Online Resource Minimization

Nicole Megow
with Lin Chen and Kevin Schewior

EURANDOM "Scheduling under Uncertainty"
Eindhoven, June 2015

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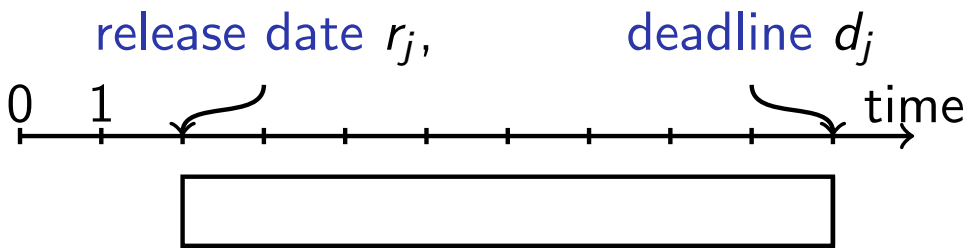

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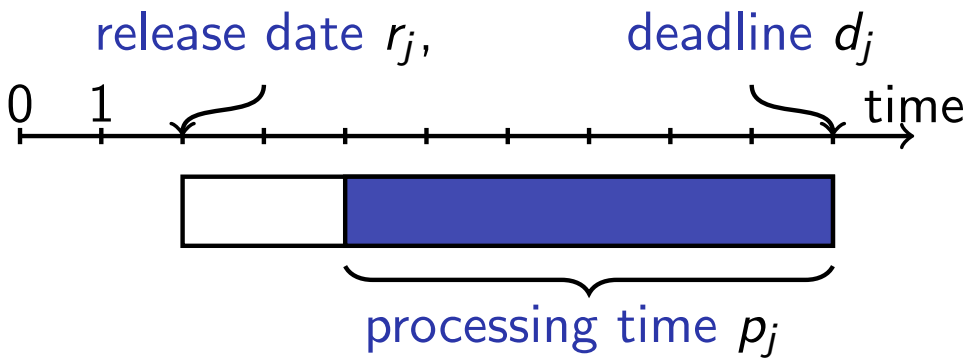

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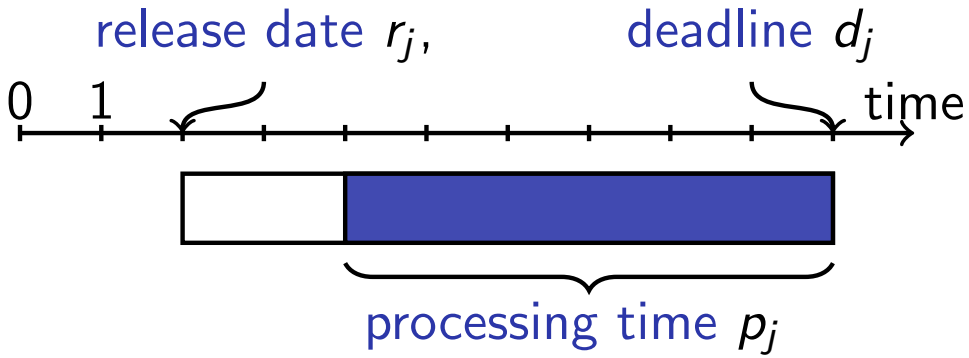

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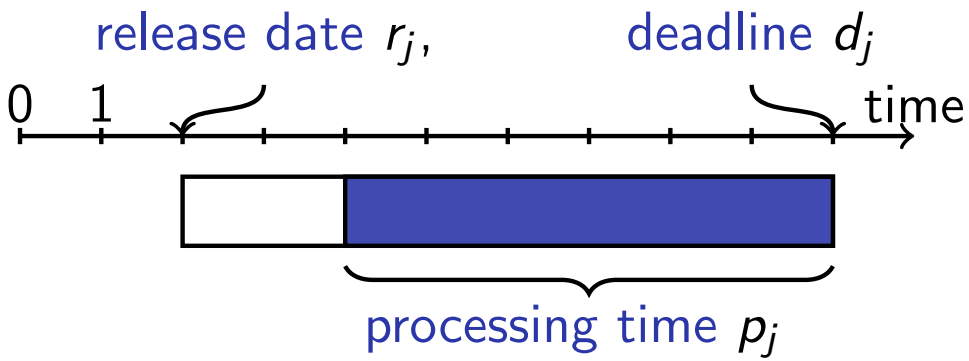

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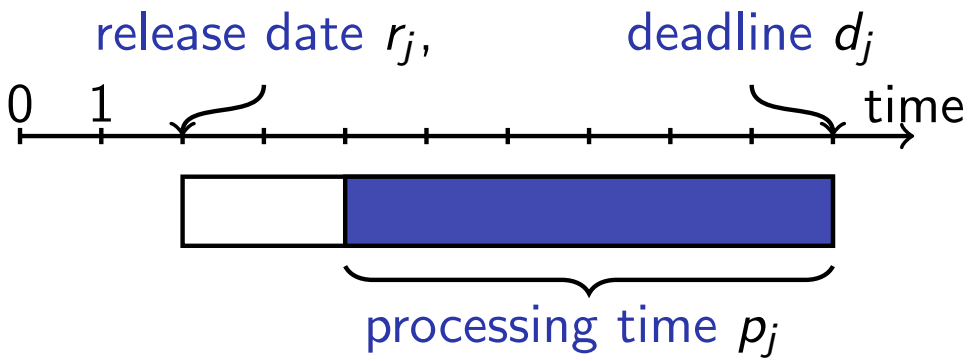

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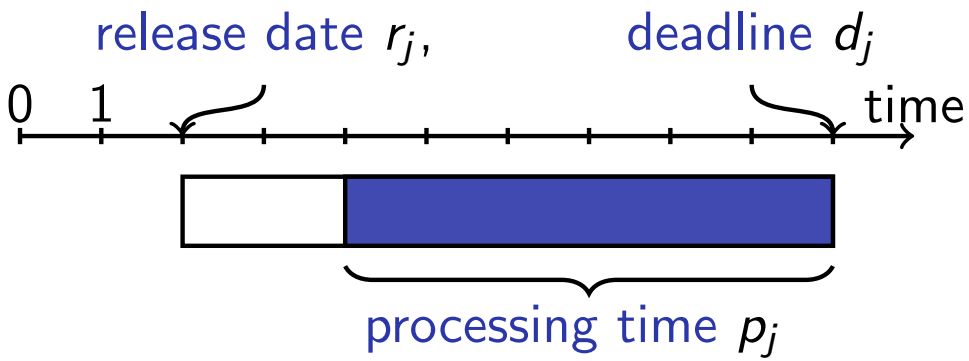

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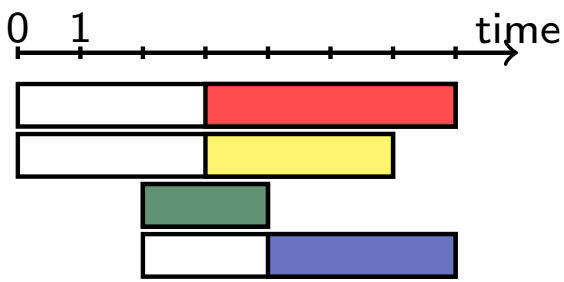

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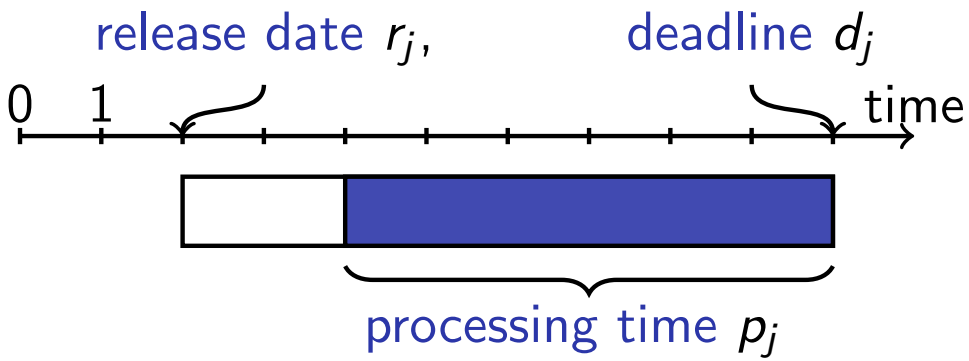

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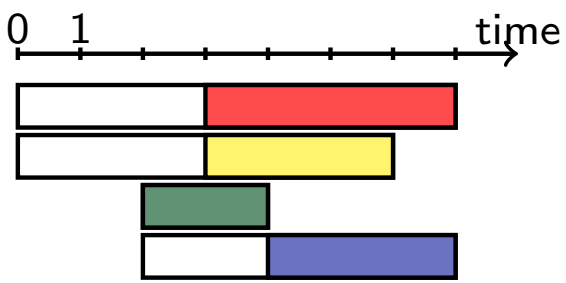

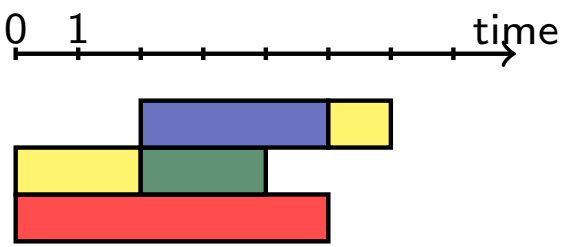

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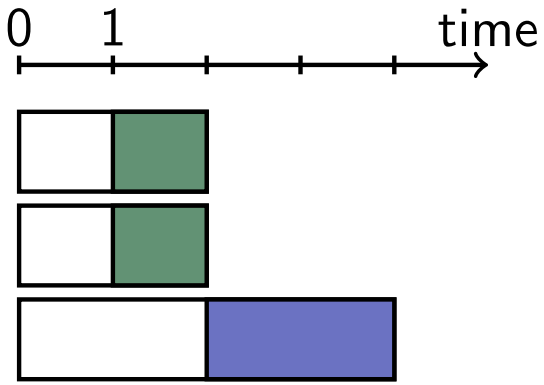

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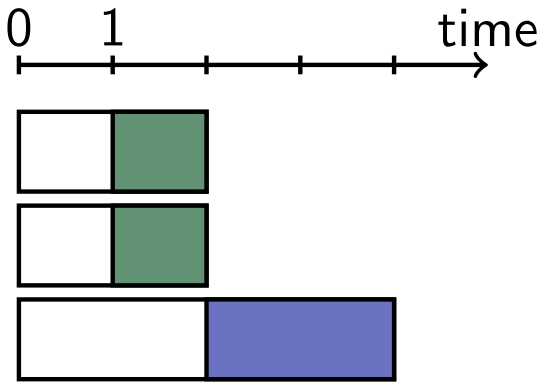

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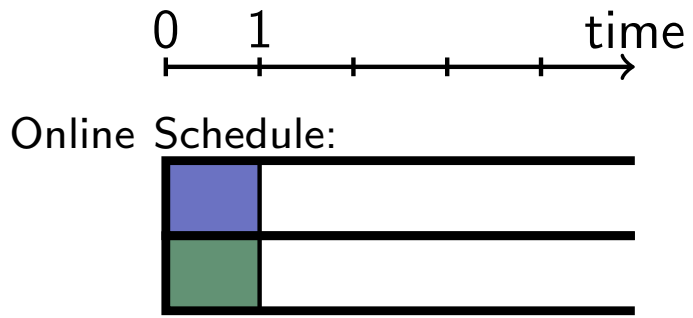
The blue job is scheduled to some extent in $[0, 1]$.



