

Data Analytic Aspects of the Canonical Decomposition Theorem for Finite Groups

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EURANDOM- TU/e, Eindhoven, The Netherlands, December 13th, 2006

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PURPOSE

This seminar will discuss the synthetic relation among symmetry arguments in experimental designs, the canonical decomposition theorem and the identification, interpretation and statistical inference of experimental hypotheses consistent with and derived from those arguments. The algebraic tools are those of group algebras, representations and Fourier analysis over finite groups. The classes of experimental applications include those of data indexed by finite sets and groups, within which particular examples will be discussed, including symmetry studies for voting preferences, chart designs in visual testing, handedness (chirality) of elementary planar patterns and refraction profiles in linear optics.

THE SLOAN FONTS AND THEIR SYMMETRIES

C	O	H	Z	V
S	Z	N	D	C
V	K	C	N	R
K	C	R	H	N
Z	K	D	V	C
H	V	O	R	K
R	H	S	O	N
K	S	V	R	H
H	N	K	C	D
N	D	V	K	O
D	H	O	S	Z
V	R	N	D	O
C	Z	H	K	S
O	R	Z	S	K

Letter	Symmetries	P	Entropy
Z	1, o	0.844	0.433
N	1, o	0.774	0.535
H	1, o, v, h	0.688	0.619
V	1, v	0.636	0.656
R	1	0.622	0.663
K	1	0.609	0.669
D	1, h	0.556	0.687
S	1, o	0.516	0.693
O	1, o, v, h	0.470	0.692
C	1, h	0.393	0.673

THE MULTIPLICATION TABLE FOR $C_2 \times C_2$

*	1	v	h	o
1	1	v	h	o
v	v	1	o	h
h	h	o	1	v
o	o	h	v	1

SYMMETRY STUDIES

- A set V of indices or labels (s) upon which certain symmetry relations can be defined;
- A finite group (G) with g elements acts on V and determines a linear representation (ρ) in the data vector space (\mathcal{V});
- A set of algebraically orthogonal projections of the form

$$\mathcal{P} = \frac{1}{g} \sum_{\tau \in G} \chi(\tau^{-1}) \rho(\tau),$$

one for each irreducible representation in dimension of n and character χ of G ;

- Applying the decomposition $I = \mathcal{P}_1 + \mathcal{P}_2 + \dots + \mathcal{P}_h$ to study

$$(x|y) = (x|\mathcal{P}_1 y) + (x|\mathcal{P}_2 y) + \dots + (x|\mathcal{P}_h y);$$

within the context of statistical inference for quadratic forms.

THE CANONICAL PROJECTIONS FOR $C_2 \times C_2$

In the present case

- $V = G$;
- $x' = (1, o, v, h)$.

This leads to

$$\mathcal{P}_1 = 1/4 \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}, \quad \mathcal{P}_2 = 1/4 \begin{bmatrix} 1 & 1 & -1 & -1 \\ 1 & 1 & -1 & -1 \\ -1 & -1 & 1 & 1 \\ -1 & -1 & 1 & 1 \end{bmatrix},$$

$$\mathcal{P}_3 = 1/4 \begin{bmatrix} 1 & -1 & 1 & -1 \\ -1 & 1 & -1 & 1 \\ 1 & -1 & 1 & -1 \\ -1 & 1 & -1 & 1 \end{bmatrix}, \quad \mathcal{P}_4 = 1/4 \begin{bmatrix} 1 & -1 & -1 & 1 \\ -1 & 1 & 1 & -1 \\ -1 & 1 & 1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix}.$$

THE CANONICAL INVARIANTS

The canonical invariants $\mathcal{I} = \mathcal{P}_X$ are given by

$$\mathcal{I}_1 = \pm(1 + o + v + h), \quad \mathcal{I}_o = \pm(1 + o - v - h),$$

$$\mathcal{I}_v = \pm(1 + v - o - h), \quad \mathcal{I}_h = \pm(1 + h - o - v).$$

ASSIGNING DATA TO THE SYMMETRIES IN G

$$\mathbf{fix}(\tau) = \{s \in V; \varphi(\tau, s) = s\} \subseteq V$$

$$\underline{x}(\tau) = \frac{1}{|\mathbf{fix}(\tau)|} \sum_{s \in \mathbf{fix}(\tau)} x(s).$$

