Dynamics of public opinion under different conditions

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OUTLINE OF THE TALK

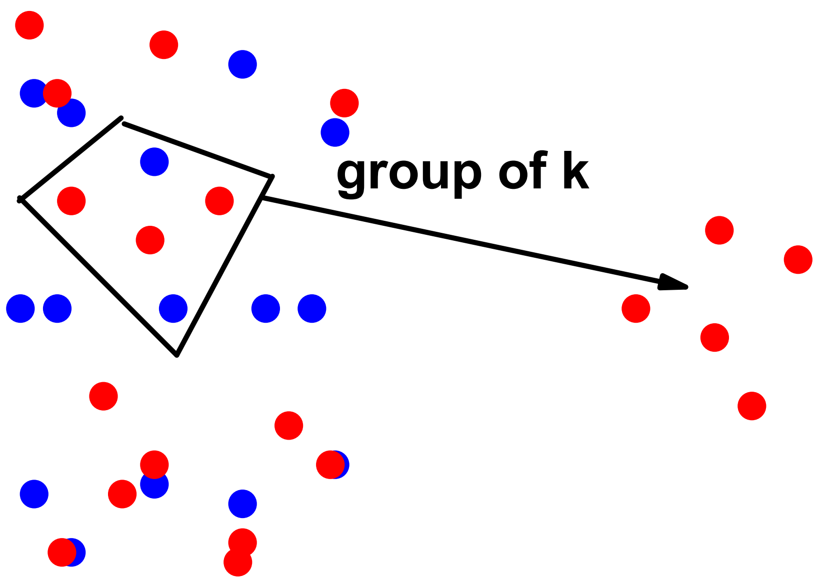
- Short review of Galam's models
- Rational choice model
- Discussion group ot two people, $k = 2$, case: fixed points and dynamics.
- Rational choice model with inflexibles.
- Case $k = 2$ with inflexibles: fixed points and dynamics.
- Conclusions.

Basic Galam's model for binary-state agents

N agents at time t:

the 'reds' fraction is p_t, the 'blues' fraction is $(1 - p_t)$.

The local majority view.



If the group is at par, no changes occur.

Difference equation (discrete map)

EXAMPLE, $k = 3$: $p_{t+1} = p_t^3 + 3p_t^2\,(1 - p_t)$

Three fixed points:

- $p_0^\star = 0$ and $p_1^\star = 1$ are attractive.
- $p_2^\star = 1/2$ is repelling.

“Reds” fraction disappears if $p_0 < 1/2$
“Blues” fraction disappears if $p_0 > 1/2$

Extensions of the model:

► Contrarian effect S. Galam, Physica A (2004).

There exist a specific (minority) of fraction a that takes the opinion opposite to the majority:

- At low $a < a_c$, both opinions coexist, two attracting fixed points and one repeling
- For $a > a_c$, the two attracting fixed points merge into one (attracting) fixed point.
- a_c depends on group's size; e.g. for $k = 3$ $a_c = 1/6$

Extensions of the model:

▶ Effect of infllexeble minorities, S.Galam and F. Jacobs, Physica A (2007); S. Galam, Physica A (2010).

For group's size $k = 3$.

- Not trivial attractors $p_0^\star > r_0$, $p_1^\star < 1 - b_0$.
- Disappearance of one of the attractors.
- Creating a single attractor.

Extensions of the model:

► Majority-minority model, M. Mobilia and S. Redner. Phys. Rev. E, (2003)
With probability π the group adopts the local majority view
With probability $1-\pi$ the group adopts the local minority view

The results are obtained for groups of size $k=3$.