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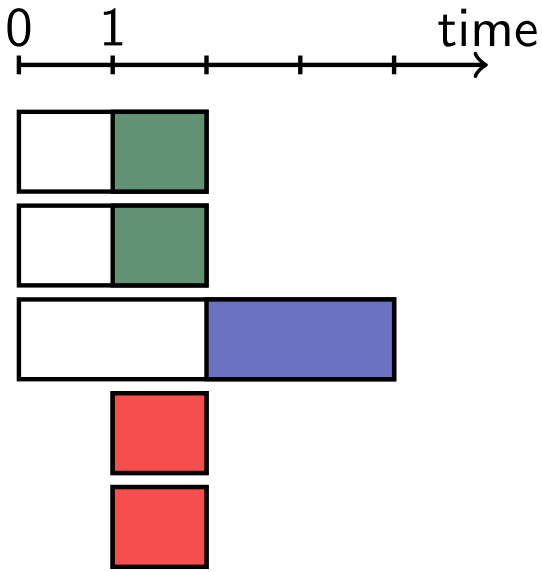

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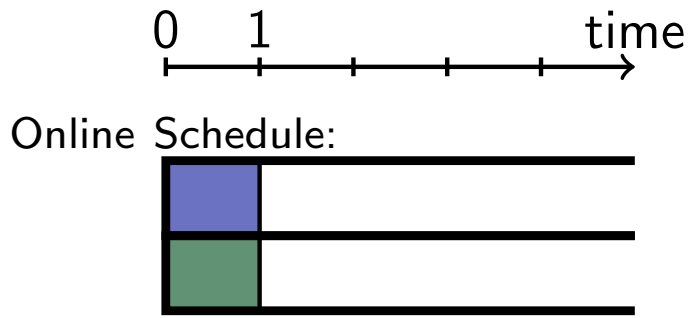

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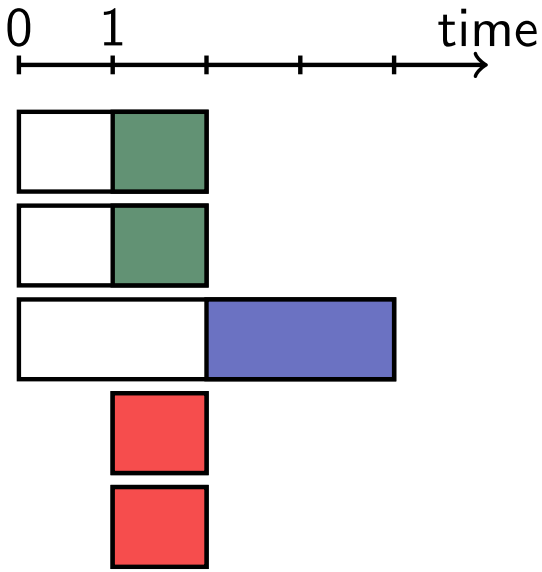

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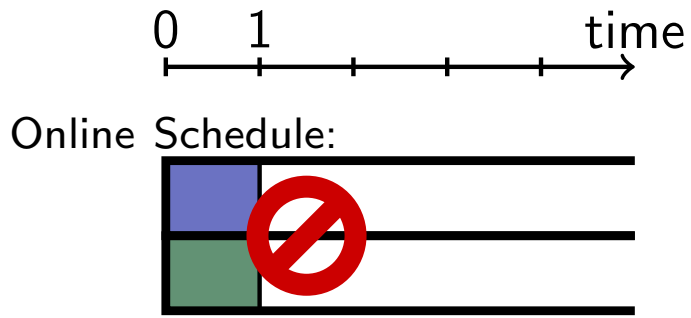

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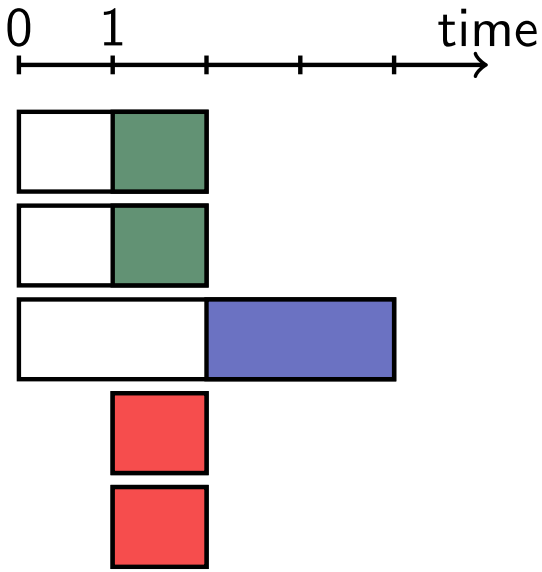

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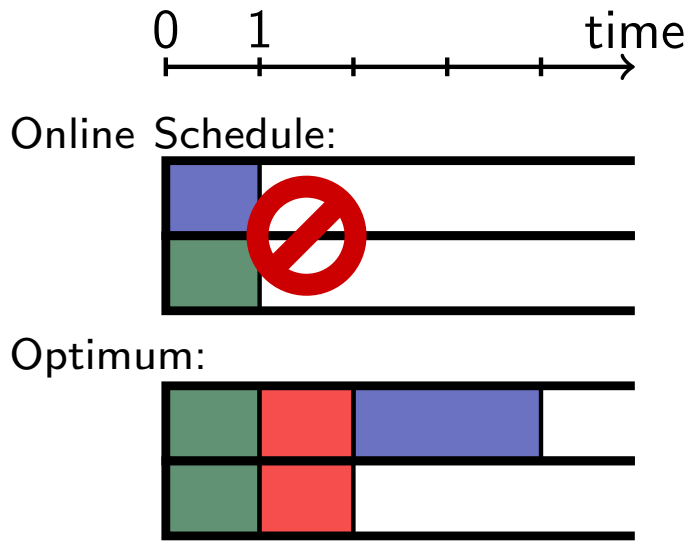

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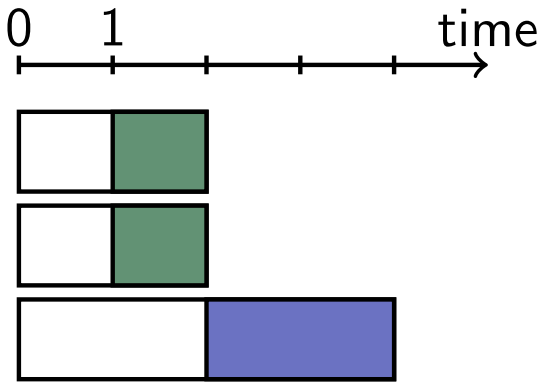

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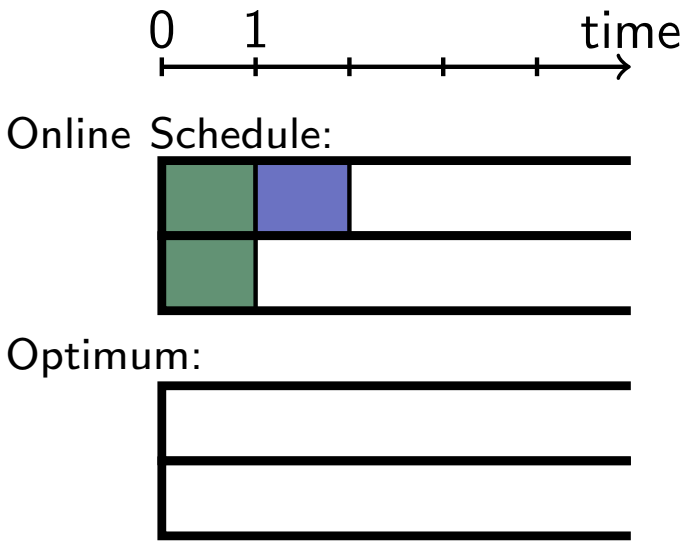

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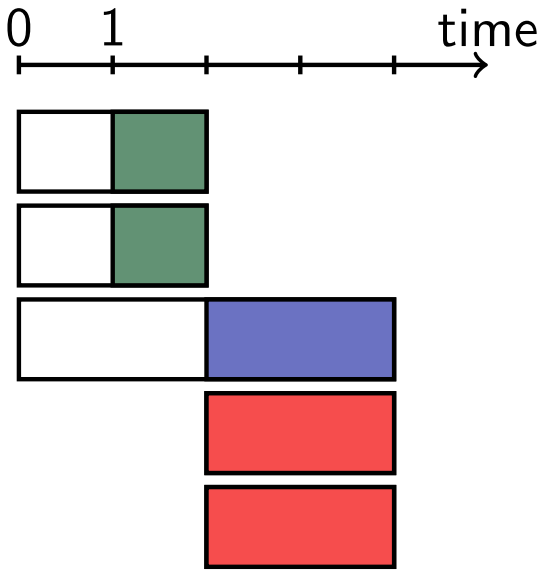

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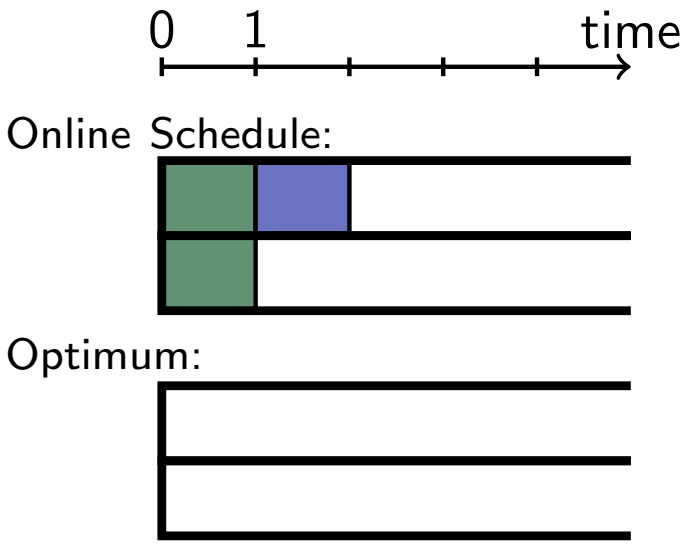

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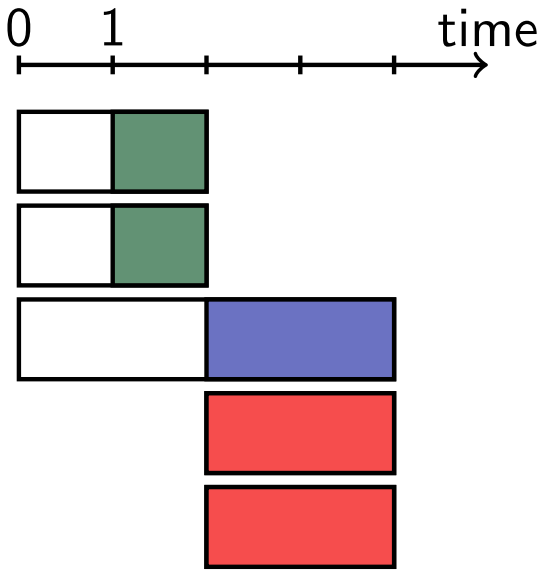

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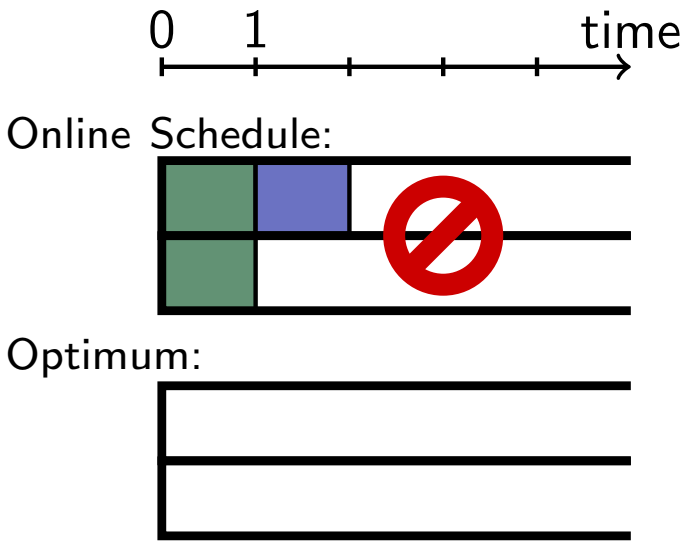

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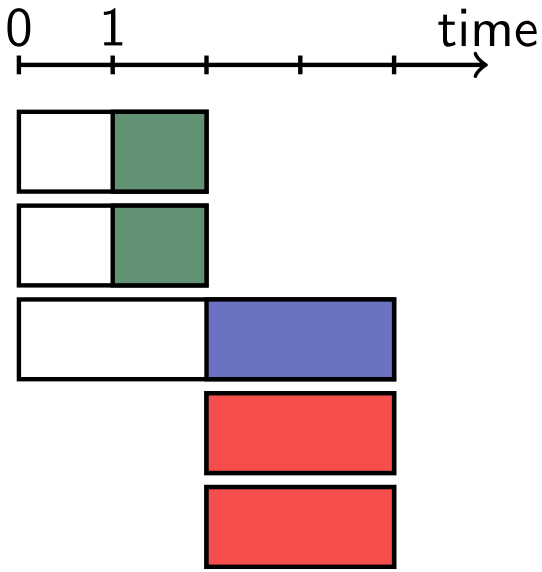

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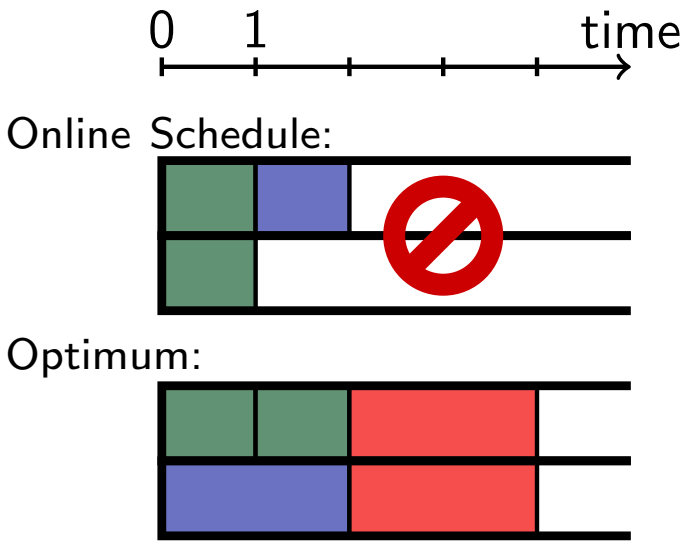

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Competitive analysis: An online algorithm ALG is c-competitive if

$$\mathrm{ALG}(I) \leq c \cdot m(I),$$

for all instances I with a feasible offline schedule on $m(I)$ machines.

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  - General lower bound is $5/4$. Philips et al. (STOC 1996)
  - $p_j = 1$: There is an optimal e-competitive online algorithm.
    Bansal, Kimbrel, Pruhs (J. ACM 2007)
    Devanur et al. (arxiv 2014)

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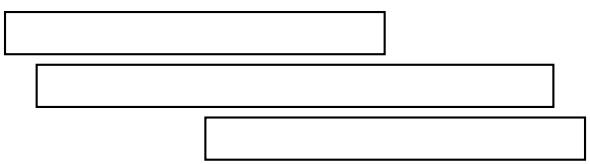
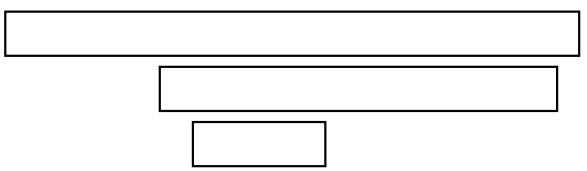
Agreeable instances

for any two jobs j, k:
$r_j < r_k$ implies $d_j \leq d_k$



Laminar instances

for any two intersecting time windows I_j, I_k: either $I_j \subseteq I_k$ or $I_k \subseteq I_j$



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Idea: Guess-and-Double.

