

# **COST Action MP0108**

**Eindhoven, The Netherlands**

May 1-22, 2011

***Classical, Semiclassical and Quantum Models for  
Understanding Human Systems***

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***with the outstanding participation of  
Marcel Ausloos***

# Abstract

The intense use of Information and Communications Technologies (ICT) into individuals life make that old problems (like **socioeconomic uncertainty**, labor stress, together with new problems as the digital divide and its consequent inequality, data protection, privacy, security, intellectual copyright ) emerge strongly, demanding **the construction of new paradigms**. It is necessary to **develop tools for evaluation and prospecting** of the complex dynamics of the IS as well as **to understand how political and economical decisors behave** considering that **technological advances are faster than the individuals psychological adaptation** capacities. Pointing to the analysis of decision making processes, we describe **some models** coming from classical as well as quantum physics to **provide a theoretical framework** for at least some aspects of human behavior.



# Basis for the new paradigms

- \* The IS behaves as a complex system
- \* The impact of the ICT in Society is irreversible and unavoidable.
- \* In this uncertain times decision making should be supported by theoretical frameworks able to prospect the impact of every decision.
- \* When monitoring information for potential events incoming information provides uncertain and conflicting evidence, but at some point in time, a decision must be made. Incorrect decisions could result in deadly consequences, or in missing opportunities.

## Models and Behavior

The presumption that investors act rationally was formally questioned since the Tversky- Kahneman and Smith works (Nobel Prize 2002 ).

The psychological aspects of decision making under uncertainty play a central role financial and economics decisions (behavioral finance domain) which unfortunately affect the whole society.

***Short review: Classical and full  
quantum Hamiltonian***

***More details on about the  
Su(2) dissipative nonlinear  
Hamiltonian***

## a) A classical model

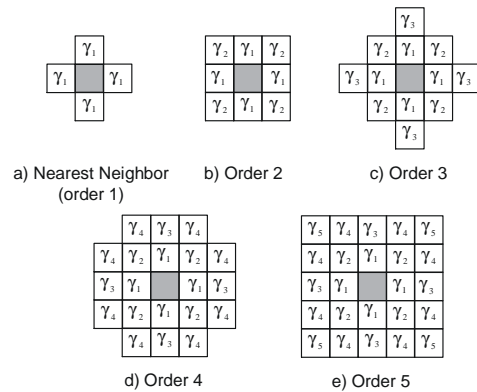
The well known prey-predator model, in a generalized version, typically describes the case of  $n$  agents (companies, governments, investors) competing for or collaborating to get some common resource :

$$\dot{s}_i = \alpha_i s_i (\beta_i - s_i) - \sum_{i \neq j} \gamma(s_i, s_j) s_i s_j \quad i = 1, \dots, n$$

where  $s_i$  is the size of agent  $i$ ;  $\alpha_i$  is the growth rate of agent  $i$ , if no interaction is present;  $\beta_i$  is the maximum capacity of agent  $i$  and  $\gamma(s_i, s_j)$  is the interaction between the agent  $s_i$  and the agent  $s_j$ . The sign of  $\gamma(s_i, s_j)$  defines if agents are in cooperation (-) or competition (+).

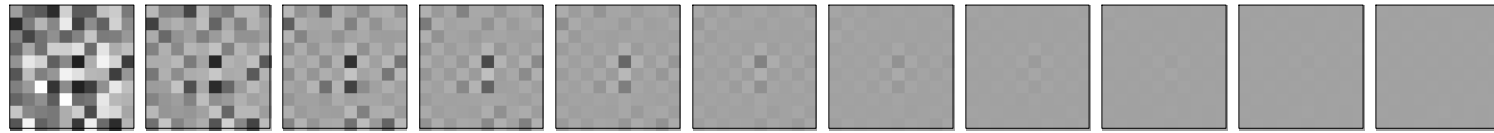
# Case 1

## Introducing a neighbor hierarchy

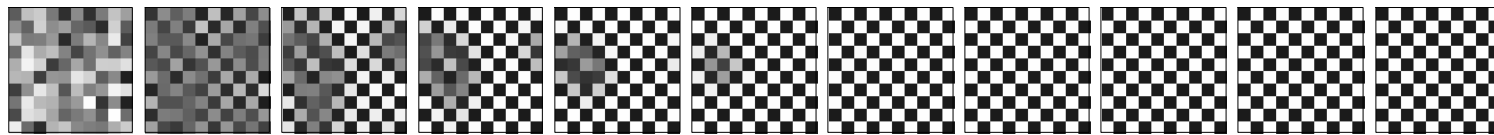


## Neighbor

and working in 2D-dimensions for certain parameters values a hung scenario appears dynamically in competitive scheme



a) Simulation with parameters in Region B (all agents are non zero)