FONT ENTROPY DISTRIBUTION BY SYMMETRY TYPE

Sloan chart line	$\underline{x}(1)$	$\underline{x}(o)$	$\underline{x}(v)$	$\underline{x}(h)$
COH Z V	0.614	0.580	0.657	0.660
VK C NR	0.636	0.535	0.656	0.673
KCR H N	0.632	0.575	0.619	0.645
Z KDVC	0.624	0.433	0.656	0.680
HVORK	0.660	0.655	0.657	0.655
R H SON	0.640	0.635	0.655	0.655
K S VRH	0.660	0.655	0.640	0.619
H N KCD	0.636	0.575	0.619	0.660
N DVKO	0.648	0.615	0.675	0.690
D H OS Z	0.624	0.608	0.655	0.667
VR N DO	0.648	0.615	0.675	0.690
C Z H K S	0.618	0.580	0.619	0.645
OR Z SK	0.630	0.603	0.692	0.692

Font	Symm.	P	Ent.
Z	1, o	0.844	0.433
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C	1, h	0.393	0.673

o: 2.5 ± 0.804 , h: 2.0 ± 0.698 , v: 1.5 ± 0.741 .

Fonts (Z,N,S) in red have symmetries of {1, o} only.

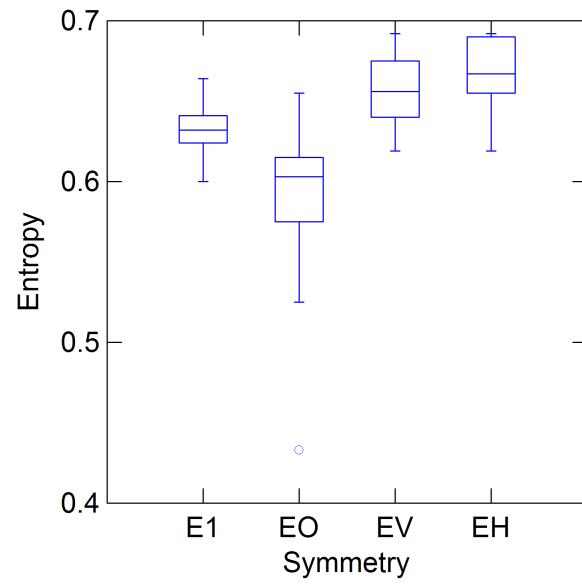
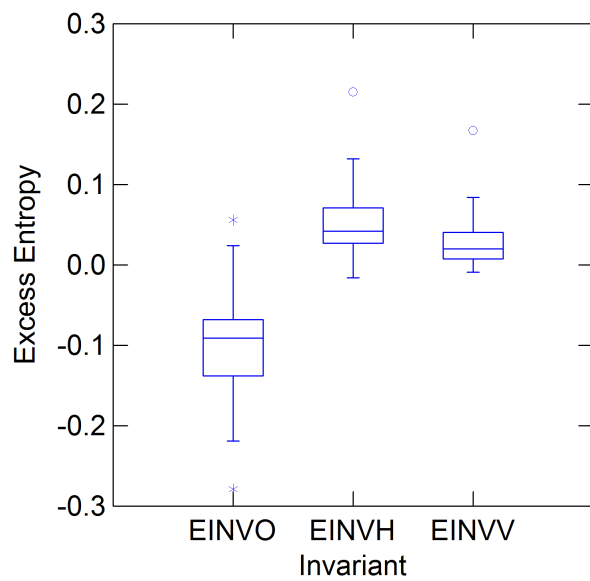
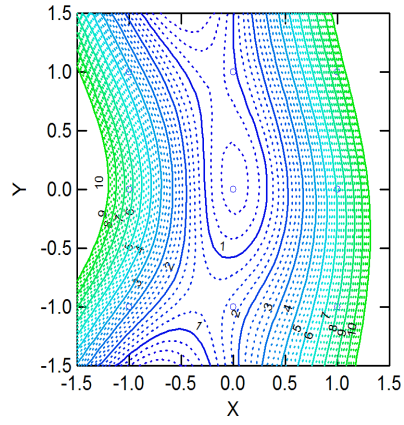
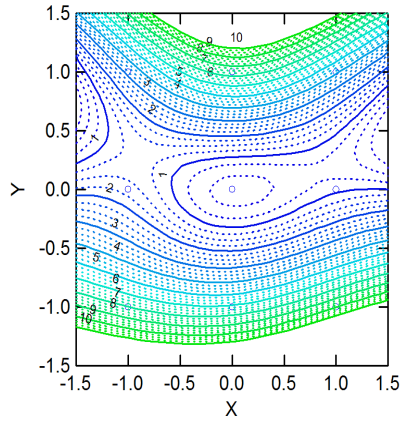
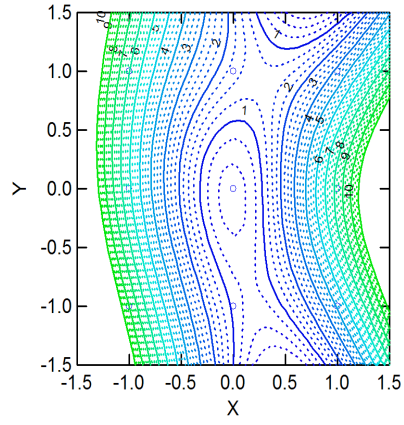
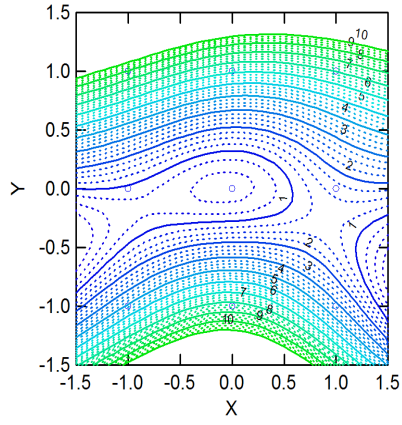
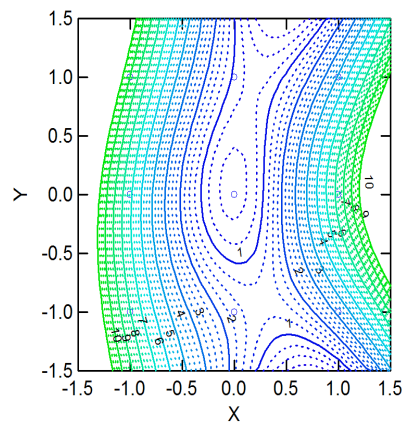
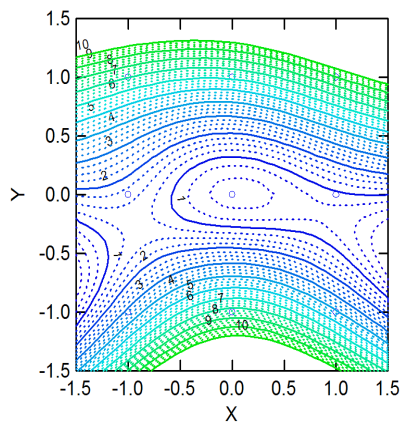
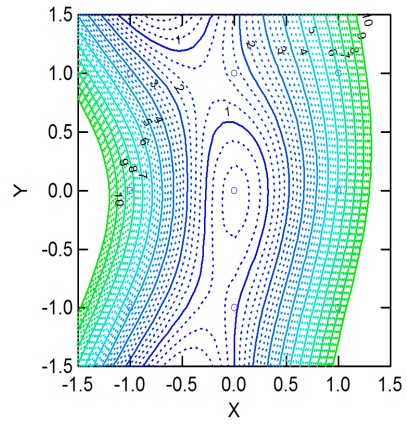
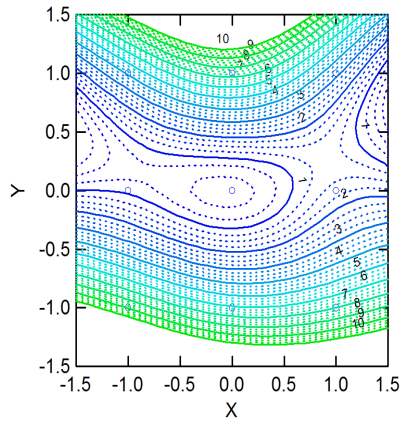
FIGURE 1. Mean line entropy distribution classified by font symmetry $E_\tau \equiv \underline{x}(\tau)$.