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Algorithm $\text{EDF}_{m'}$: At any time schedule m available jobs with minimum deadline and preempt other jobs if necessary.

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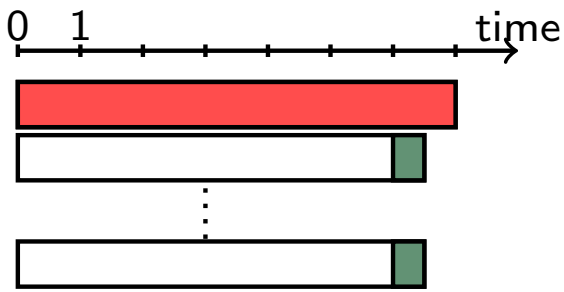

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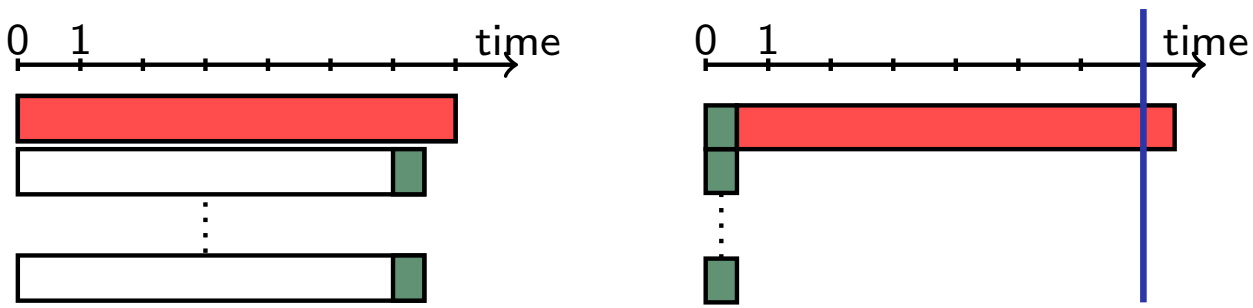

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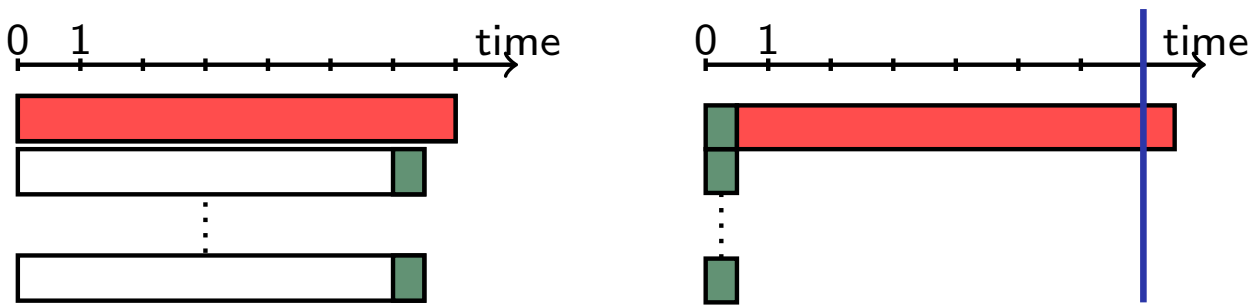

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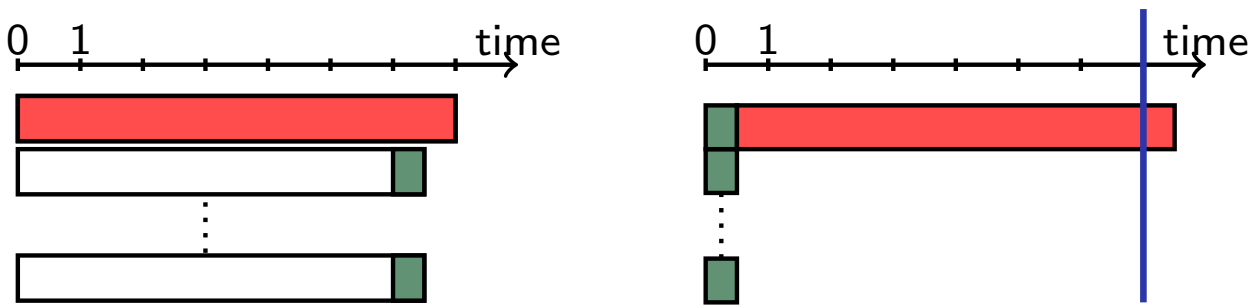

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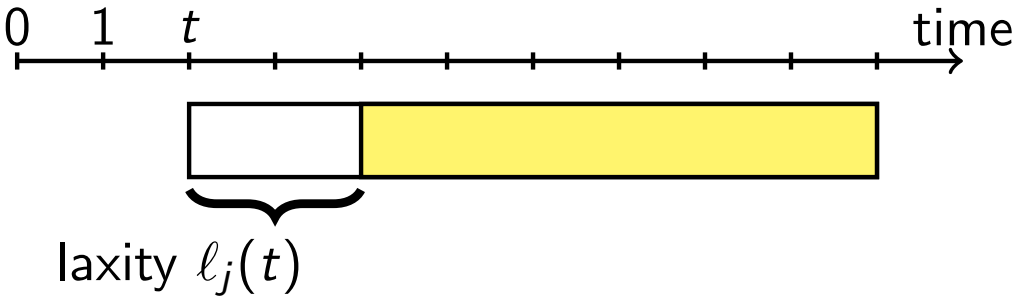
Lemma

If every job is α-loose, then $\text{EDF}_\star$ is $\frac{1}{(1-\alpha)^2}$-competitive.

Least Laxity First (LLF)

Laxity of a job



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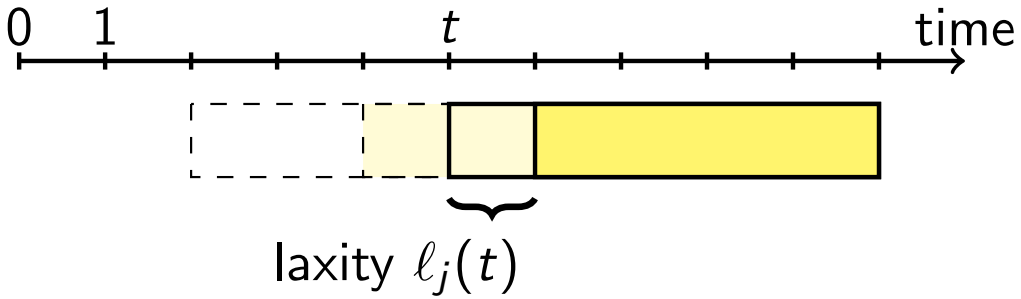

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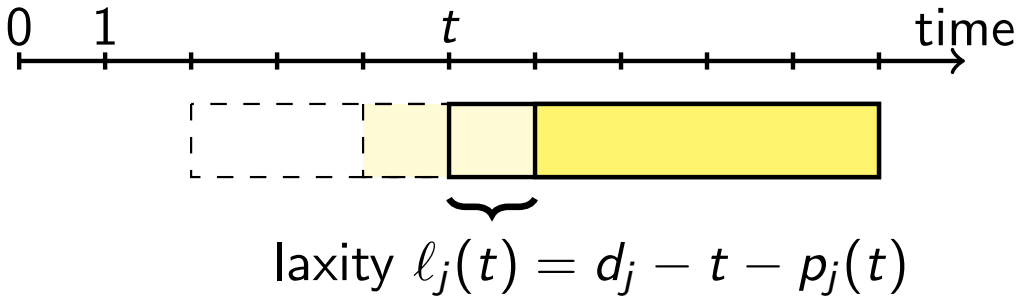

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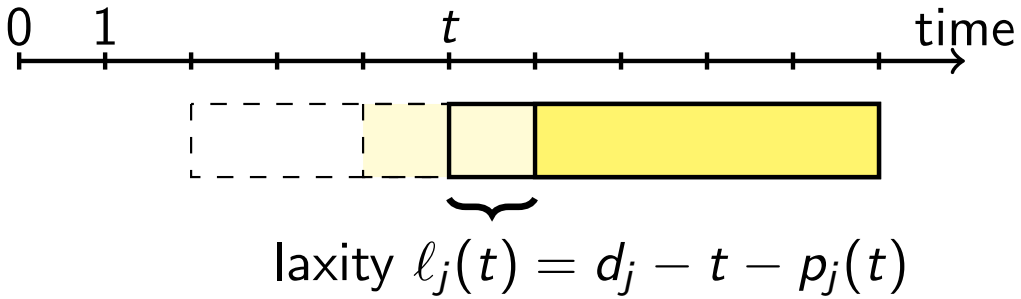

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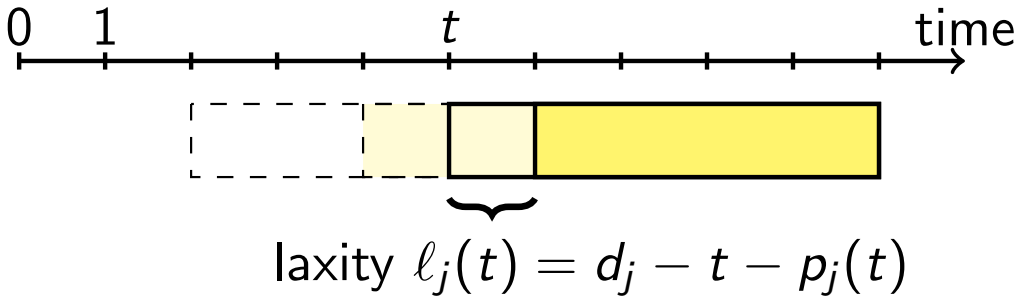

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LLF may fail on any number of machines only dependent on OPT.

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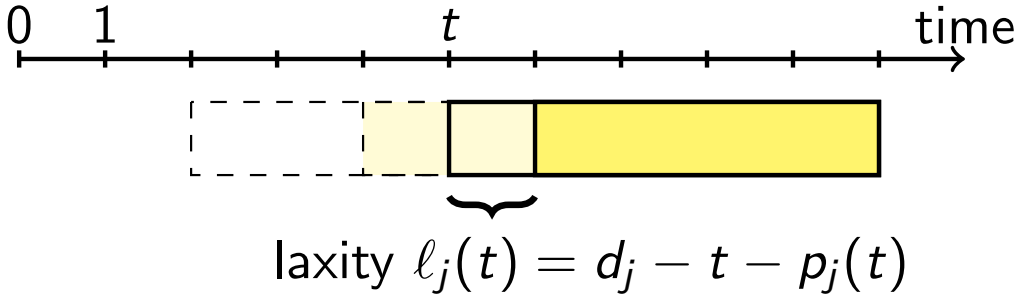

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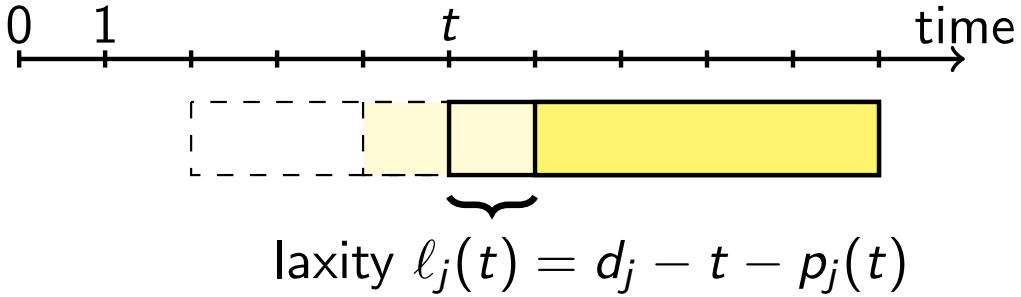
Theorem

There are instances on which LLF fails on $o(n^{\frac{1}{3}})$ machines.

Least Laxity First (LLF)

Laxity of a job



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Lemma. LLF on α-loose jobs again $\frac{1}{(1-\alpha)^2}$-competitive.

Agreeable instances

The setting:

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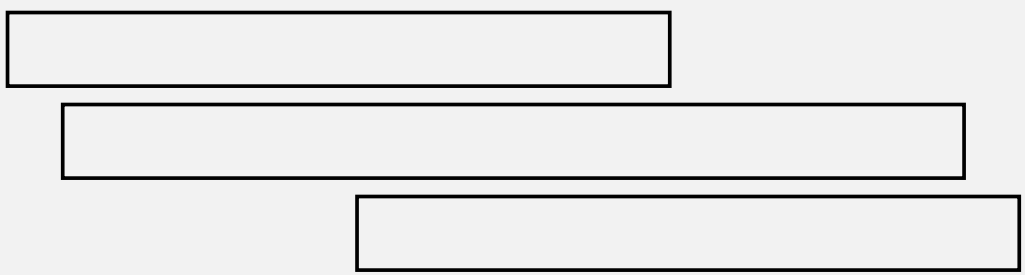
For any two jobs j, k: $r_j < r_k$ implies $d_j \leq d_k$.