Case b) Simulation with parameters in Region D (chessboard)

***Hung scenarios (S. G. "contrarians") emerge dynamically as the equilibrium stage in competition situations***




## Case 2

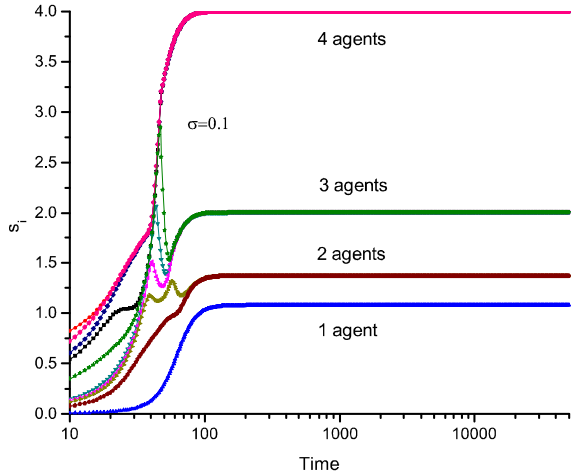
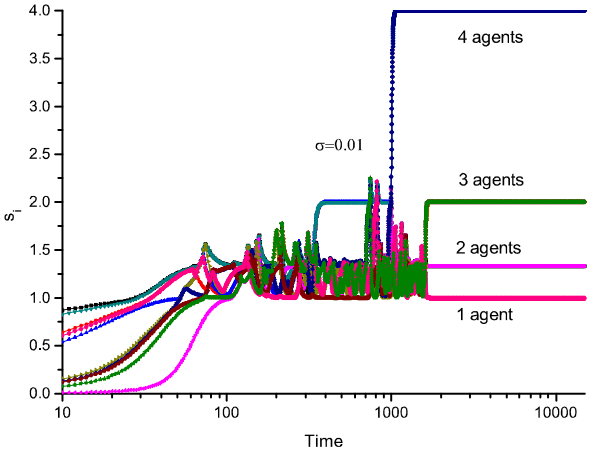
Introducing a dynamic agent size through

$$\gamma(s_i, s_j) = \exp\left[-\left(\frac{s_i - s_j}{\sigma}\right)^2\right]$$

When we have a competition scheme related to the agents' sizes, this acts as constraint allowing that **only "players" of similar sizes truly compete with each other**. In market language, this represents the natural segmentation into big, medium, and small players.

Also means that decisions (global politics, financial rules, INTERNET management) **are taken for only few players**, which is not very fair. A **full cooperative attitude** taken by the agents, allows that agents groups configure clusters whose collective capacity is higher than the individual capacity, initially defined by the  $\beta$  value. 