FIGURE 2. Distribution of the canonical invariants $\text{EINVO} \equiv \mathcal{I}_o$, $\text{EINVV} \equiv \mathcal{I}_v$ and $\text{EINVH} \equiv \mathcal{I}_h$ for the entropy data indexed by the planar font symmetries.







Rotations.

$$\text{reference } s_0 = \begin{bmatrix} 9 & 6 & 7 \\ 1 & 0 & 2 \\ 4 & 8 & 3 \end{bmatrix} \rightarrow x(s) = 0;$$

$$90 \text{ deg : } s = \begin{bmatrix} 7 & 2 & 3 \\ 6 & 0 & 8 \\ 9 & 1 & 4 \end{bmatrix} \rightarrow x(s) = \|s - s_0\| = 19.11;$$

$$180 \text{ deg : } \begin{bmatrix} 3 & 8 & 4 \\ 2 & 0 & 1 \\ 7 & 6 & 9 \end{bmatrix} \rightarrow x(s) = 11.11;$$

$$270 \text{ deg : } \begin{bmatrix} 4 & 1 & 9 \\ 8 & 0 & 6 \\ 3 & 2 & 7 \end{bmatrix} \rightarrow x(s) = 19.11;$$

Reflections (reversals).

horizontal reflection: $\begin{bmatrix} 4 & 8 & 3 \\ 1 & 0 & 2 \\ 9 & 6 & 7 \end{bmatrix} \rightarrow x(s) = 10.00;$

45 deg line reflection: $\begin{bmatrix} 3 & 2 & 7 \\ 8 & 0 & 6 \\ 4 & 1 & 9 \end{bmatrix} \rightarrow x(s) = 22.44;$

vertical reflection: $\begin{bmatrix} 7 & 6 & 9 \\ 2 & 0 & 1 \\ 3 & 8 & 4 \end{bmatrix} \rightarrow x(s) = 1.33;$

135 deg line reflection: $\begin{bmatrix} 9 & 1 & 4 \\ 6 & 0 & 8 \\ 7 & 2 & 3 \end{bmatrix} \rightarrow x(s) = 15.55.$

$$G = \underbrace{\{1, \eta, \eta^2, \eta^3\}}_{\text{rotations}}, \underbrace{\{\tau, \eta\tau, \eta^2\tau, \eta^3\tau\}}_{\text{reversals}}$$

$$\left[\begin{array}{c|cccccccc} & s & 1 & \eta & \eta^2 & \eta^3 & \tau & \eta\tau & \eta^2\tau & \eta^3\tau \\ \hline x(s) & & u & r & R & \rho & h & d & v & D \end{array} \right],$$

In the numerical example,

$$x' = (0, 19.1, 11.1, 19.1, 10, 22.4, 1.3, 15.5).$$

Form the canonical projections:

$$\mathcal{P}_i = \frac{\chi_i(1)}{8} \sum_{\sigma \in D_4} \bar{\chi}_i(\sigma) \nu(\sigma), \quad i = 1, \dots, 5.$$

There are as many projections as the number (h) of irreducible characters in the group of symmetries. Moreover, we have

- $I = \mathcal{P}_1 + \mathcal{P}_2 + \dots + \mathcal{P}_h$, (reduction of the identity)
- $\mathcal{P}_i \mathcal{P}_j = 0$ for $i \neq j$, (algebraic orthogonality)
- $\mathcal{P}_i^2 = \mathcal{P}_i$, $i = 1, \dots, h$ (projection matrices).

These are the resulting canonical projections:

$$\mathcal{P}_2 = \frac{1}{8} \begin{bmatrix} 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 \\ 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 \\ 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 \\ 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 \\ -1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 \\ -1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 \\ -1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 \\ -1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 \end{bmatrix}, \quad \mathcal{P}_3 = \frac{1}{8} \begin{bmatrix} 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 \\ -1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 \\ 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 \\ -1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 \\ 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 \\ -1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 \\ 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 \\ -1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 \end{bmatrix},$$

$$\mathcal{P}_4 = \frac{1}{8} \begin{bmatrix} 1 & -1 & 1 & -1 & -1 & 1 & -1 & 1 \\ -1 & 1 & -1 & 1 & 1 & -1 & 1 & -1 \\ 1 & -1 & 1 & -1 & -1 & 1 & -1 & 1 \\ -1 & 1 & -1 & 1 & 1 & -1 & 1 & -1 \\ -1 & 1 & -1 & 1 & 1 & -1 & 1 & -1 \\ 1 & -1 & 1 & -1 & -1 & 1 & -1 & 1 \\ -1 & 1 & -1 & 1 & 1 & -1 & 1 & -1 \\ 1 & -1 & 1 & -1 & -1 & 1 & -1 & 1 \end{bmatrix}, \quad \mathcal{P}_5 = \frac{1}{2} \begin{bmatrix} 1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 & -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 1 \end{bmatrix}.$$