Agreeable Instances

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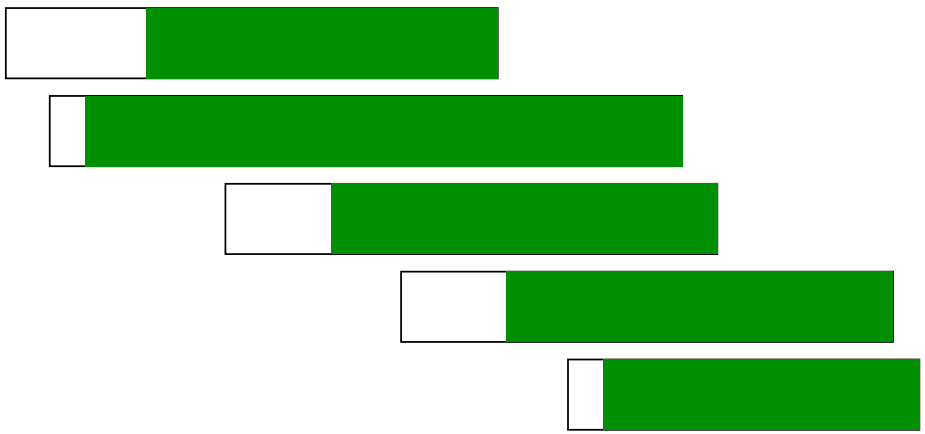
Schedule each job in the middle of its time window.



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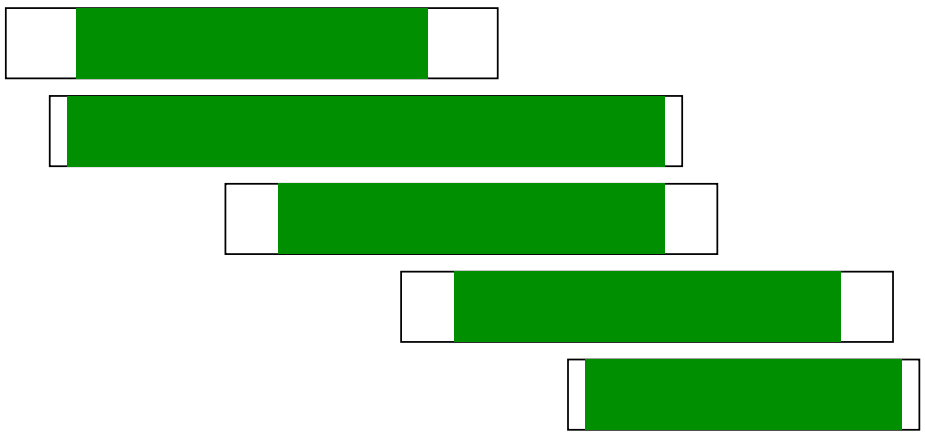

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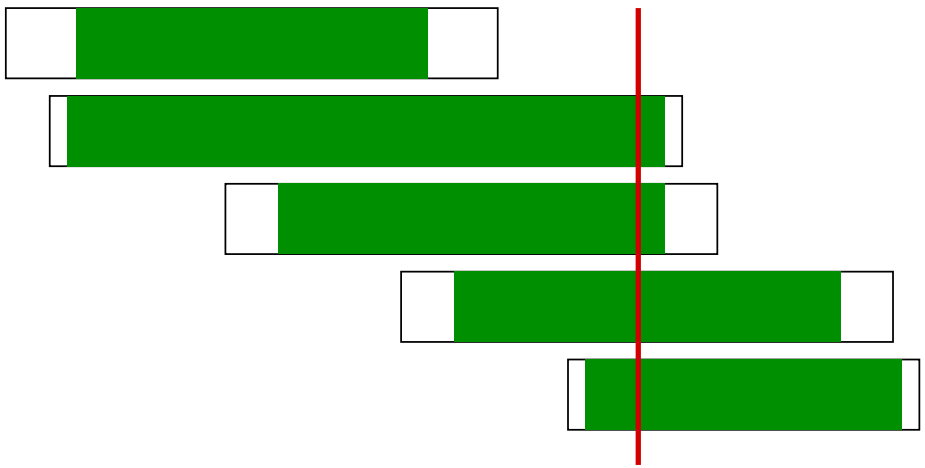

Lemma. At any time t, MediumFit is processing at most $\mathcal{O}(m)$ jobs.

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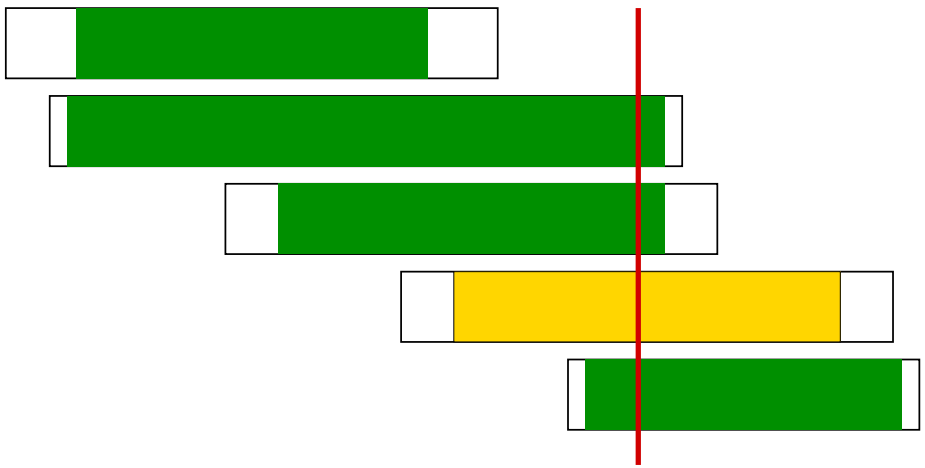

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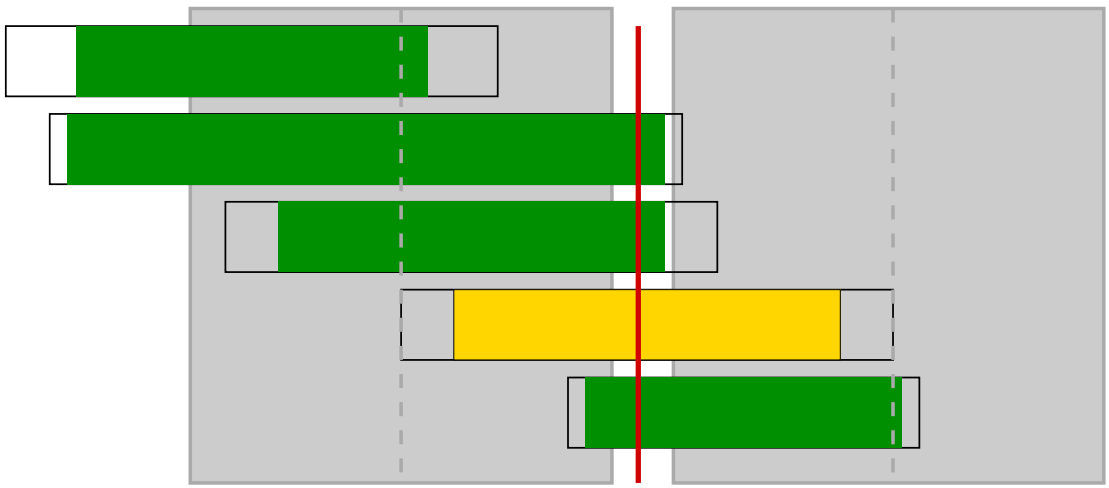

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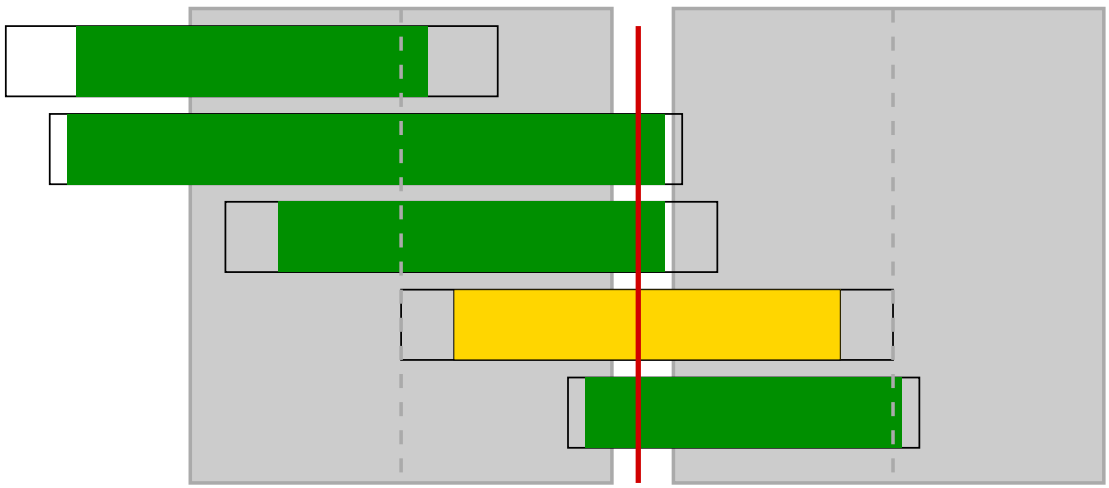

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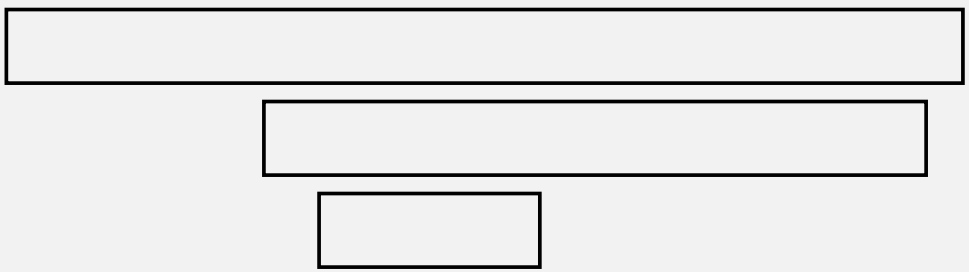
Theorem

EDF + MediumFit is a (non-)preemptive $\mathcal{O}(1)$-competitive algorithm.

Laminar instances

The setting:

For any two jobs j, k with intersecting time windows I_j, I_k:
either $I_j \subseteq I_k$ or $I_k \subseteq I_j$.



- Assume m is known.
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Laminar instances: High-level procedure

Open $m' = \mathcal{O}(m \log m)$ machines.

1. Assignment procedure
   - Upon arrival, a job is assigned to a machine and never migrates.
   - Jobs with larger time interval get assigned first (laminar jobs).
   - Fit full time interval into laxity of smallest already assigned job.
2. Scheduling procedure
   - Single machine scheduling: smallest job first (= EDF)

Laminar instances: Assignment procedure

Upon arrival of job j with time window $[r_j, d_j]$:

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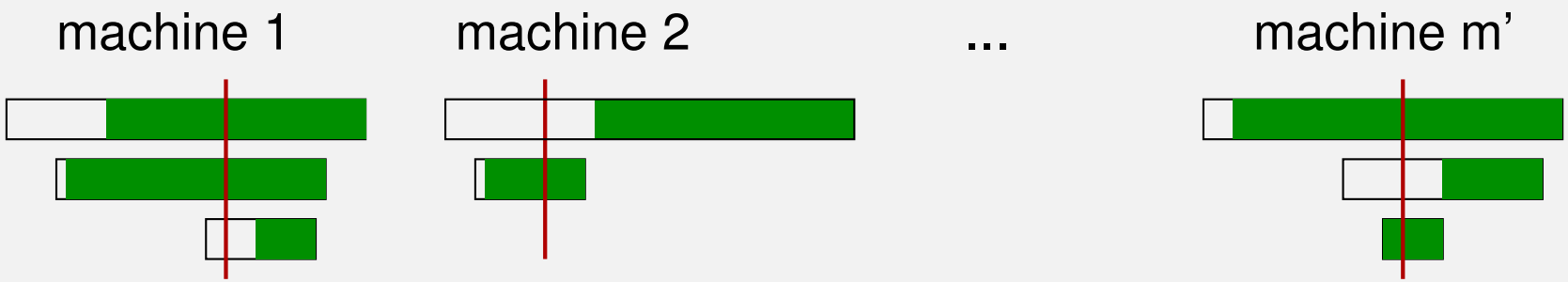

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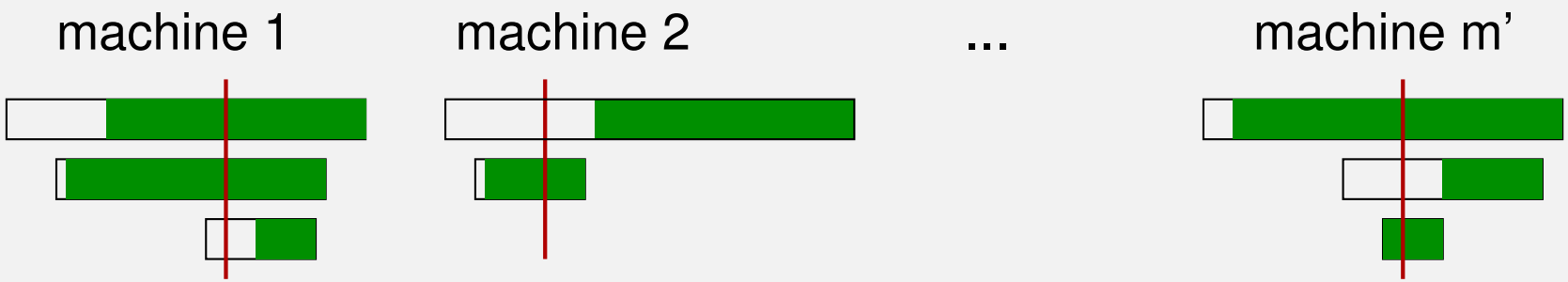