# An example of cooperative scenario for agents with coupled through size $\sigma = 0.01, \sigma = 0.1$



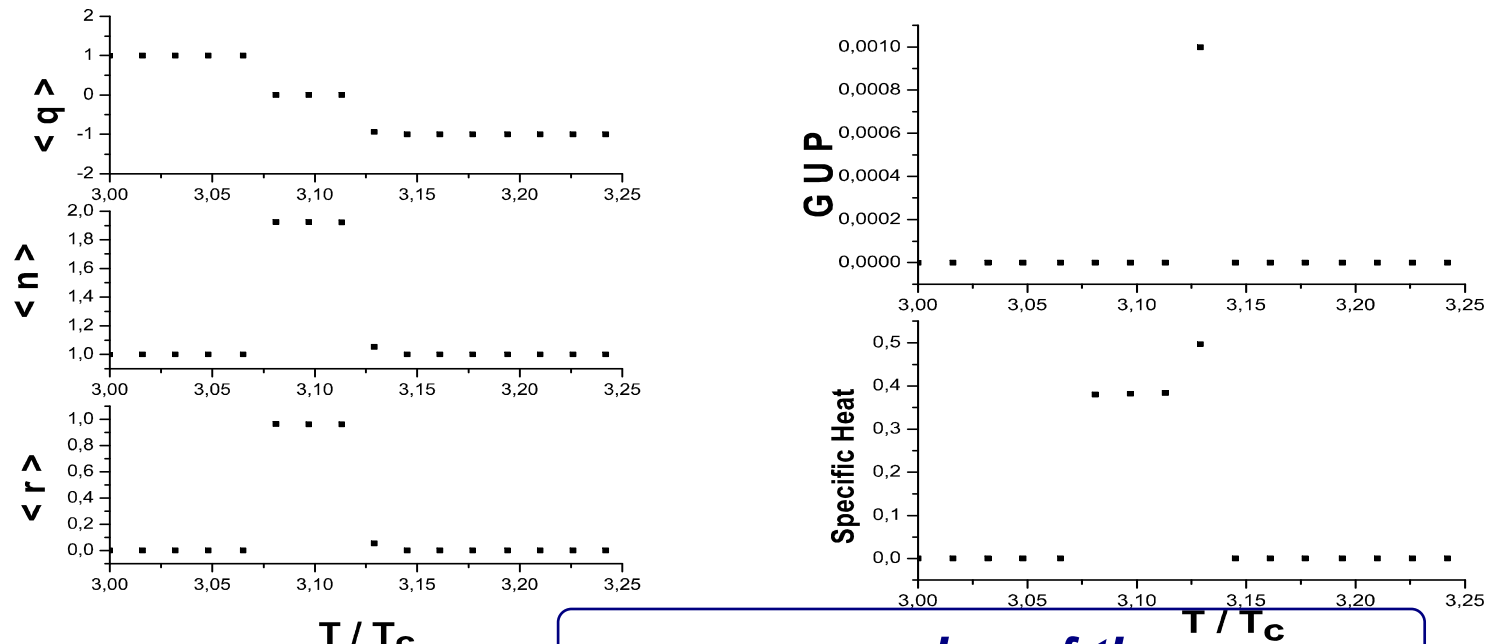
- Dynamical emergence of contrarians in a 2-D Lotka – Volterra lattice C. F. Caiafa and A. N. Proto, International Journal on Modern Physics C, Vol. 17 (2006) 385-394
- Dynamic Peer-to-Peer Competition L.F. Caram, C.F. Caiafa, A.N. Proto and M. Ausloos, Physica A 389 (2010) 2628\_2636, ISSN:0378-4371 -HOLLAND
- Dynamics of a Multiagent system with size-coupled interaction, L.F. Caram, A.N. Proto to be published in Adv. & Appl. Stat. Sci.

## A full quantum model: the Hubbard Hamiltonian

Spin systems, as the Ising model, are very useful to describe the yes-no processes that precede an opinion formation, or to describe social situations. The Hubbard-like Hamiltonian, describes a strongly correlated systems in the spontaneous colossal magneto resistance regime has three possible states/ opinions / attitudes: +1, 0 (stand by), -1. We used the Maximum Entropy Principle (*MEP*) formalism which provides us with a method to evaluate the specific heat and the Generalized Uncertainty Principle (*GUP*).

The specific heat, defined as the variation of the information (entropy) exchange among individuals respect to the social temperature, increases substantially when the system is at the 'stand by' (0) attitude as well as the *GUP* increases when the social system is near to jump from neutral, indifferent or stand by to negative opinion.

This last point is in line with the description given in cite: buse, in the sense that there exists a quantum component in the decision making process. We consider that our main contribution comes from the ability of *MEP* formalism for evaluating the *SH* and *GUP*.



**$\langle q \rangle$  mean value of the op. representing opinion**  
 **$\langle n \rangle$  mean value of the op. representing number of "sites"**  
 **$\langle r \rangle$  mean value of the op. representing interaction**

## b) Semiquantum non linear Independent-time Systems

For the semiquantum non linear independent-time systems of the form

$$\hat{H} = \sum_{j=1}^3 a_j(q,p) \hat{\sigma}_j + \frac{p^n}{2m} + V(q),$$

(n =2, here) the quantum degrees of freedom,  $\hat{\sigma}_j$ , belong to the generators of the  $SU(2)$

Lie algebra  $\{\hat{\sigma}_x, \hat{\sigma}_y, \hat{\sigma}_z\}$

They constitute a **(CSNCO)**. The classical degrees of freedom are the canonically conjugate variables  $(q,p)$ . The nonlinearity of  $\hat{H}$  in the semiquantum context is included in the  $a_j(q,p)$  set (property of MEP procedure) and the term  $\frac{p^2}{2m} + V(q)$  is a purely classical one.