The resulting **canonical invariants** are:

$$\mathcal{P}_{1x} = 1/8 \begin{bmatrix} u+r+R+\rho+h+d+v+D \\ u+r+R+\rho+h+d+v+D \\ u+r+R+\rho+h+d+v+D \\ u+r+R+\rho+h+d+v+D \\ u+r+R+\rho+h+d+v+D \\ u+r+R+\rho+h+d+v+D \\ u+r+R+\rho+h+d+v+D \\ u+r+R+\rho+h+d+v+D \end{bmatrix}, \quad \mathcal{P}_{2x} = 1/8 \begin{bmatrix} u+r+R+\rho-h-d-v-D \\ u+r+R+\rho-h-d-v-D \\ u+r+R+\rho-h-d-v-D \\ u+r+R+\rho-h-d-v-D \\ u+r+R+\rho-h-d-v-D \\ -u-r-R-\rho+h+d+v+D \\ -u-r-R-\rho+h+d+v+D \\ -u-r-R-\rho+h+d+v+D \\ -u-r-R-\rho+h+d+v+D \end{bmatrix},$$

$$\mathcal{P}_{3x} = 1/8 \begin{bmatrix} u-r+R-\rho+h-d+v-D \\ -u+r-R+\rho-h+d-v+D \\ u-r+R-\rho+h-d+v-D \\ -u+r-R+\rho-h+d-v+D \\ u-r+R-\rho+h-d+v-D \\ -u+r-R+\rho-h+d-v+D \\ u-r+R-\rho+h-d+v-D \\ -u+r-R+\rho-h+d-v+D \end{bmatrix}, \quad \mathcal{P}_{4x} = 1/8 \begin{bmatrix} u-r+R-\rho-h+d-v+D \\ -u+r-R+\rho+h-d+v-D \\ u-r+R-\rho-h+d-v+D \\ -u+r-R+\rho+h-d+v-D \\ -u+r-R+\rho+h-d+v-D \\ u-r+R-\rho-h+d-v+D \\ -u+r-R+\rho+h-d+v-D \\ u-r+R-\rho-h+d-v+D \end{bmatrix},$$

and

$$\mathcal{P}_{5x} = 1/2 \begin{bmatrix} u-R \\ r-\rho \\ -u+R \\ -r+\rho \\ h-v \\ d-D \\ -h+v \\ -d+D \end{bmatrix}.$$

Their dimensions, or degrees of freedom (df), are, respectively, 1, 1, 1, 1 and 4.

The resulting **Analysis of Variance**, or decomposition

$$\|x\|^2 = x'x = x'\mathcal{P}_1x + \dots + x'\mathcal{P}_5x$$

of the sum of squares $x'x$, is

source	$x'\mathcal{P}_x$	df
\mathcal{P}_1	$(u + r + R + \rho + h + d + v + D)^2 / 8$	1
\mathcal{P}_2	$(u + r + R + \rho - h - d - v - D)^2 / 8$	1
\mathcal{P}_3	$(-u + r - R + \rho - h + d - v + D)^2 / 8$	1
\mathcal{P}_4	$(-u + r - R + \rho + h - d + v - D)^2 / 8$	1
\mathcal{P}_5	$[(u - R)^2 + (r - \rho)^2 + (h - v)^2 + (d - D)^2] / 2$	4
total	$u^2 + r^2 + R^2 + \rho^2 + h^2 + d^2 + v^2 + D^2$	8

Here is the numerical evaluation for the structured data

$$x' = (0, 19.1, 11.1, 19.1, 10, 22.4, 1.3, 15.5) :$$

source	$x'\mathcal{P}_x$	df
\mathcal{P}_1	1216.47700	1
\mathcal{P}_2	0.00001	1
\mathcal{P}_3	361.40100	1
\mathcal{P}_4	0.02531	1
\mathcal{P}_5	123.06300	4
total	1700.94130	8