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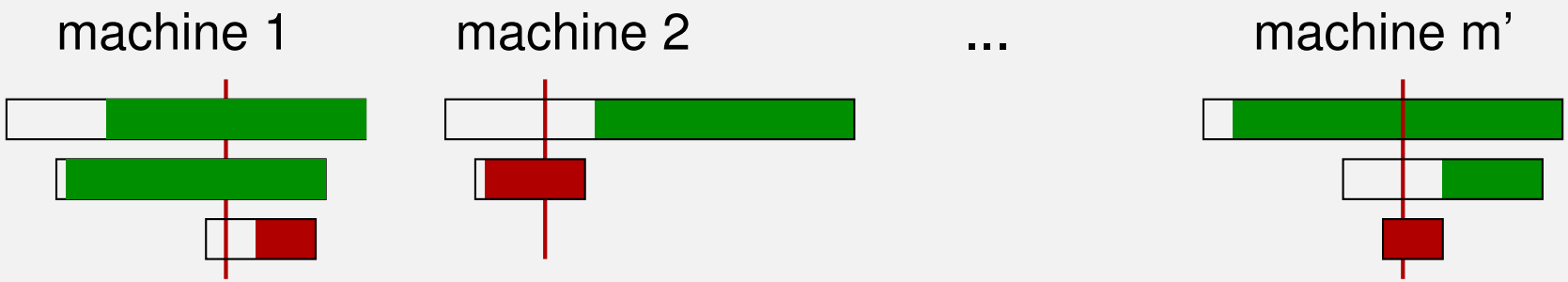

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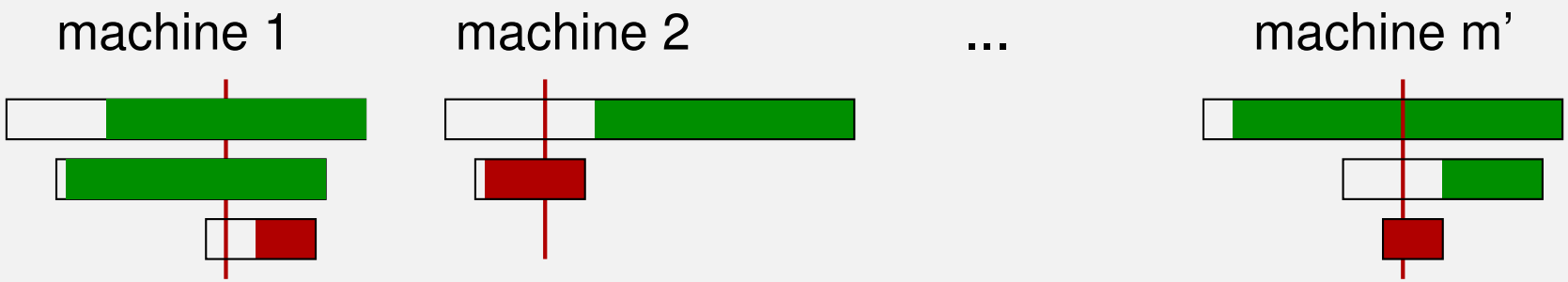
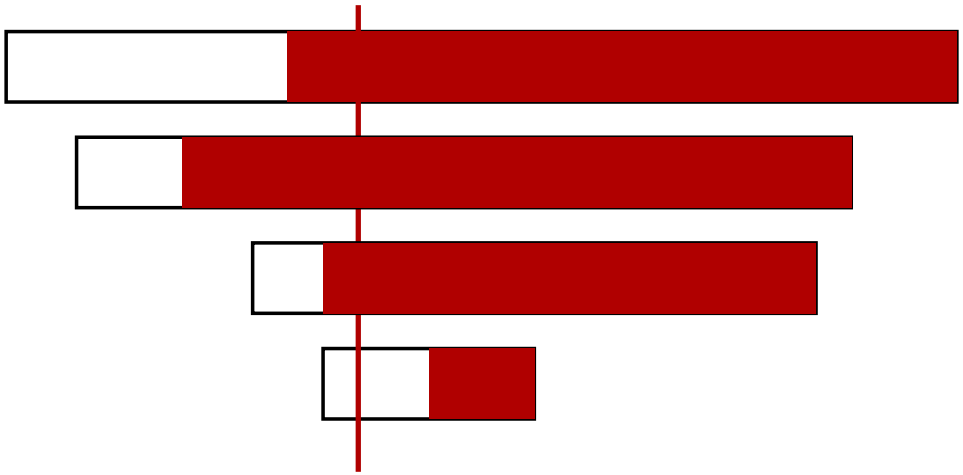


- On each machine consider inclusion minimal job.
- These laminar jobs form an inclusion chain.

Laminar instances: Assignment procedure

- Inclusion minimal jobs



Idea:

Laminar instances: Assignment procedure

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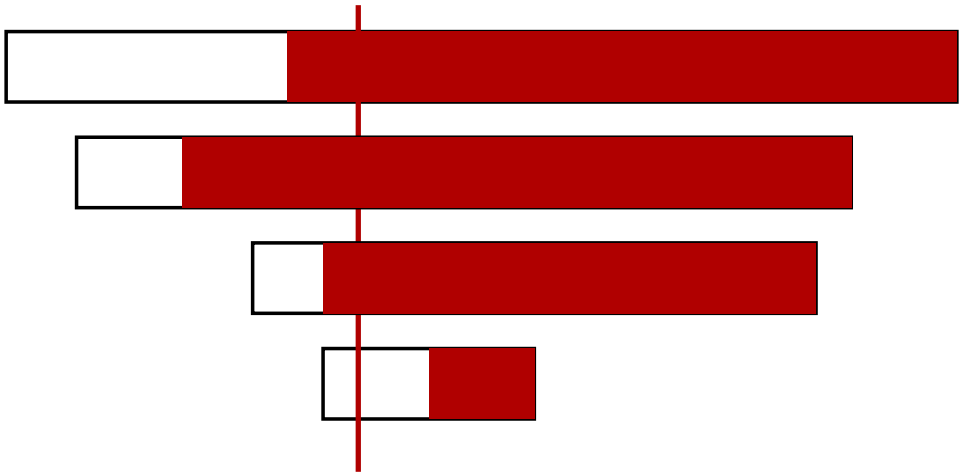

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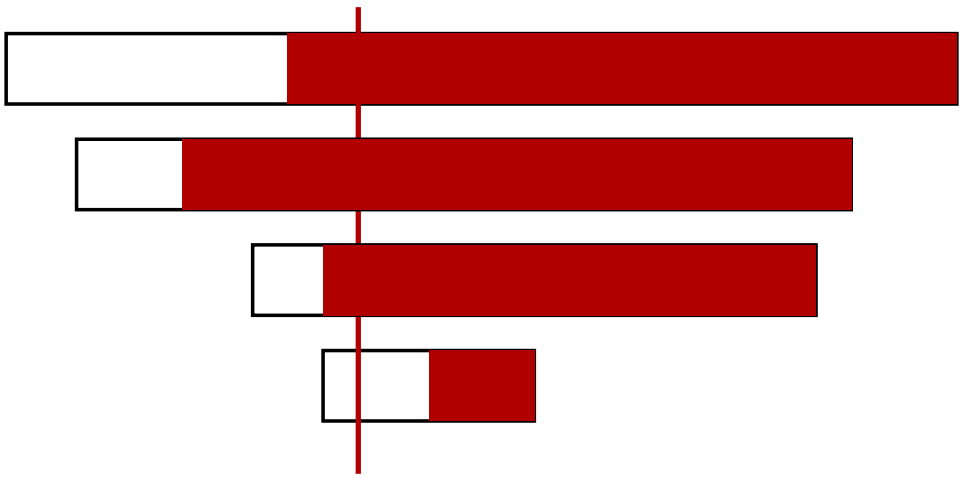

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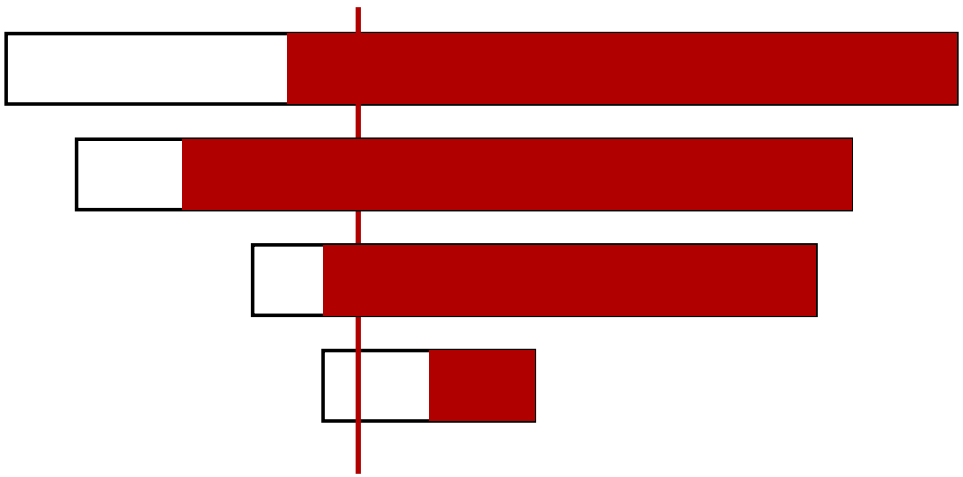

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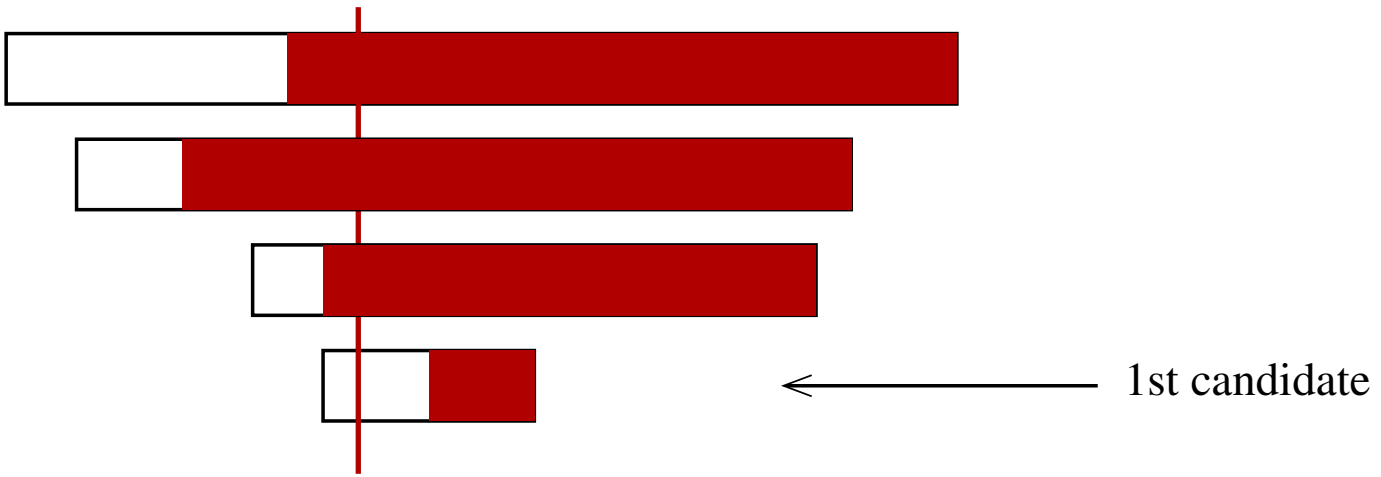

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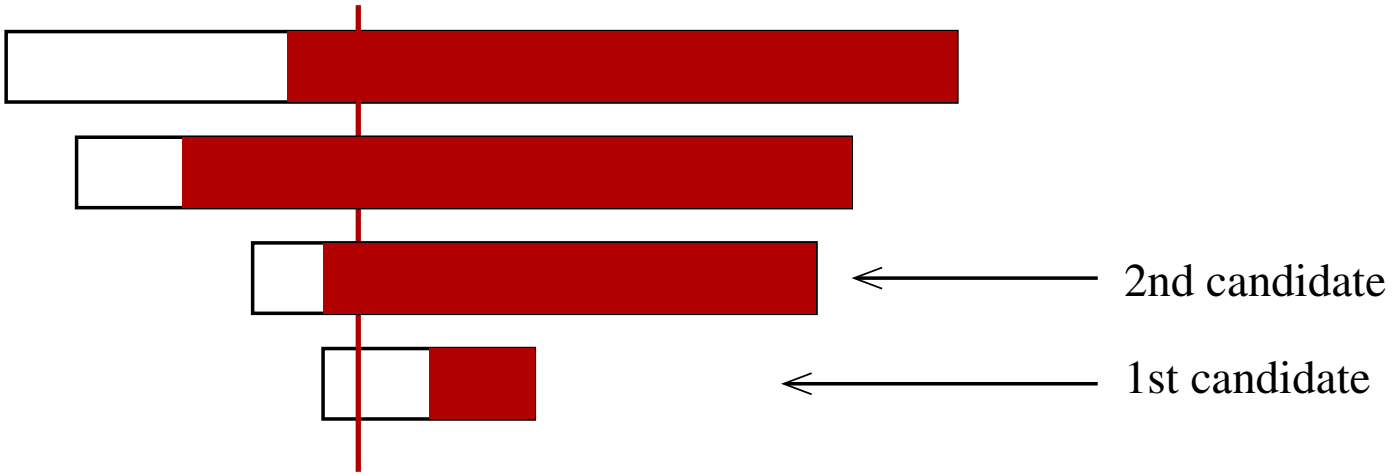

