)</td>
</tr>
</tbody>
</table>

Performance: The average current revenue, 45,649 USD/Month. The average revenue based on optimal design, 52,693 USD/M. Improvement +14.68%
Apply the basic model to design van Buren Public Storage Warehouse

- **Management insight 1**: Increase the number of units with the small size “5×10" to fit to students' demand in the UIC (see nearby UIC). “5×10" mapped to market segment “studio or one-bedroom”, a typical student’s room.

- **Management insight 2**: Increase units with the large size “25×10" “30×10" to fit to increasing demand from CBD (see nearby Sears Tower in CBD).
Design Warehouses with upgrade operations

### Table: Design for warehouses with upgrade operations

<table>
<thead>
<tr>
<th>Warehouse</th>
<th>Items</th>
<th>Class 1</th>
<th>Class 2</th>
<th>Class 3</th>
<th>Class 4</th>
<th>Class 5</th>
<th>Class 6</th>
<th>Class 7</th>
<th>Class 8</th>
<th>Revenue</th>
</tr>
</thead>
<tbody>
<tr>
<td>S.P. Rotterdam&lt;sup&gt;a&lt;/sup&gt;</td>
<td>Type (m&lt;sup&gt;2&lt;/sup&gt;)</td>
<td>3</td>
<td>6</td>
<td>9</td>
<td>12</td>
<td>15</td>
<td>18</td>
<td>22</td>
<td>27</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Prices (€)</td>
<td>109</td>
<td>132</td>
<td>177</td>
<td>225</td>
<td>254</td>
<td>372</td>
<td>436</td>
<td>468</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Demand</td>
<td>31(2)&lt;sup&gt;b&lt;/sup&gt;</td>
<td>31(2)</td>
<td>33(2)</td>
<td>13(2)</td>
<td>7 (3)</td>
<td>8(3)</td>
<td>2(4)</td>
<td>2(4)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Old design</td>
<td>34</td>
<td>44</td>
<td>58</td>
<td>25</td>
<td>18</td>
<td>27</td>
<td>3</td>
<td>4</td>
<td>38577€</td>
</tr>
<tr>
<td></td>
<td>New design</td>
<td>15(0)&lt;sup&gt;c&lt;/sup&gt;</td>
<td>11(7)</td>
<td>21(16)</td>
<td>53(14)</td>
<td>2(0)</td>
<td>23(13)</td>
<td>9(6)</td>
<td>20(5)</td>
<td>57882€</td>
</tr>
<tr>
<td>N.D. Philly&lt;sup&gt;d&lt;/sup&gt;</td>
<td>Type (ft&lt;sup&gt;2&lt;/sup&gt;)</td>
<td>5 ×</td>
<td>5×</td>
<td>10 ×</td>
<td>10 ×</td>
<td>10 ×</td>
<td>10 ×</td>
<td>10 ×</td>
<td>10 ×</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Prices($)</td>
<td>65</td>
<td>79</td>
<td>132</td>
<td>227</td>
<td>222</td>
<td>255</td>
<td>326</td>
<td>396</td>
<td>67017$</td>
</tr>
<tr>
<td></td>
<td>Demand</td>
<td>34(2)</td>
<td>90(2)</td>
<td>70(2)</td>
<td>50(2)</td>
<td>40(2)</td>
<td>30(3)</td>
<td>9(3)</td>
<td>2(4)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Old design</td>
<td>78</td>
<td>180</td>
<td>144</td>
<td>22</td>
<td>54</td>
<td>22</td>
<td>24</td>
<td>4</td>
<td>67017$</td>
</tr>
<tr>
<td></td>
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<td>72</td>
<td>172</td>
<td>130</td>
<td>102</td>
<td>65</td>
<td>7</td>
<td>1</td>
<td>0</td>
<td>72872$</td>
</tr>
</tbody>
</table>
Sample-based sensitivity analysis

- W Chicago Warehouse. 1000 demand samples.
- With probability of 98.9 %, we can improve the revenue.
Sample-based sensitivity analysis

- W Chicago Warehouse. 1000 demand samples.
- With probability of 95.1 %, the old design for 10ft × 10ft is too large.
Apply the basic model to design warehouses in the high demand environment
Apply the upgrade model to design warehouses in the moderate demand environment