- Two attractors 0 and 1 for $\pi > 2/3$
- Creating a single attractor at $p_2^\star = 1/2$ for $\pi < 2/3$

Rational choice model: local rules

In any event of discussion, that is the sample include at least one of the "reds" and one of the "blues":

π_r,	probability for	"blue" $\longmapsto$ "red"
π_b,	probability for	"red" $\longmapsto$ "blue"

and hence,

$1 - \pi_r$,	probability for	"blue" remains "blue"
$1 - \pi_b$,	probability for	"red" remains "red"

Dynamics in the case of groups of two people, $k = 2$

At time t:

0 *r*-'reds' p_t *b*-'blues' 1

Draw *rr* with probability p_t^2. At time $t + 1 \longmapsto$ *r r*
Draw *bb* with probability $(1 - p_t)^2$. At $t + 1 \longmapsto$ *bb*

Draw *r b* with probability $2p_t(1 - p_t)$, and count 'reds' at time $t + 1$:

$$\left[\underbrace{(1-\pi_b)\pi_r}_{rr} + \underbrace{(1-\pi_b)(1-\pi_r)\left(\frac{1}{2}\right)}_{rb} + \underbrace{\pi_b \pi_r \left(\frac{1}{2}\right)}_{br} \right]$$

Difference equation in the case $k = 2$

Introducing the difference of probabilities:

$\boxed{\lambda = \pi_r - \pi_b}$, $-1 \leq \lambda \leq 1$

$$\boxed{p_{t+1} = f_2(\lambda, p_t) = (1 + \lambda)p_t - \lambda p_t^2}$$

Setting, $\lambda = 0$, the map reduces to the 'basic Galam's model' for $k = 2$.

Difference equation in the case $k = 2$

Comparing with the "general Galam's model" (Europhysics Lett. 2005).

$$p_{t+1} = m_{2,2} p_t^2 + 2 m_{2,1} p_t \left(1 - p_t\right) + m_{2,0} \left(1 - p_t\right)^2$$

$$p_{t+1} = (1 + \lambda) p_t - \lambda p_t^2$$

$m_{2,2} = 1$, $m_{2,1} = \dfrac{1}{2}\left(1 + \lambda\right)$ and $m_{2,0} = 0$.

Fixed points in the case $k = 2$

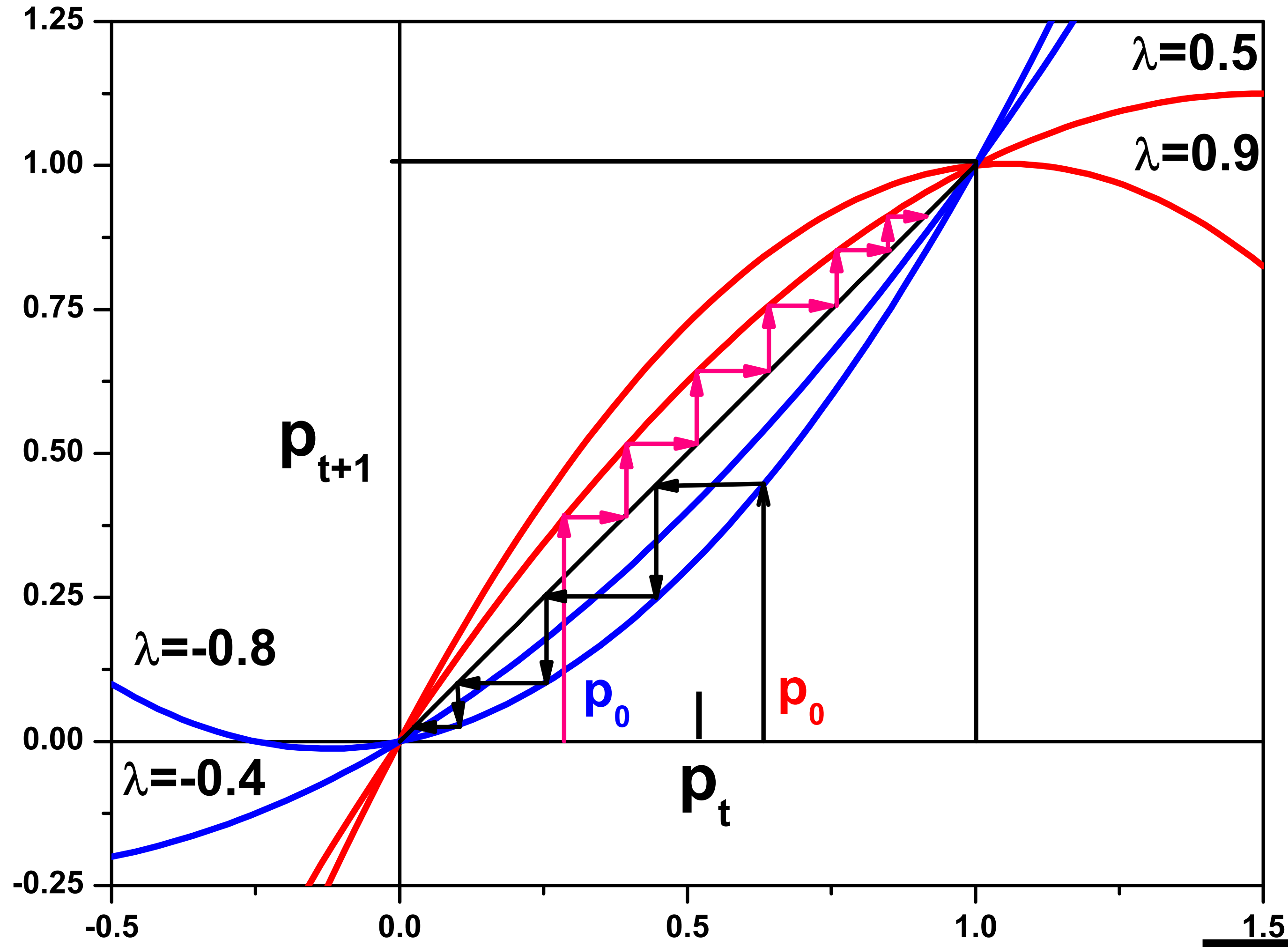
Two fixed points: $p_0^\star = 0$ and $p_1^\star = 1$