# The Uncertainty Principle for the SU(2)

The Generalized Uncertainty Principle,  $I^H$ , for a CSNCO of  $q$  elements is

$$I^H = \sum_{j,k=1}^q \left\{ (\Delta \hat{O}_j)^2 (\Delta \hat{O}_k)^2 - \left[ \langle \hat{L}_{jk} \rangle - \langle \hat{O}_j \rangle \langle \hat{O}_k \rangle \right]^2 \right\} \geq \\ \geq -\frac{1}{4} \sum_{j,k=1}^q \langle [\hat{O}_j, \hat{O}_k] \rangle^2$$

(with  $\langle \hat{L}_{jk} \rangle = \frac{1}{2} \langle \hat{O}_j \hat{O}_k + \hat{O}_k \hat{O}_j \rangle$  and  $j < k$ ). We have demonstrated that if

the dynamic matrix  $G(q,p)$  is an antisymmetric one, then  $I^H$  (GUP) is:

$$I^H = 3 - 2 \left( \langle \hat{\sigma}_x \rangle^2 + \langle \hat{\sigma}_y \rangle^2 + \langle \hat{\sigma}_z \rangle^2 \right) = 3 - 2 \langle \hat{\sigma} \rangle^2$$

and it is a constant of motion. As the uncertainty principle must hold for the  $SU(2)$  Lie algebra GUP

$$0 < \langle \hat{\sigma} \rangle^2 = \langle \hat{\sigma}_x \rangle^2 + \langle \hat{\sigma}_y \rangle^2 + \langle \hat{\sigma}_z \rangle^2 < 1$$

which defines the Bloch sphere of the system.

**ALL HAMILTONIANS WHICH INCLUDES  
THE S(U)2 LIE GROUP  
SHARE THE UP SAME FORM AND IT IS ALSO A  
DYNAMICAL INVARIANT**

## ***Our dissipative SU(2) decision model***

Our model will always be able to be represented by a Hamiltonian of the form

$$\hat{H} = B \hat{\sigma}_z + C q^n \hat{\sigma}_j + \frac{p^2}{2m} + V(q),$$

$\hat{\sigma}_j$  stands for  $\hat{\sigma}_x$ ,  $\hat{\sigma}_y$  or  $\hat{\sigma}_z$  and  $C$  is the coupling constant between the quantum and classical subsystems.

**Quantum subsystem:**  $\hat{H}_q = B\hat{\sigma}_z$ , where,  $\hat{\sigma}_z$  is the  $z$  –component of a  $1/2$  spin particle;  $B$  parameter is the external magnetic field (parallel to  $z$  –direction) which oblige the  $\hat{\sigma}_z$  spin's component to be aligned in the  $z$  –direction. **The  $B$  parameter represents any “mandatory” statement** which could come from authority or some “extreme collective situation” and the quantum part of the system (quantum subsystem)  $B\hat{\sigma}_z$  represents the “opinion-attitude-decision” taken by a group of individuals, decision makers, leaders, and  $\langle \hat{\sigma}_z \rangle$ , the physical magnitude which represents the “opinion-attitude-decision”.

**Classical subsystem:**  $H_{cl} = \frac{p^2}{2m} + V(q)$ , represents the individuals as a whole or the society, where the term  $\frac{p^2}{2m}$  contains collective interests, projects, expectations, beliefs and the potential  $V(q)$  represents the degree of *cohesion* energy between



individuals belonging to a collective group. Here  $V(q) = \frac{q^4}{4}$

**Interaction:** The group of individuals (decision makers, leaders) represented by  $\langle \hat{\sigma}_z \rangle$  interacts with society, through  $C q^n \hat{\sigma}_j$  ( here  $n = 2$ ) and also should take into account the “external-social pressure-market-expectations-limit situations-global context” under which the “opinion-attitude-decision” is taken. This interaction, will be modeled introducing dissipation in the classical subsystem (via the parameter  $\mathcal{E}$ ) by means of an ad hoc term,  $-\mathcal{E}p$ .

The  $SU(2)$  Lie algebra  $\{\hat{\sigma}_x, \hat{\sigma}_y, \hat{\sigma}_z\}$  is the  $CSNCO$ , and leads to the antisymmetric matrix:

$$G = \begin{pmatrix} 0 & -2B & 0 \\ 2B & 0 & -2Cq \\ 0 & 2Cq & 0 \end{pmatrix}$$

Besides  $\langle \hat{H} \rangle = B \langle \hat{\sigma}_z \rangle + C q \langle \hat{\sigma}_x \rangle + \frac{p^2}{2m} + D \frac{q^4}{4}$ , and so

$$\begin{aligned} \frac{d\langle \hat{\sigma}_x \rangle}{dt} &= -2B \langle \hat{\sigma}_y \rangle, \\ \frac{d\langle \hat{\sigma}_y \rangle}{dt} &= 2B \langle \hat{\sigma}_x \rangle - 2Cq \langle \hat{\sigma}_z \rangle \\ \frac{d\langle \hat{\sigma}_z \rangle}{dt} &= 2Cq \langle \hat{\sigma}_y \rangle, \\ \frac{dq}{dt} &= \frac{p}{m} \\ \frac{dp}{dt} &= -[C \langle \hat{\sigma}_x \rangle + Dq^3 + \xi p] \end{aligned}$$

where,  $-\xi p$  is an “ad hoc” term, proportional to the velocity as it is usual in classical mechanics to include a dissipative term.