DATA ANALYTIC ASPECTS OF CHIRALITY

$$\begin{aligned} M_1 &= \begin{bmatrix} 2 & 4 & 3 \\ 5 & 0 & 5 \\ 3 & 4 & 2 \end{bmatrix}, & M_2 &= \begin{bmatrix} 4 & 5 & 3 \\ 2 & 0 & 5 \\ 3 & 2 & 4 \end{bmatrix}, & M_3 &= \begin{bmatrix} 5 & 4 & 5 \\ 4 & 0 & 3 \\ 2 & 2 & 3 \end{bmatrix}, \\ M_4 &= \begin{bmatrix} 3 & 4 & 5 \\ 2 & 0 & 2 \\ 5 & 4 & 3 \end{bmatrix}, & M_5 &= \begin{bmatrix} 2 & 5 & 2 \\ 3 & 0 & 3 \\ 4 & 5 & 4 \end{bmatrix}, & M_6 &= \begin{bmatrix} 2 & 5 & 4 \\ 3 & 0 & 3 \\ 2 & 5 & 4 \end{bmatrix}. \end{aligned}$$

The test for chirality is bound by planar (mirror) reflections. The mirrors are parallel to the dihedral axes, namely $y = \pm x$, $x = 0$ and $y = 0$.

Maps M_1 and M_4 have the symmetry of C_2 (or a center of symmetry), maps M_2 , M_5 and M_6 have the symmetry of a single axial reflections and M_3 is fixed by the identity alone. Consequently, the maps with single axial symmetries when reflected in a mirror parallel to their axis coincide, and are achiral.

FIGURE 5. The maps M_1, \dots, M_6

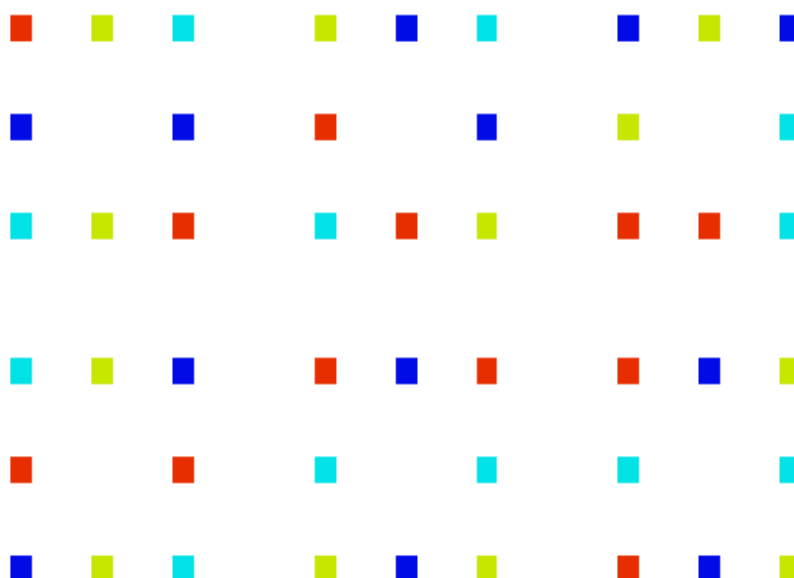
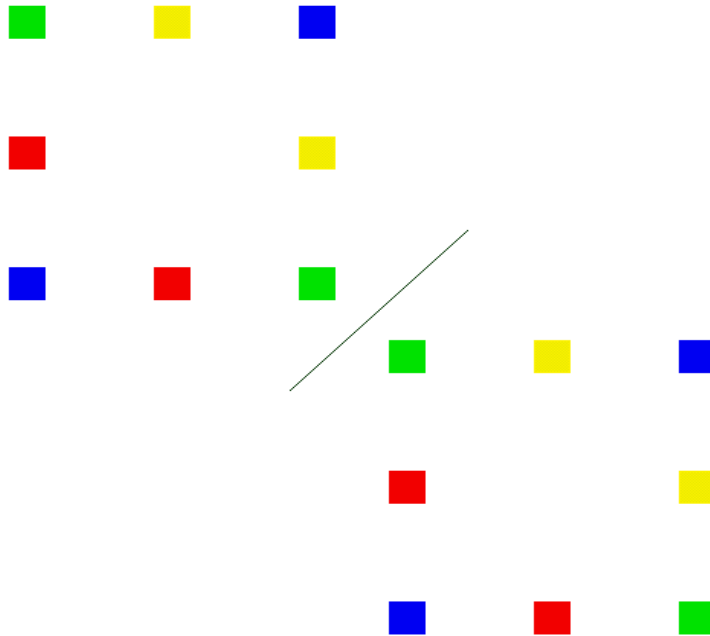