Sensitivity analysis

### Table: Sensitivity analysis of the design for SP Rotterdam warehouse

<table>
<thead>
<tr>
<th>Demand</th>
<th>Items</th>
<th>Class 1</th>
<th>Class 2</th>
<th>Class 3</th>
<th>Class 4</th>
<th>Class 5</th>
<th>Class 6</th>
<th>Class 7</th>
<th>Class 8</th>
<th>Revenue</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Type ($m^2$)</td>
<td>3</td>
<td>6</td>
<td>9</td>
<td>12</td>
<td>15</td>
<td>18</td>
<td>22</td>
<td>27</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Prices</td>
<td>109</td>
<td>132</td>
<td>177</td>
<td>225</td>
<td>254</td>
<td>372</td>
<td>436</td>
<td>468</td>
<td></td>
</tr>
<tr>
<td>Old design</td>
<td></td>
<td>34</td>
<td>44</td>
<td>58</td>
<td>25</td>
<td>18</td>
<td>27</td>
<td>3</td>
<td>4</td>
<td></td>
</tr>
<tr>
<td>07 Spring</td>
<td>New design</td>
<td>70(0)\textsuperscript{a}</td>
<td>4(13)</td>
<td>17(13)</td>
<td>46(0)</td>
<td>2(12)</td>
<td>20(6)</td>
<td>10(6)</td>
<td>21(0)</td>
<td>49999(36049)</td>
</tr>
<tr>
<td>07 Summer</td>
<td>New design</td>
<td>61(0)</td>
<td>3(15)</td>
<td>19(11)</td>
<td>45(0)</td>
<td>2(15)</td>
<td>23(6)</td>
<td>10(5)</td>
<td>20(0)</td>
<td>44030(32622)</td>
</tr>
<tr>
<td>07 Autumn</td>
<td>New design</td>
<td>15(7)</td>
<td>11(14)</td>
<td>18(16)</td>
<td>47(0)</td>
<td>2(13)</td>
<td>42(0)</td>
<td>4(6)</td>
<td>15(0)</td>
<td>50434(35709)</td>
</tr>
<tr>
<td>07 Winter</td>
<td>New design</td>
<td>62(0)</td>
<td>4(13)</td>
<td>17(12)</td>
<td>45(0)</td>
<td>2(14)</td>
<td>22(6)</td>
<td>10(6)</td>
<td>21(0)</td>
<td>50201(35724)</td>
</tr>
<tr>
<td>08 Spring</td>
<td>New design</td>
<td>14(7)</td>
<td>12(13)</td>
<td>18(13)</td>
<td>53(0)</td>
<td>2(14)</td>
<td>23(6)</td>
<td>10(6)</td>
<td>20(0)</td>
<td>63426(39449)</td>
</tr>
<tr>
<td>08 Summer</td>
<td>New design</td>
<td>15(7)</td>
<td>11(16)</td>
<td>21(14)</td>
<td>53(0)</td>
<td>2(13)</td>
<td>23(6)</td>
<td>9(5)</td>
<td>20(0)</td>
<td>57882(38577)</td>
</tr>
<tr>
<td>08 Autumn</td>
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<td>3(3)</td>
<td>11(0)</td>
<td>53998(37557)</td>
</tr>
<tr>
<td>08 Winter</td>
<td>New design</td>
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<td>11(13)</td>
<td>18(12)</td>
<td>40(0)</td>
<td>2(12)</td>
<td>41(0)</td>
<td>3(5)</td>
<td>20(0)</td>
<td>46499(34919)</td>
</tr>
</tbody>
</table>

\textsuperscript{a} Revenue: 07 Spring New design 70(0)\textsuperscript{a} 4(13) 17(13) 46(0) 2(12) 20(6) 10(6) 21(0) 49999(36049)

Suggestion - ↓ ↓ ↑ ↓ ↓ ↑ ↑

Revenue Management and Public Storage Warehouses
The motivation of robust design is to reduce the loss from the variance of demand patterns to the least.

Let \( \Lambda \) be the set of \( D \) demand data in the last two years, we present a robust model as follows:

\[
\max \{ \min_{\lambda_d \in \Lambda, d=1, \ldots, D} \left[ \sum_{i=1}^{m} r_i v(x_i, \rho_i) + r_{i+1} \eta_{i+1}(x_i) \beta_i \left( 1 - P_{\text{rej}}(x_i, y_i) \right) \right] : \\
\sum_{i=1}^{m-1} (c_i x_i + c_{i+1} y_i) + c_m x_m \leq C, 1 \leq i \leq m \}
\]
Robust design for SP Rotterdam Warehouse

- The robust optimal revenue value 44051 (current worst revenue value 32622).
- The robust design \( x^* = \{50(7), 17(13), 16(13), 37(0), 16(14), 25(5), 10(5), 13(0)\} \).
This paper presents a new facility design approach.
This paper identifies a new logistics research direction (the interface between revenue management and warehouse operations).
The paper reports a new revenue management application industry (the public storage warehouse).