The yes-no decision making process has three kind of factors: i) the way in which the individuals interact between them (the potential  $V(q)$  ), ii) the interaction between decision makers and society  $H_{int} = Cq^n \hat{\sigma}_j$ , iii) the environment conditions through the  $\varepsilon$  parameter.

The *MEP* formalism applied to these kind of systems two advantages a) classical degrees of freedom  $q$  and  $p$  act as they were parameters in the quantum commutation operation (matrix  $G(q,p)$  generates through the *MEP* ), b) the problem of solving the dynamics of a given Hamiltonian has been translated into the language of the dynamic systems.

## Numerical Simulations

The simulations have been done for several different sets of parameters and initial conditions. For sake of simplicity and shortness, the plots selected to be shown belongs to the set:  $\langle \hat{\sigma}_x \rangle = 0$ ,  $\langle \hat{\sigma}_y \rangle = 0$ ,  $\langle \hat{\sigma}_z \rangle = 0.9$ . Free election, can be made meantime

a coherent set of  $\langle \hat{\sigma}_i \rangle$  should be selected satisfying  $I^H \leq 1$

or  $\langle \hat{\sigma} \rangle^2 \leq 1$ , for this case  $\langle \hat{\sigma} \rangle^2 \leq 0.81$

$q_0 = 0$ , (just for simplicity) and so,

**IC CANNOT BE ARBITRARILY SELECTED.  
IF SO UP COULD BE VIOLATED**

$$p_{(0)} = [2m (\langle \hat{H}_1 \rangle - B \langle \hat{\sigma}_z \rangle_{(0)} - C q_{(0)} \langle \hat{\sigma}_x \rangle_{(0)} - \frac{q^4(0)}{4})]^{1/2},$$

is evaluated through the  $\langle \hat{H} \rangle$  to obtain a coherent selection of the Initial Conditions of the nonlinear dynamic equations of the system. Parameters can be chosen without restriction.

For our plots

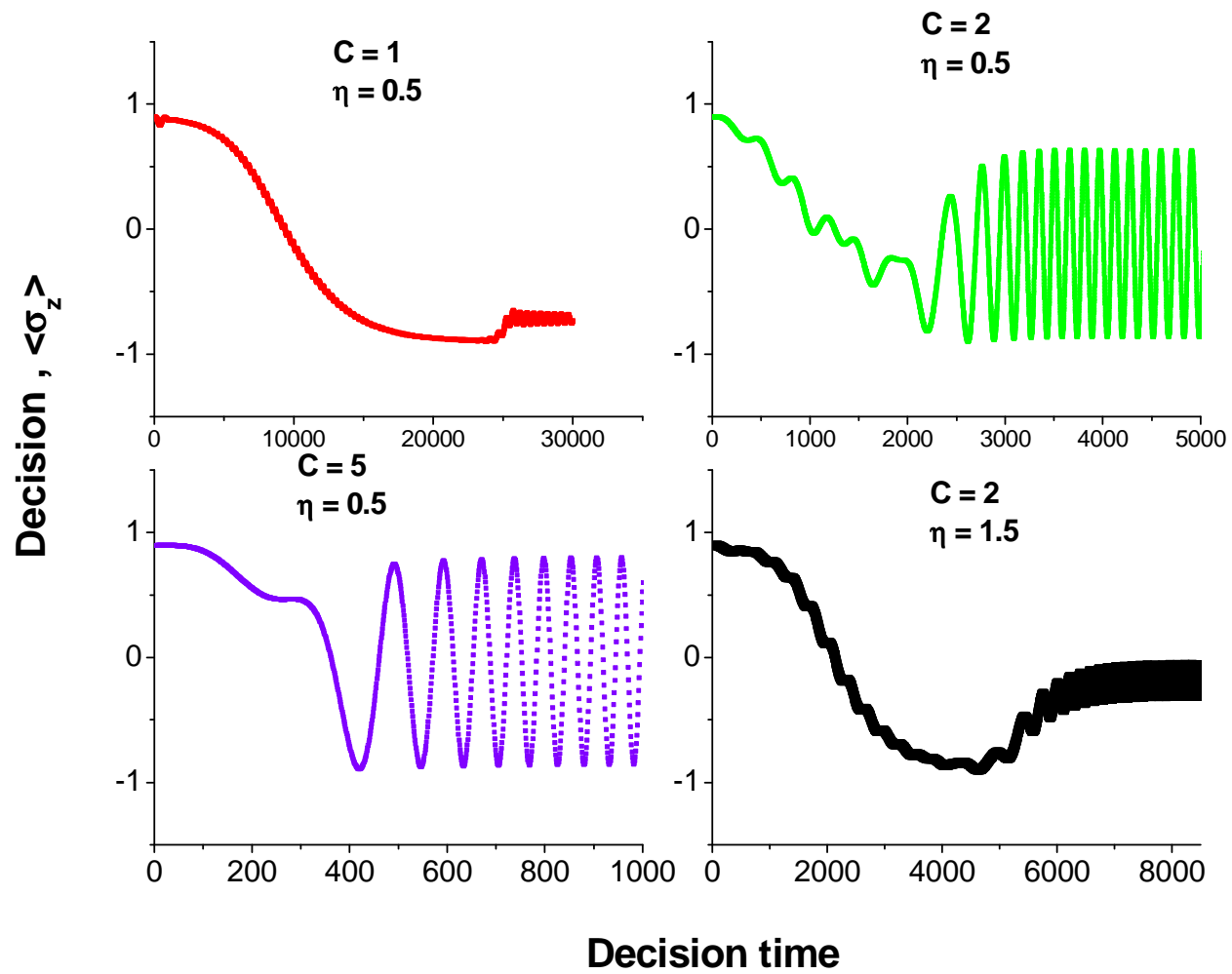
$$B = 0.5, C = 1.0, D = 1.0, m = 16, \langle \hat{H} \rangle = 0.5,$$