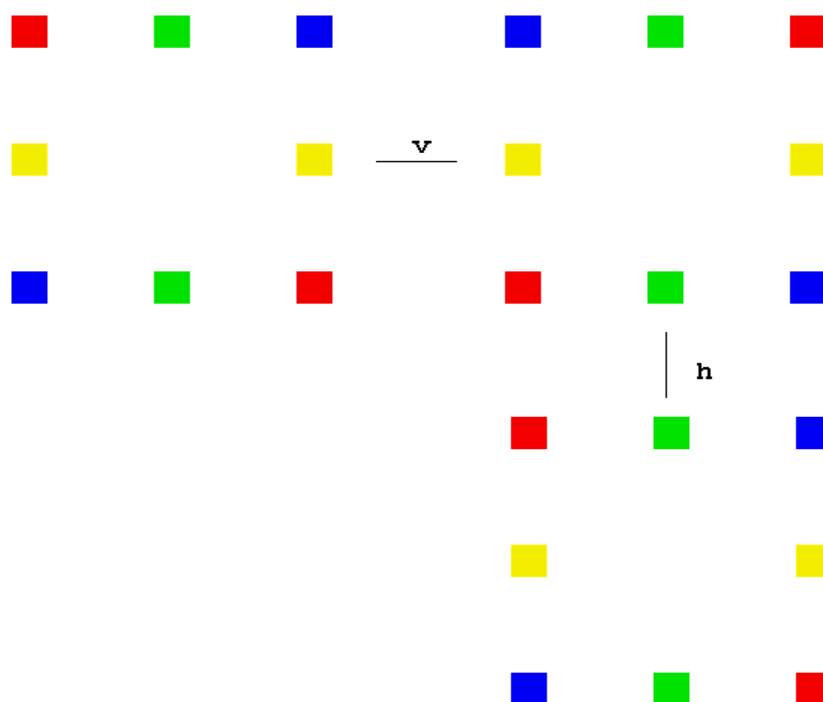
Figure 6 shows the reflection of map M_2 in its mirror.

FIGURE 6. The mirror imaging of M_2



Maps with a center of symmetry have rotational symmetry, which factors as the product of two axial reflections. They require two mirrors to recover the original image. Figure 7 illustrates the double-mirror imaging of map M_1 . These maps are also achiral. The map M_3 is chiral in that no axial reflection or point rotation can take the original map to any of its mirror images.

FIGURE 7. The double-mirror imaging of M_1



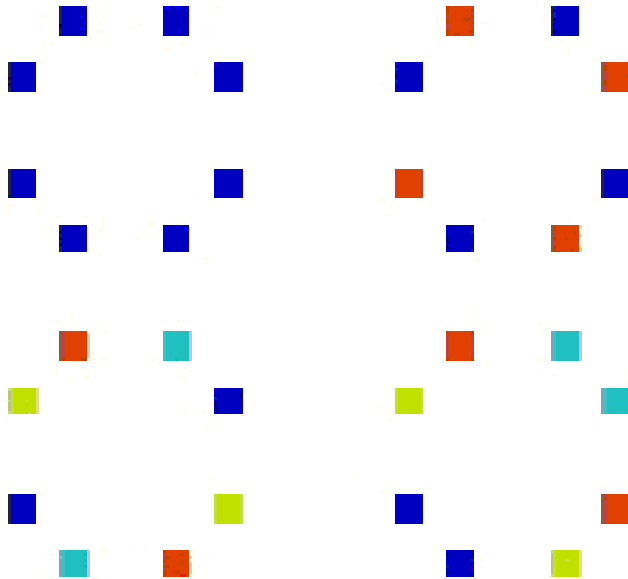
5 × 5 MAPS

These are maps of the type

$$\begin{bmatrix} 0 & a & 0 & k & 0 \\ n & 0 & 0 & 0 & b \\ 0 & 0 & 0 & 0 & 0 \\ d & 0 & 0 & 0 & \ell \\ 0 & m & 0 & c & 0 \end{bmatrix}.$$

Figure (8) shows four 5 × 5 maps, with the symmetries of D_4 , C_4 , C_2 and axial $y = x$, respectively.