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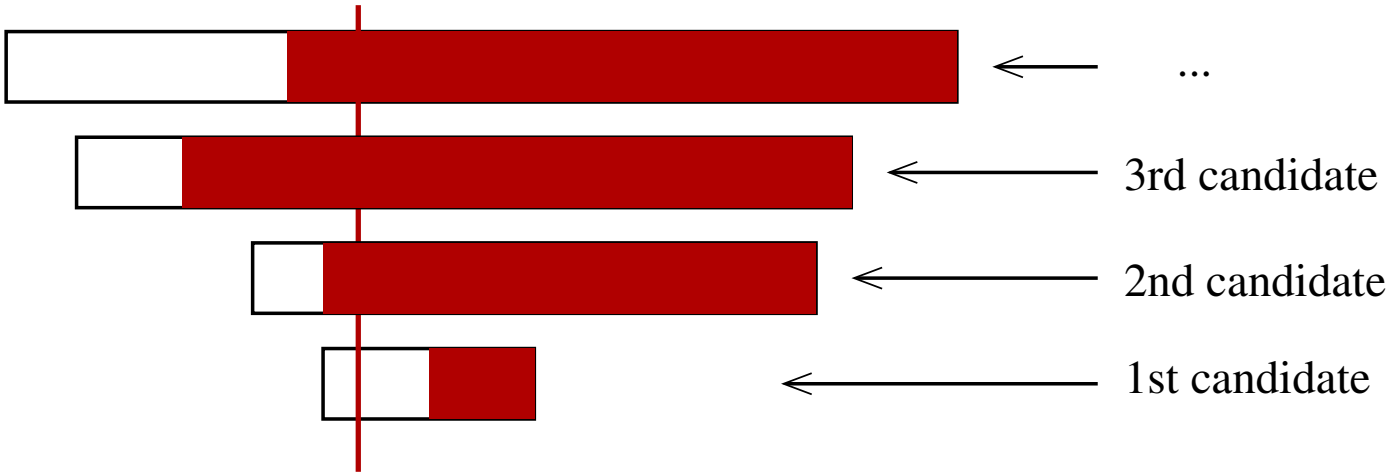

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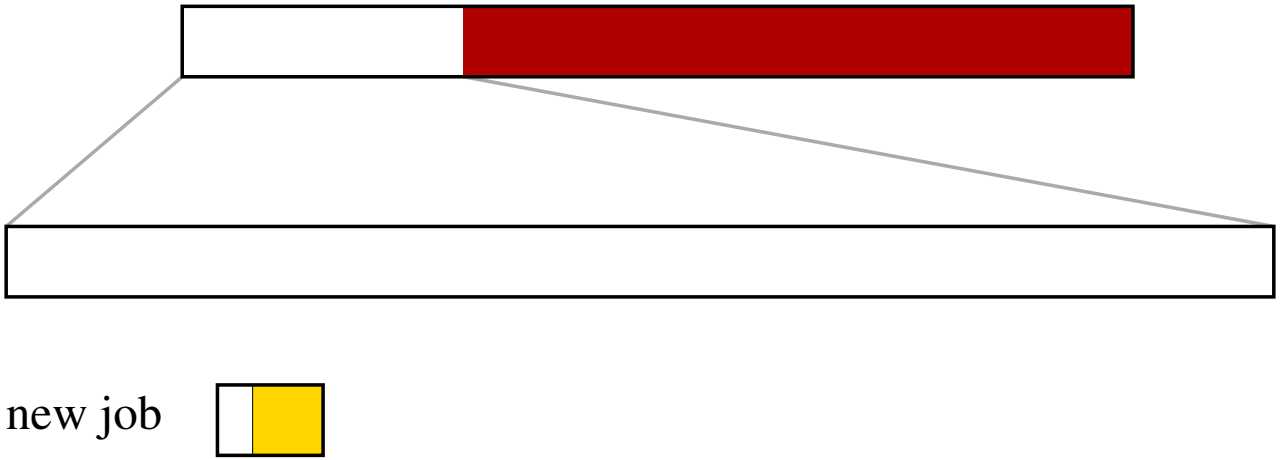

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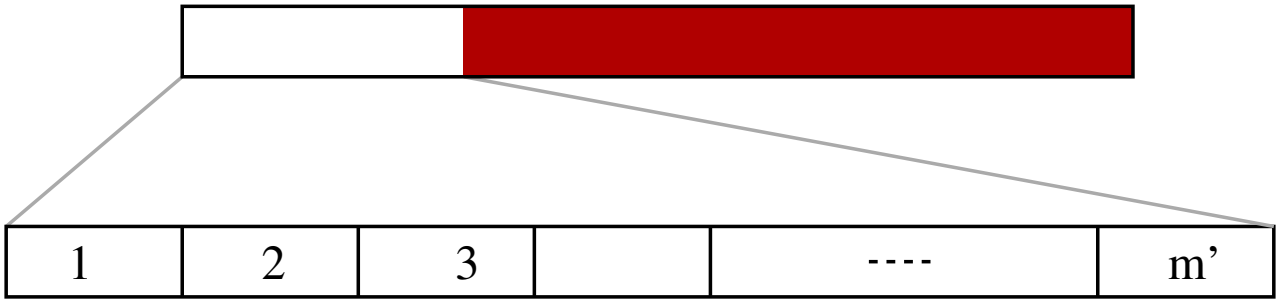

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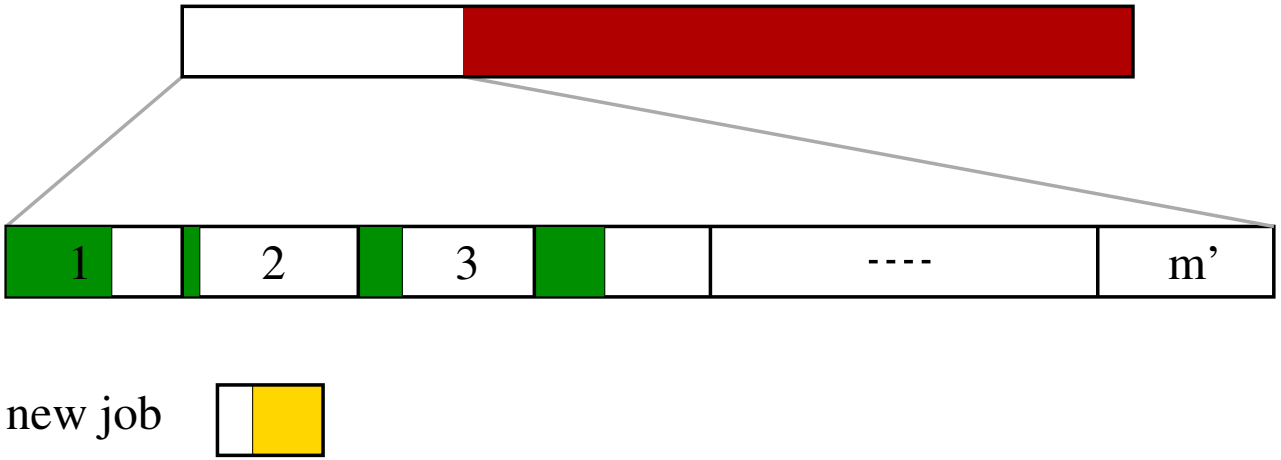

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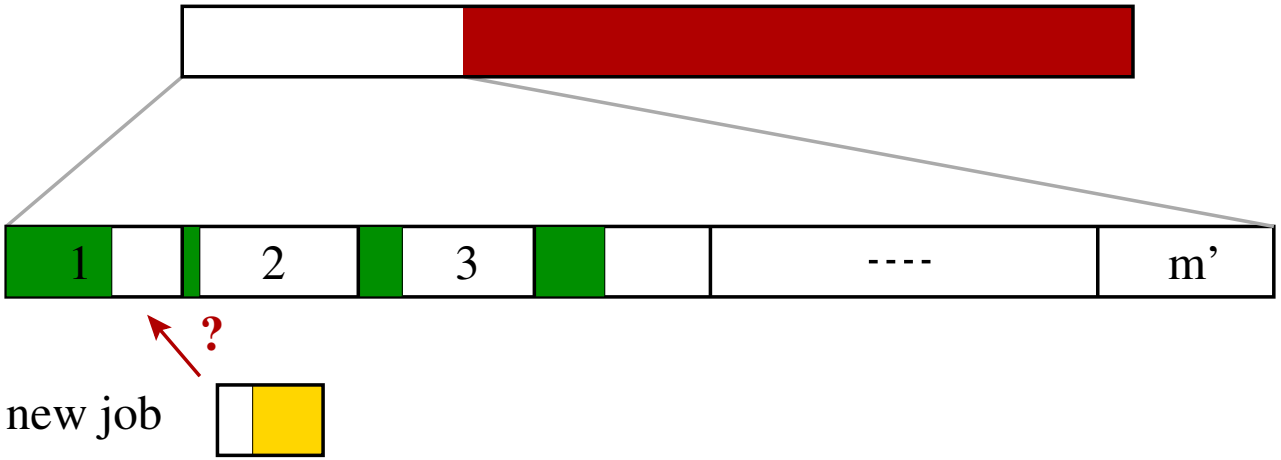

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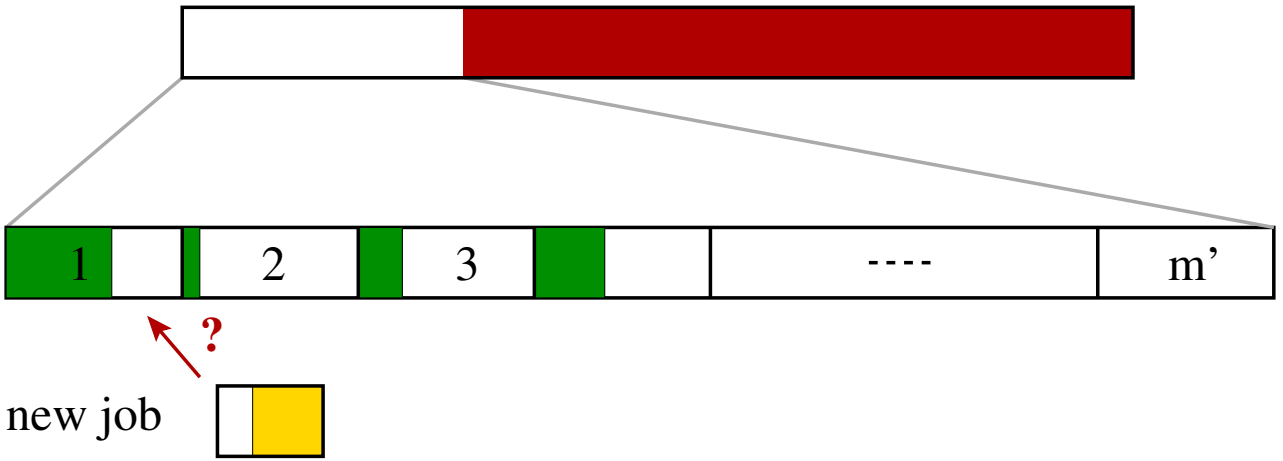

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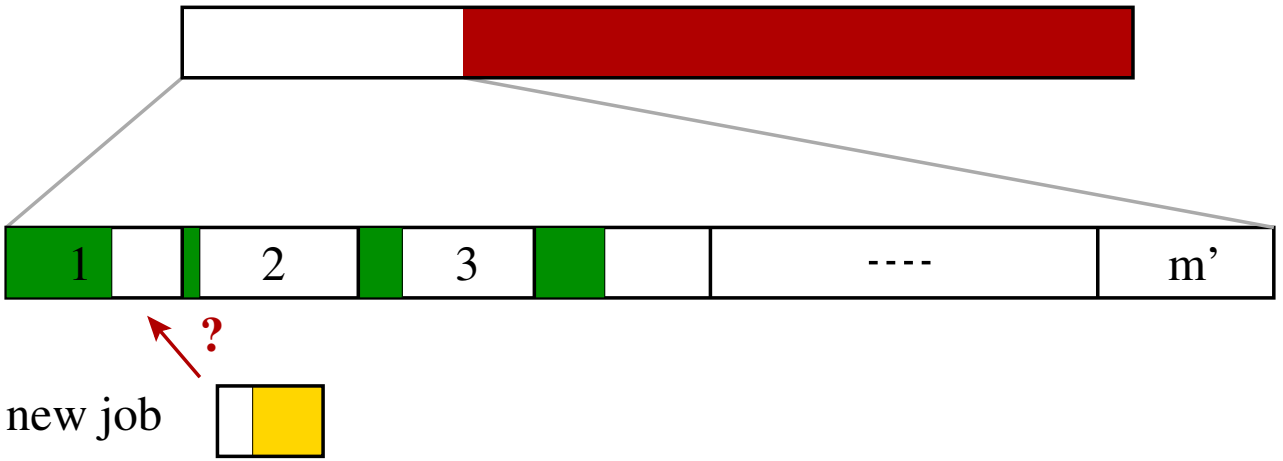

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To show: for $m' = \mathcal{O}(m \log m)$ the assignment never fails.