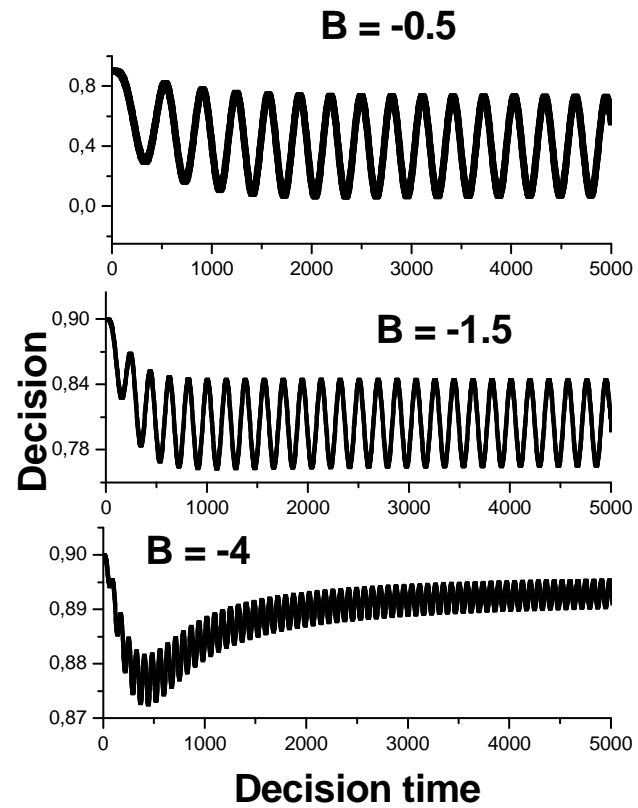
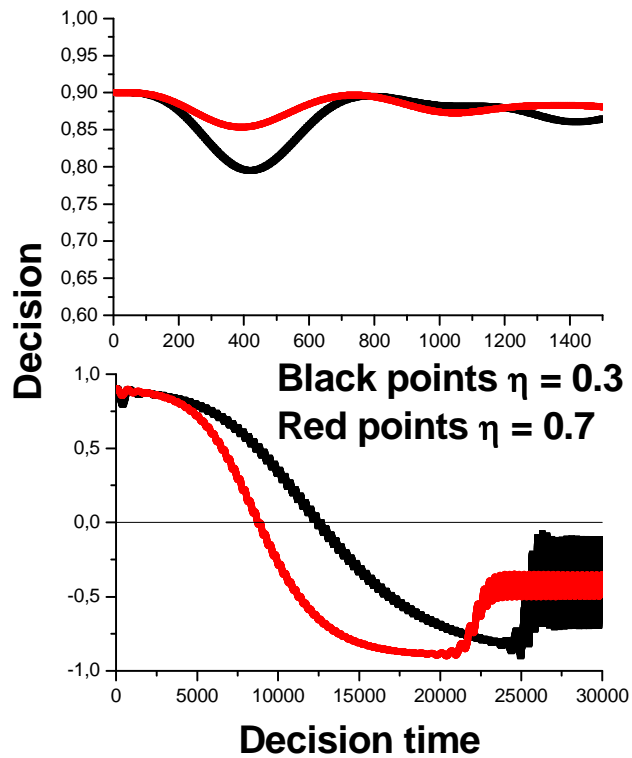
$\eta = 0.3$  (black points) and  $\eta = 0.7$  (red points). Maximum evolution time = 30000 steps.