- $p_0^\star = 0$ attractive for $\lambda < 0$ and repelling for $\lambda > 0$.
- $p_1^\star = 1$ attractive for $\lambda > 0$ and repelling for $\lambda < 0$.

REMARK: $p_t \mapsto \dfrac{1-\lambda}{\lambda} x_t + 1$ formally transforms f_2 into the logistic map, $\mu x_t (1 - x_t)$, with $\mu = 1 - \lambda$.

Graphical iterations of $f_2(\lambda, p_t) = (1+\lambda)p_t - \lambda p_t^2$

Red lines, $\lambda > 0$, blue lines $\lambda < 0$; $\lambda = \pi_r - \pi_b$



Introducing ‘inflexibles’; the case of $k = 2$

At time t:

EXAMPLE:

Draw $r_0 b_1$ with probability $2r_0(1 - b_0 - p_t)$.
This adds to the ‘reds’ fraction at time $t + 1$:

$$\left[\pi_r + (1 - \pi_r)\left(\frac{1}{2}\right)\right]$$

$r_0 r_1$ $\qquad$ $r_0 b_1$

Difference equation in the case $k = 2$ with inflexibles

Introducing: $\boxed{\lambda = \pi_r - \pi_b}$; $\boxed{\beta_r = r_0 \pi_b}$; $\boxed{\beta_b = b_0 \pi_r}$