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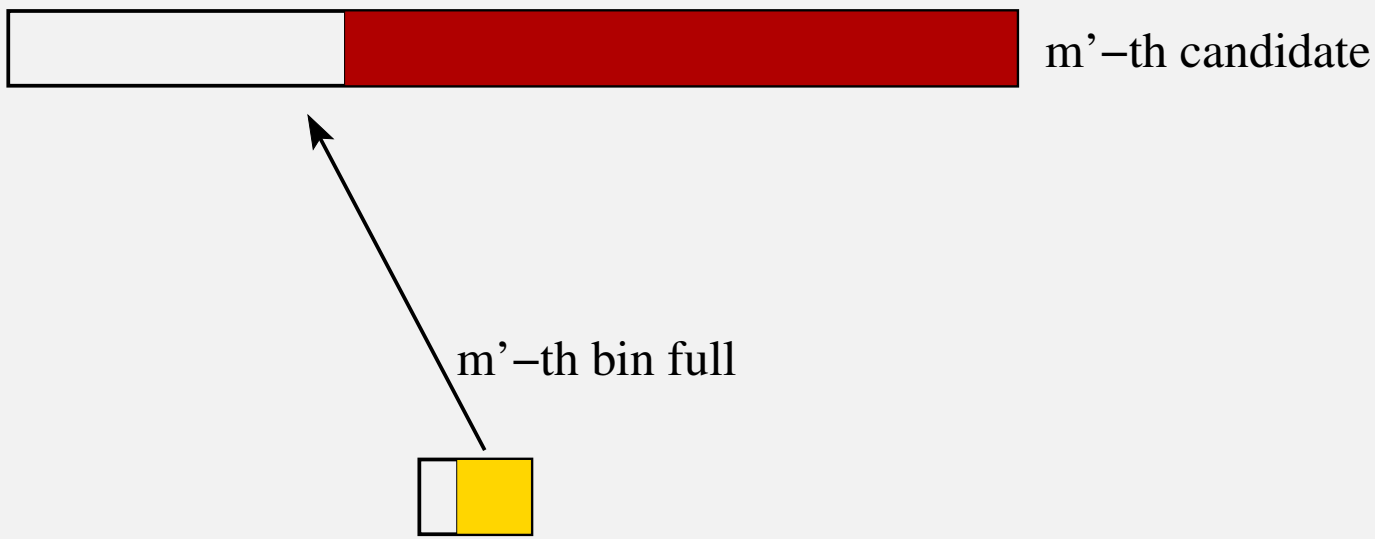

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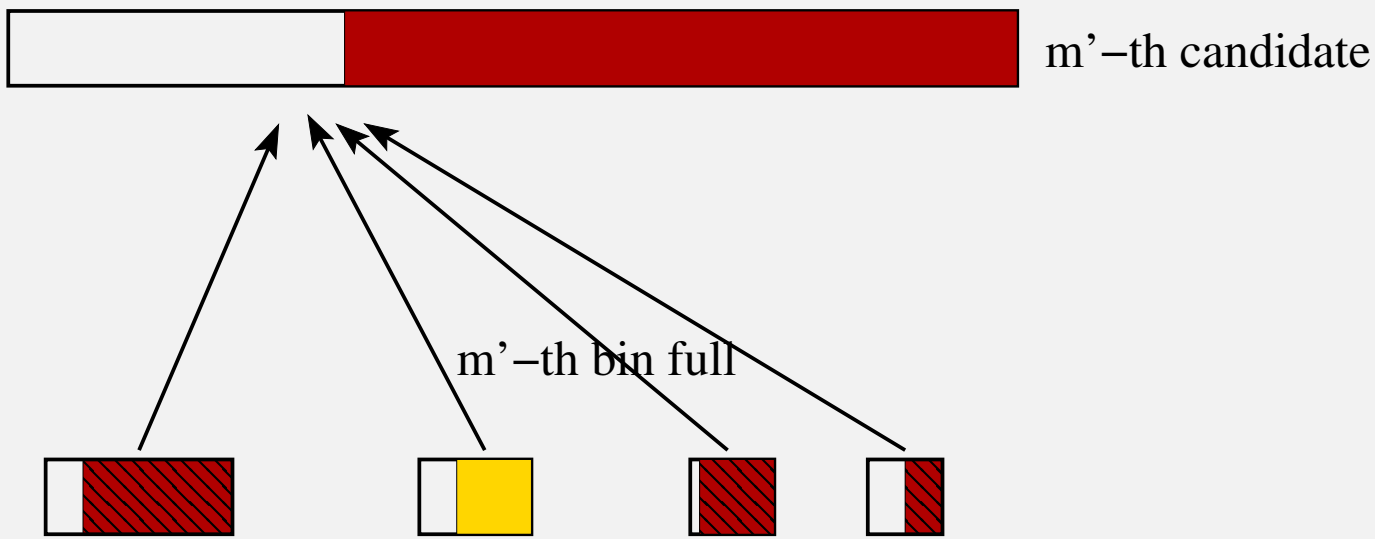

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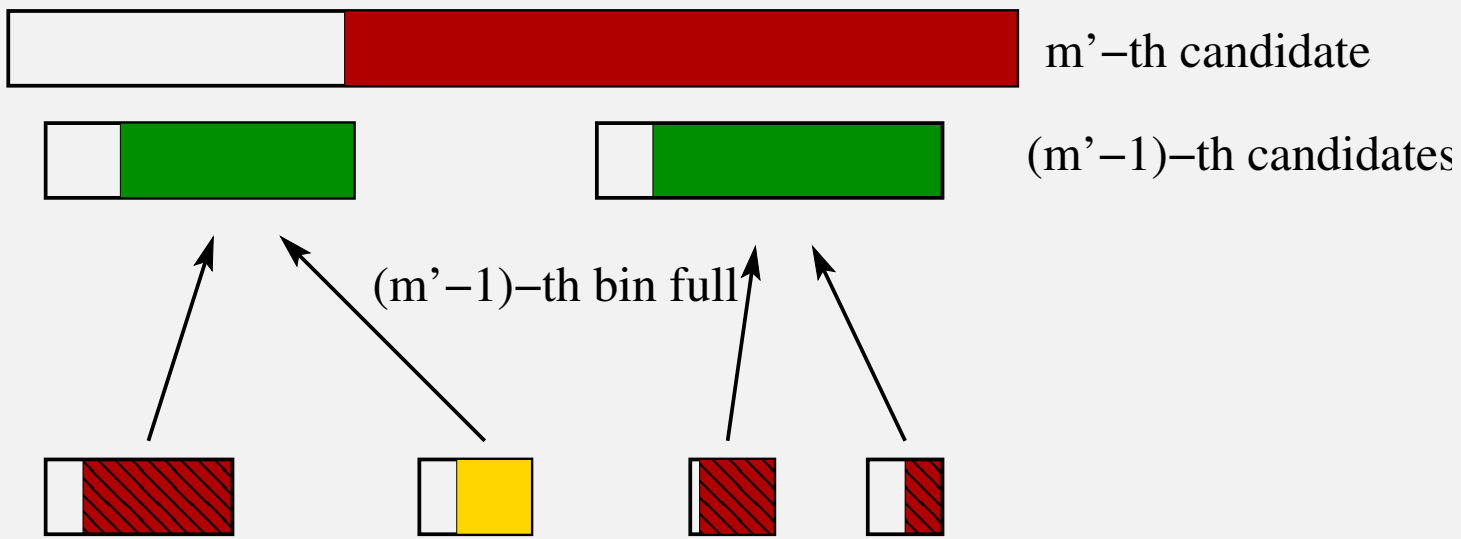

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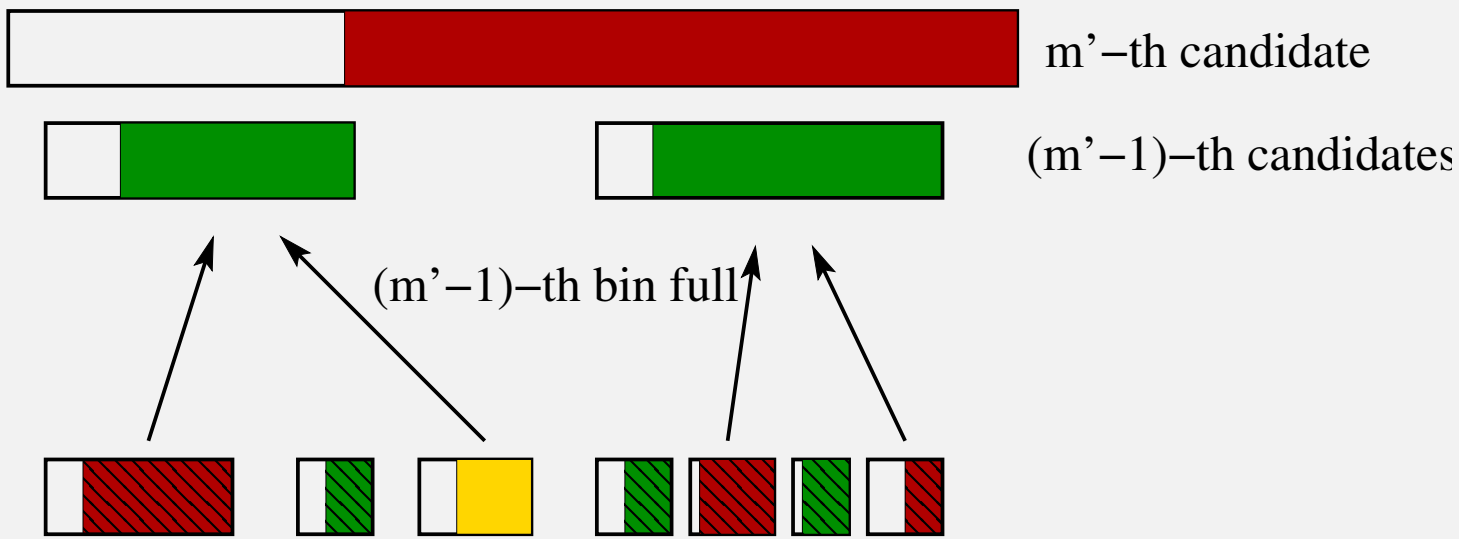

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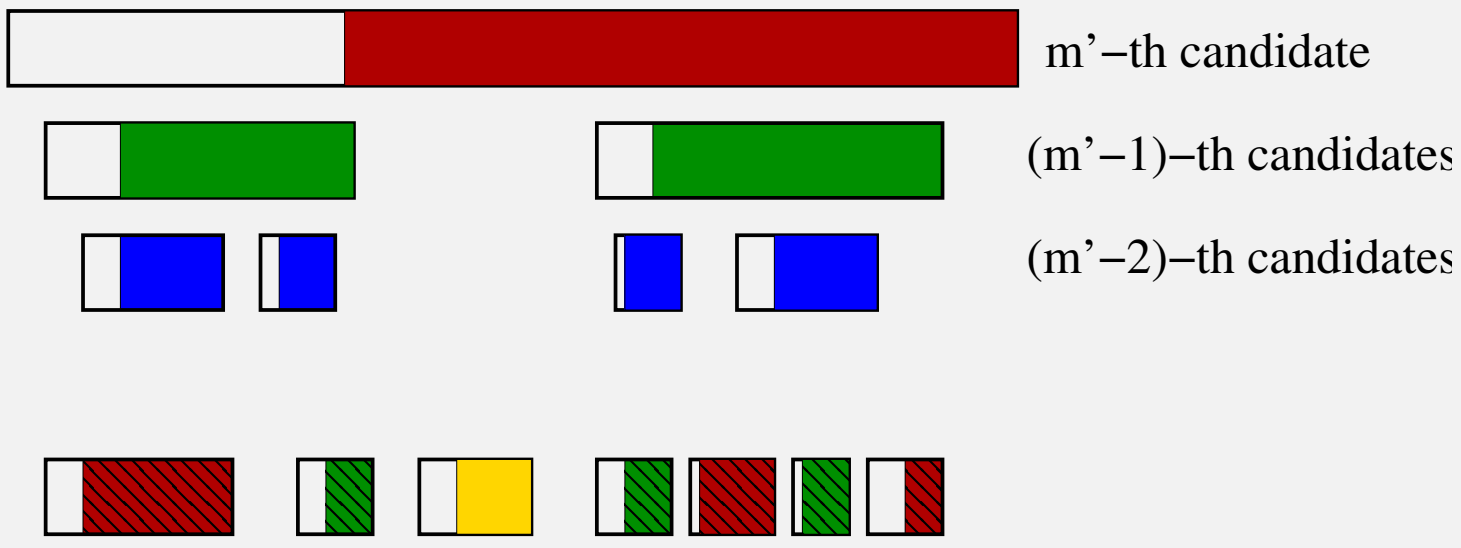