$\eta = 0.3$  (black points) and  $\eta = 0.7$  (red points). Maximum evolution time = 30000 steps.

For  $\eta = 0.3$  the decision is taken around 12400 steps, instead for  $\eta = 0.7$  the decision was taken around 8700 steps.

Simulations for the left figure  $B$  and  $\eta$  have the same sign (no matter if + or -), which means that the mandatory statement and the “external conditions-social pressure-limit situation” are not in opposition. Instead on right figure Fig. the  $B$  value (mandatory parameter) was settled as a negative number, for  $\eta = 0.7$ .





Upper part left Fig. are the first steps of the evolution

*Spin model APPLICATION TO  
DATA*

**Dedicated to my dear friend M.A.**



**1983: A law introduced contracts among enterprises in the legal system**

***Unión Transitoria de Empresas (UTE)***

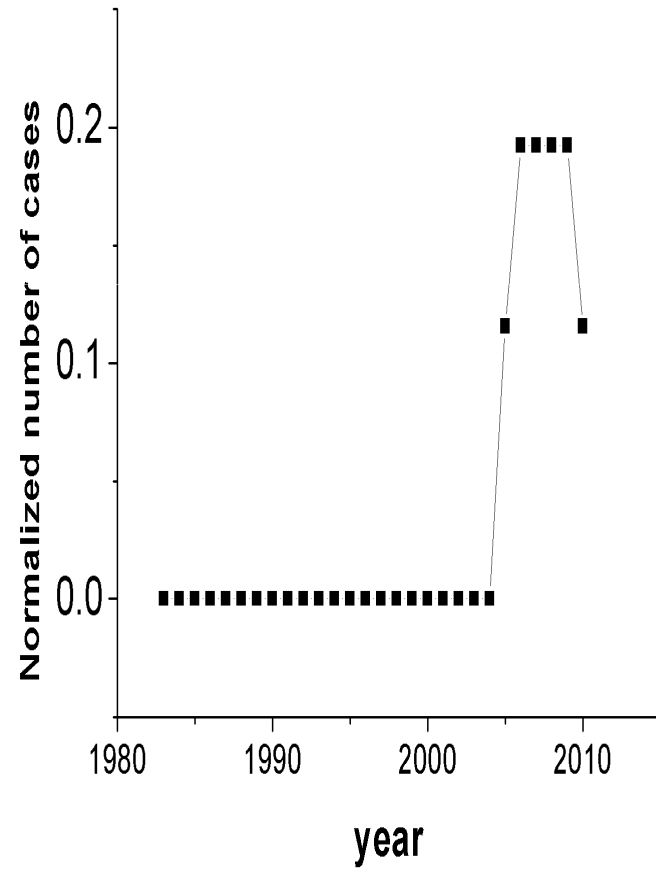
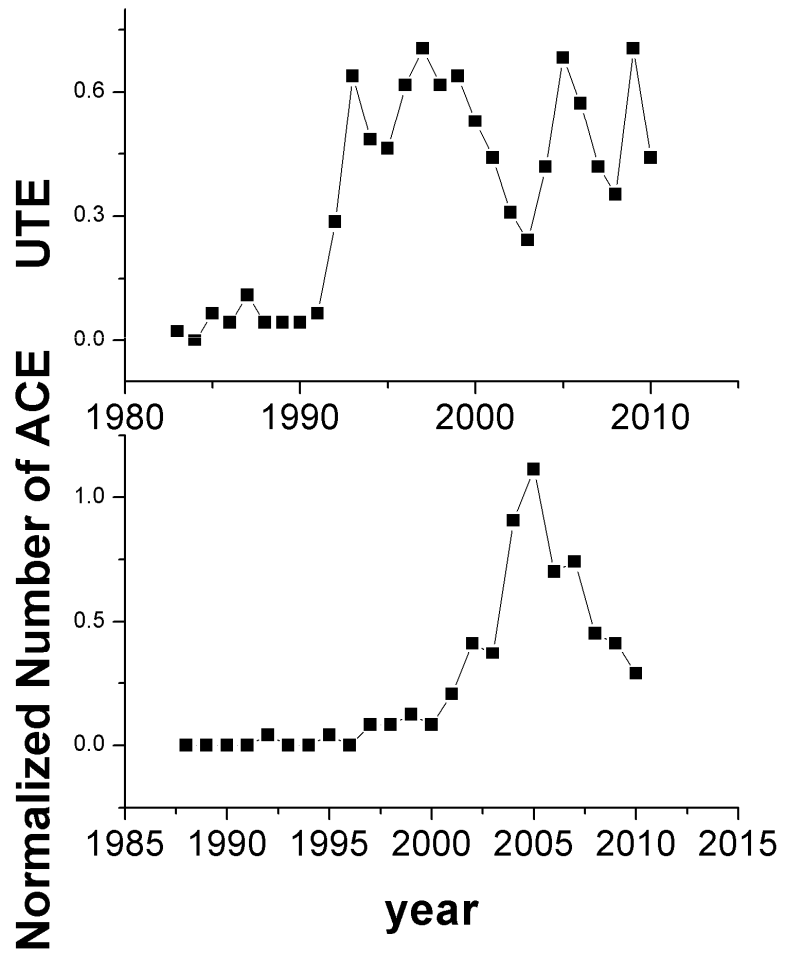
Temporary Union of Firms