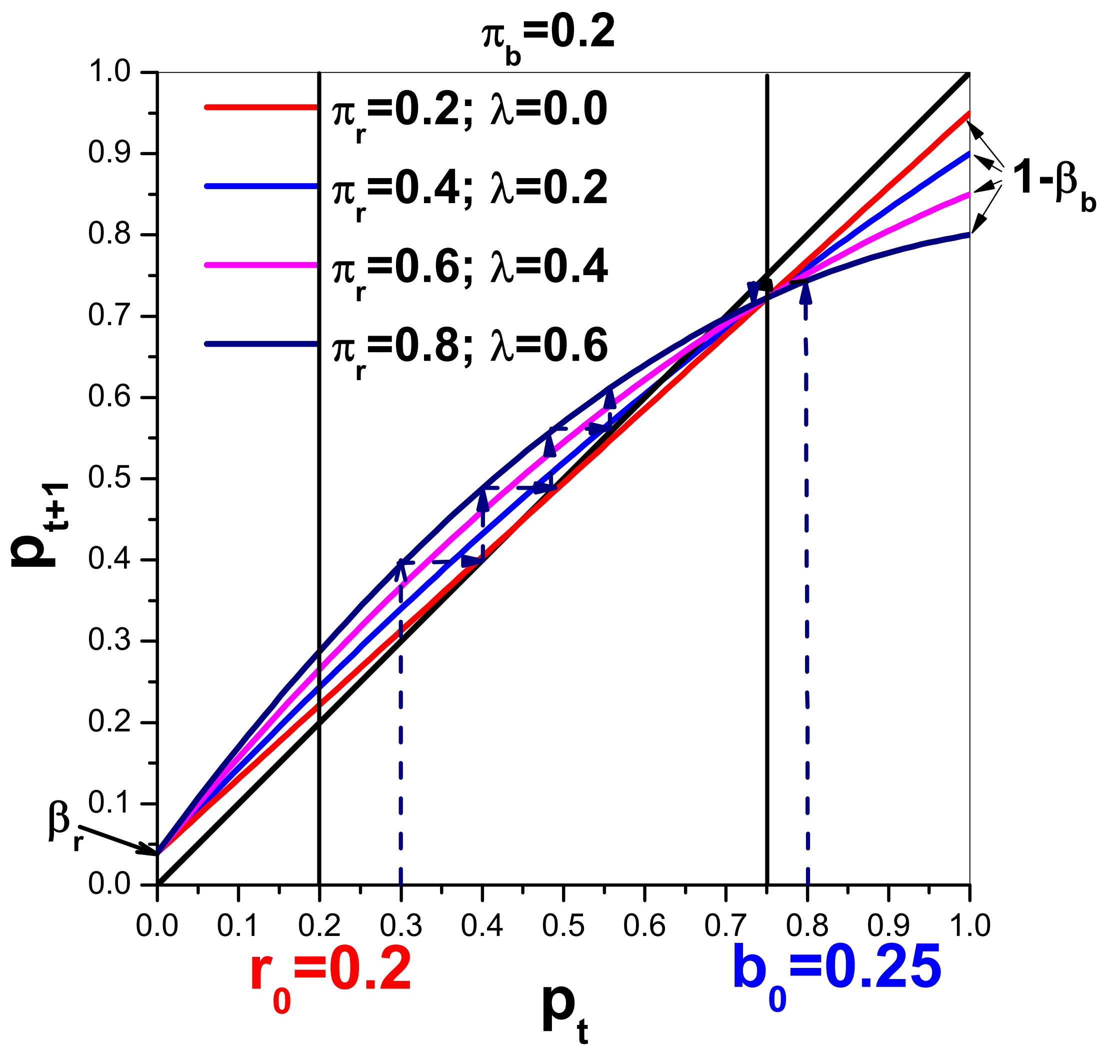
$$\boxed{p_{t+1} = f_2(\lambda, \beta_r, \beta_r; p_t) = \beta_r + (1 + \lambda - \beta_r - \beta_b)\, p_t - \lambda p_t^2}$$

Fixed points:

$$p^\star_\pm = \frac{1}{2}\left[\left(1 - \frac{\beta_r}{\lambda} - \frac{\beta_b}{\lambda}\right) \pm \sqrt{\left(1 - \frac{\beta_r}{\lambda} - \frac{\beta_b}{\lambda}\right)^2 + \frac{4\beta_r}{\lambda}}\right]$$

Iterations of $f_2(\lambda, \beta_r, \beta_b; p_t)$

$$f_2(\lambda, \beta_r, \beta_b; p_t) = \beta_r + (1 + \lambda - \beta_r - \beta_b)p_t - \lambda p_t^2$$



Iterations of $f_2(\lambda, \beta_r, \beta_b; p_t)$

$$f_2(\lambda, \beta_r, \beta_b; p_t) = \beta_r + (1 + \lambda - \beta_r - \beta_b)p_t - \lambda p_t^2$$