FIGURE 8. Maps with the symmetries of D_4 , C_4 , C_2 and axial $y = x$, respectively.



The maps with the symmetry of C_4 .

$$\begin{bmatrix} 0 & a & 0 & k & 0 \\ k & 0 & 0 & 0 & a \\ 0 & 0 & 0 & 0 & 0 \\ a & 0 & 0 & 0 & k \\ 0 & k & 0 & a & 0 \end{bmatrix}, \quad \begin{bmatrix} x'\mathcal{P}_1x = (a-k)^4 \\ x'\mathcal{P}_2x = (a-k)^4 \\ x'\mathcal{P}_3x = 0 \\ x'\mathcal{P}_4x = 0 \\ x'\mathcal{P}_5x = 0 \end{bmatrix}.$$

For $a \neq k$ these maps have four-fold (two-fold and center of) symmetry without axial symmetries.

The maps with the symmetry of C_2 .

$$\begin{bmatrix} 0 & a & 0 & k & 0 \\ \ell & 0 & 0 & 0 & b \\ 0 & 0 & 0 & 0 & 0 \\ b & 0 & 0 & 0 & \ell \\ 0 & k & 0 & a & 0 \end{bmatrix}, \quad \begin{bmatrix} x'\mathcal{P}_1x = 8((a-b)^2 + (a-k)^2 + (a-\ell)^2 + (b-k)^2 + (b-\ell)^2 + (k-\ell)^2)^2 \\ x'\mathcal{P}_2x = 8((a-b)^2 + (k-\ell)^2 - (b-\ell)^2 - (a-\ell)^2 - (a-k)^2 - (b-k)^2)^2 \\ x'\mathcal{P}_3x = 8((a-b)^2 + (k-\ell)^2 - (b-\ell)^2 + (a-\ell)^2 - (a-k)^2 + (b-k)^2)^2 \\ x'\mathcal{P}_4x = 8((a-b)^2 + (k-\ell)^2 + (b-\ell)^2 - (a-\ell)^2 + (a-k)^2 - (b-k)^2)^2 \\ x'\mathcal{P}_5x = 0 \end{bmatrix}.$$

These maps have two-fold (and center of) symmetry without axial symmetries.

Maps with vertical axis of symmetry.

$$\begin{bmatrix} 0 & a & 0 & a & 0 \\ b & 0 & 0 & 0 & b \\ 0 & 0 & 0 & 0 & 0 \\ d & 0 & 0 & 0 & d \\ 0 & c & 0 & c & 0 \end{bmatrix}, \quad \begin{bmatrix} x'\mathcal{P}_1x = 8((a-b)^2 + (a-c)^2 + (a-d)^2 + (b-c)^2 + (b-d)^2 + (c-d)^2)^2 \\ x'\mathcal{P}_2x = 0 \\ x'\mathcal{P}_3x = 8((a-b)^2 - (a-c)^2 + (a-d)^2 + (b-c)^2 - (b-d)^2 + (c-d)^2)^2 \\ x'\mathcal{P}_4x = 0 \\ x'\mathcal{P}_5x = 8((a-c)^2 + (b-d)^2)^2 \end{bmatrix}.$$

Maps with $y = x$ axis of symmetry.

$$\begin{bmatrix} 0 & a & 0 & b & 0 \\ c & 0 & 0 & 0 & b \\ 0 & 0 & 0 & 0 & 0 \\ d & 0 & 0 & 0 & a \\ 0 & d & 0 & c & 0 \end{bmatrix}, \quad \begin{bmatrix} x'\mathcal{P}_1x = 8((a-b)^2 + (a-c)^2 + (a-d)^2 + (b-c)^2 + (b-d)^2 + (c-d)^2)^2 \\ x'\mathcal{P}_2x = 0 \\ x'\mathcal{P}_3x = 0 \\ x'\mathcal{P}_4x = 8((a-b)^2 - (a-c)^2 + (a-d)^2 + (b-c)^2 - (b-d)^2 + (c-d)^2)^2 \\ x'\mathcal{P}_5x = 8((a-c)^2 + (b-d)^2)^2 \end{bmatrix}.$$

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