***Agrupación de Colaboración Empresarial (ACC)***

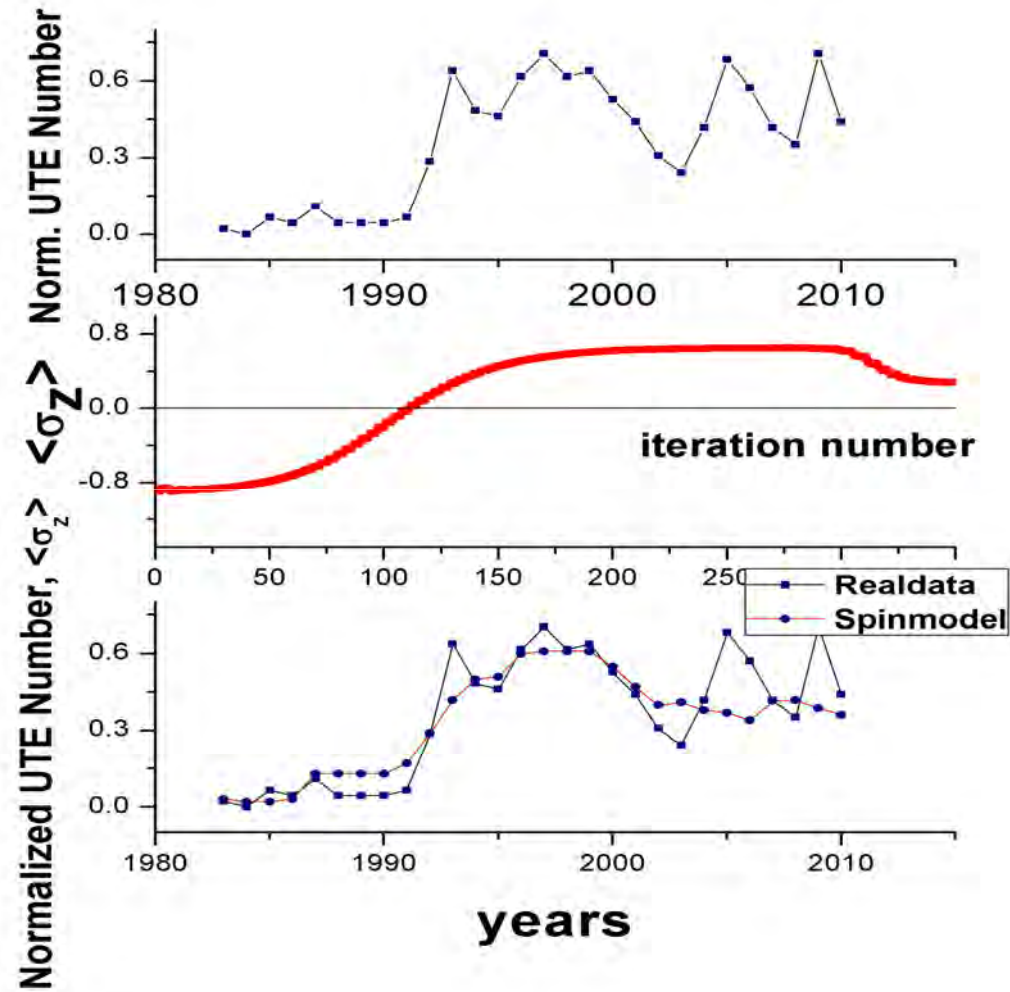
Group of Collaborating Firms

***Consortios de Cooperación Empresarial (CCE)***

Consortium of Cooperating Firms



# Case 1: UTE







Request on references  
are welcome





*Thank you for your attention!!*

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