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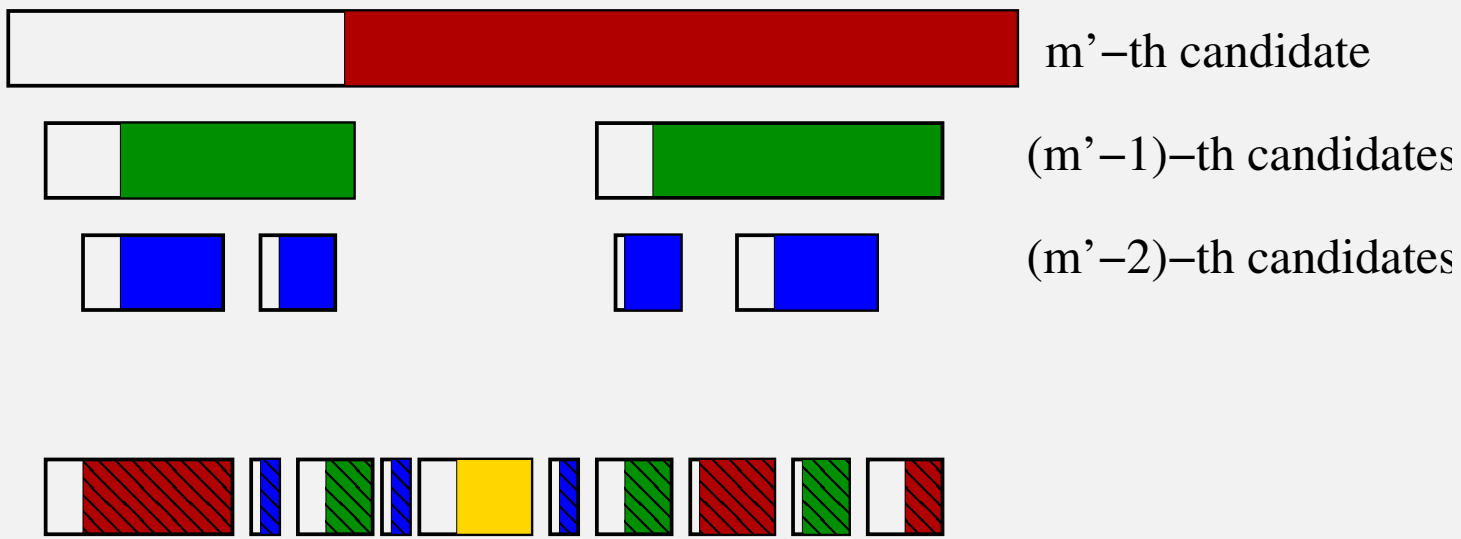

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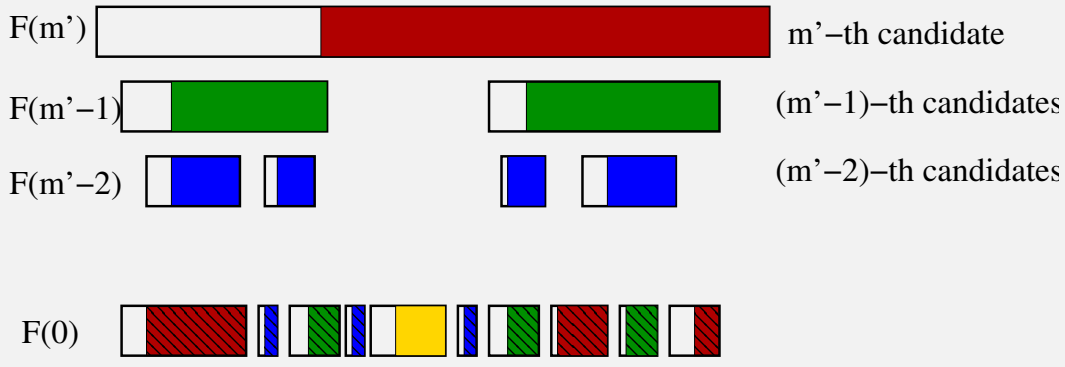

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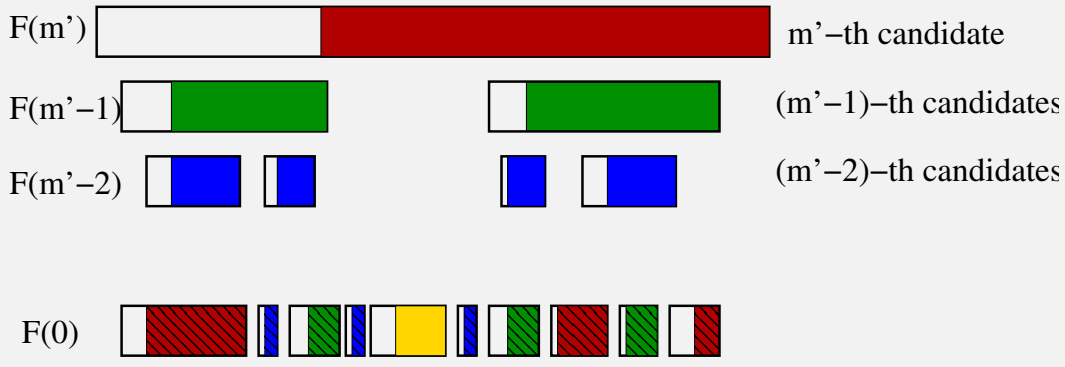

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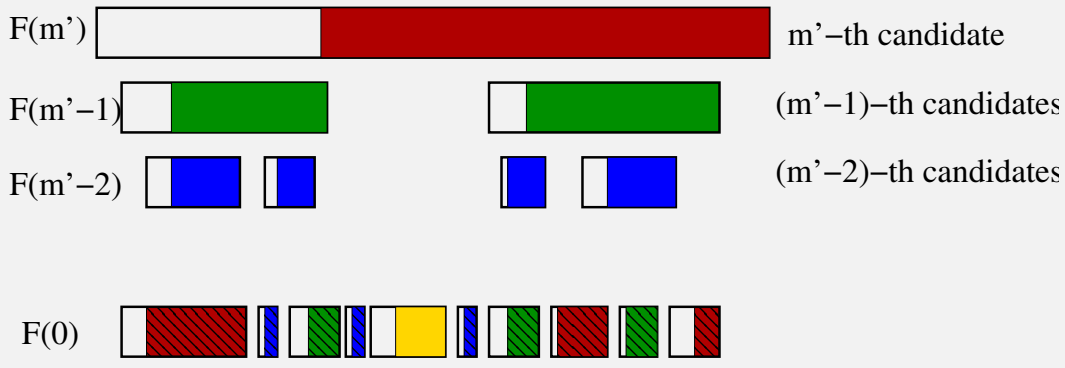

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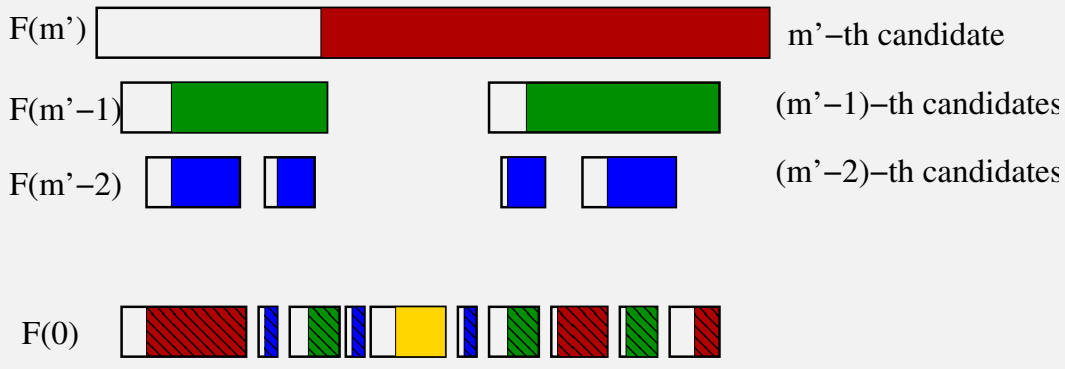

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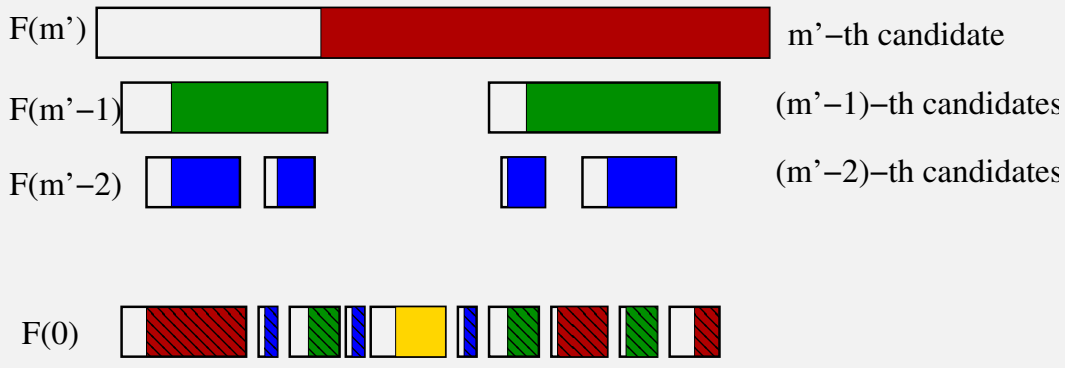

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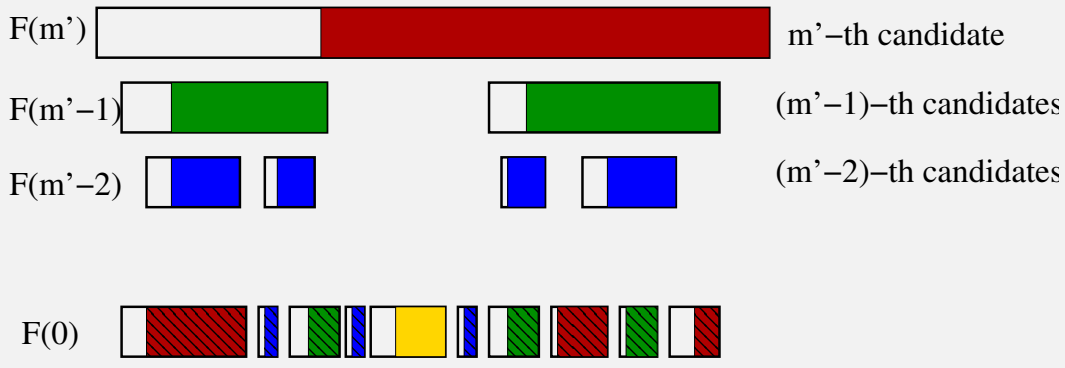

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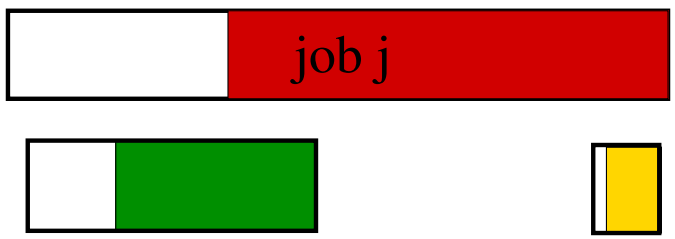
Theorem

The algorithm is $\mathcal{O}(\log m)$-competitive for laminar instances.

Outlook on a generalization

No clear separation into laminar or agreeable subinstances possible.



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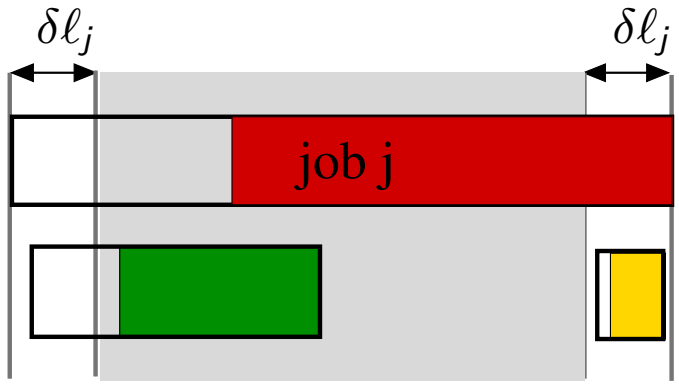

Job j δ-dominates j' if the time window of j' intersects with $[r_j + \delta\ell_j, d_j - \delta\ell_j]$.

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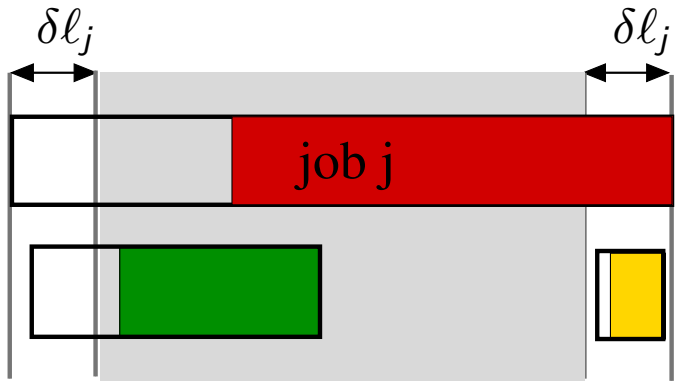
No clear separation into laminar or agreeable subinstances possible.



Job j δ-dominates j' if the time window of j' intersects with $[r_j + \delta\ell_j, d_j - \delta\ell_j]$.

Lemma. There are at most $\mathcal{O}(m)$ jobs that dominate the same job but do not dominate each other.

Outlook on a generalization

1. Assignment procedure
   - Jobs are assigned to $g = \mathcal{O}(m^2 \log m)$ groups.

2. Scheduling procedure
   - Schedule each group on $\mathcal{O}(m)$ machines.
   - At any time t schedule inclusion minimal jobs among those whose "relevant" time window contains t.

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Theorem

There is a preemptive $\mathcal{O}(m^2 \log m)$-competitive algorithm.

Summary & Open questions

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- General $\mathcal{O}(m^2 \log m)$-competitive preemptive algorithm.
  - Agreeable instances: $\mathcal{O}(1)$-competitive (even non-preemptive)
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- Stochastics: Random arrival model? Stochastic processing times?