About the structure of the abelian group defined by the sandpile model on graphs

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In this talk I will present a survey of old and recent results on the structure of the group of recurrent configurations of the sandpile model for different families of (finite, non oriented) graphs. For this abelian group whose order is the number of spanning trees of the graph (called its critical group or its sandpile group), only very few results show a relationship between the combinatorial properties of the graph and the algebraic structure of the group. One of them is the fact that the minimal number of generators for this group, which is also the number of the coefficients greater than 1 in the Smith normal form of the Laplacian, has some nice properties with respect of the classical operations of deletion and contraction of the edges on the graph. These properties were used recently by D. Lorenzini in order to give the structure of the group for various families of graphs. It would be of interest to know the probability for a graph to have a cyclic sandpile group. An old result of K. A. Berman relates the sandpile group to the bicycles of a graph, this result will be considered in connection to the above problem.