Roadblocks revisited

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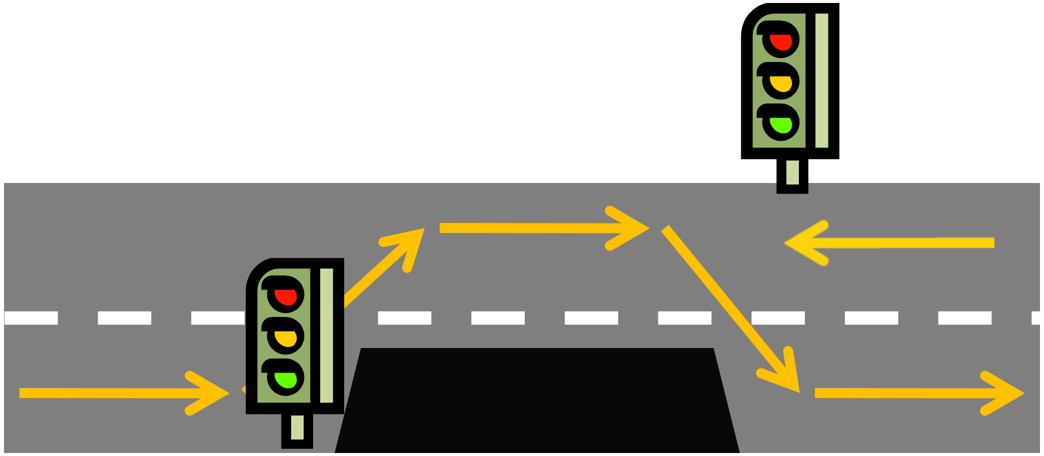
Outline

- Introduction
- Fixed green times
- Exhaustive control policy
- Maximum green time

Introduction

- **First traffic light: 10 december 1868, London**
- **First intersection with multiple traffic lights: 1914, Cleveland**
- **Orange**
- **Vehicle**

Roadblock



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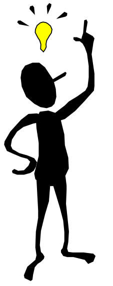
Roadblock

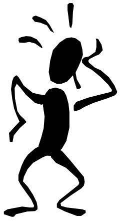
- **Model 1: fixed cycle time**

  → slightly difficult



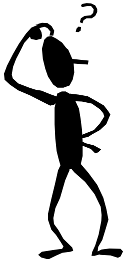
- **Model 2: stay green until queue has vanished**

  → more difficult



- **Model 3: stay green for limited amount of time**

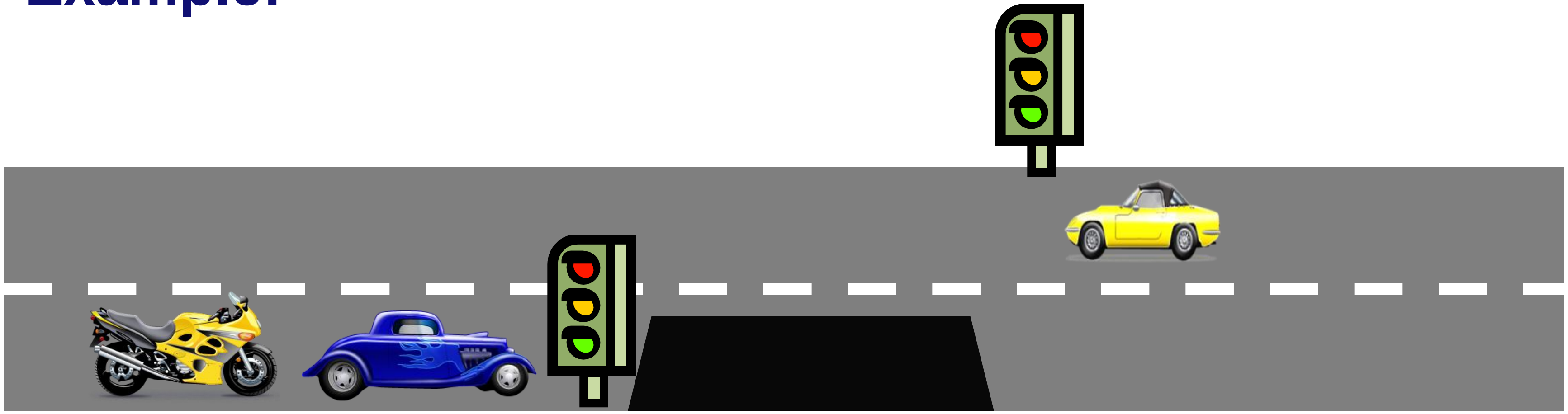
  → impossible



Roadblock – Model 2

Model 2: stay green until queue has vanished

Example:



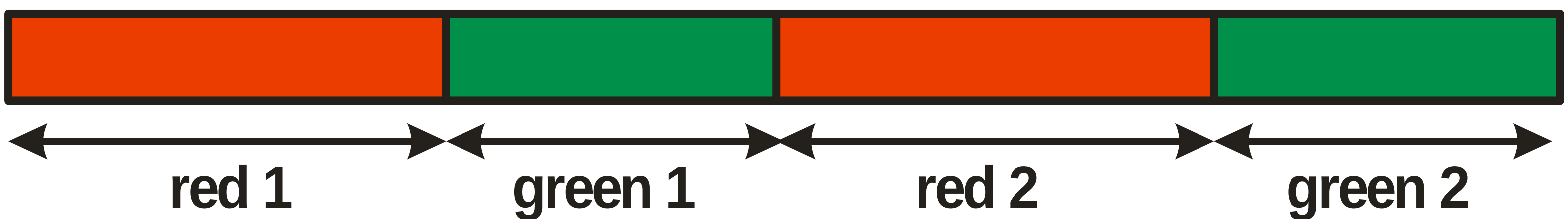
λ_1 = 300 vehicles per hour
μ_1 = 1800 vehicles per hour
r_1 = 8 seconds red time

λ_2 = 600 vehicles per hour
μ_2 = 1800 vehicles per hour
r_2 = 8 seconds red time

Roadblock – Model 2

- **Model 2: stay green until queue has vanished**

  Cycle:



- **Stability condition?**

$$\rho_1 = \frac{\lambda_1}{\mu_1}, \rho_2 = \frac{\lambda_2}{\mu_2}, \rho = \rho_1 + \rho_2 < 1$$

- **Example:** $\rho_1 = \frac{300}{1800} = 0.167, \rho_2 = \frac{600}{1800} = 0.333, \rho = 0.5$

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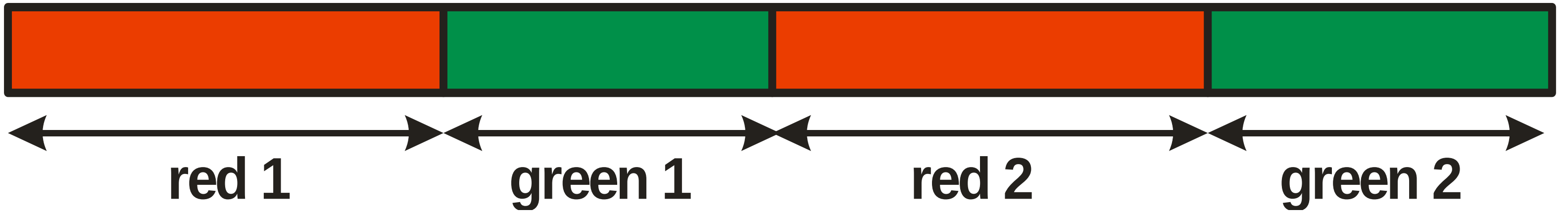
Roadblock – Model 2

- **Model 2: stay green until queue has vanished**

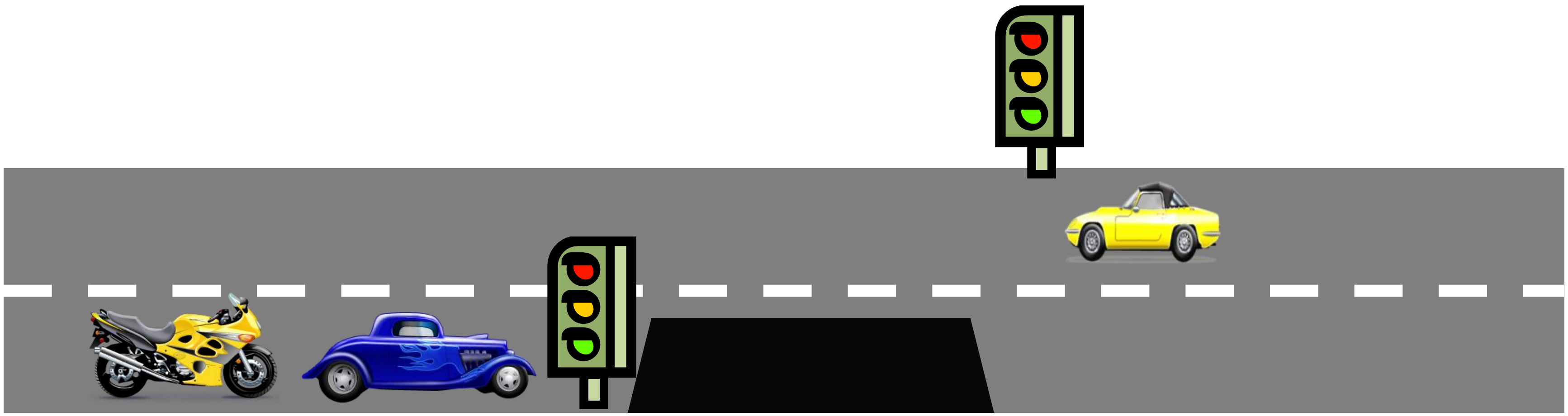
Cycle:



- **Mean cycle time?**

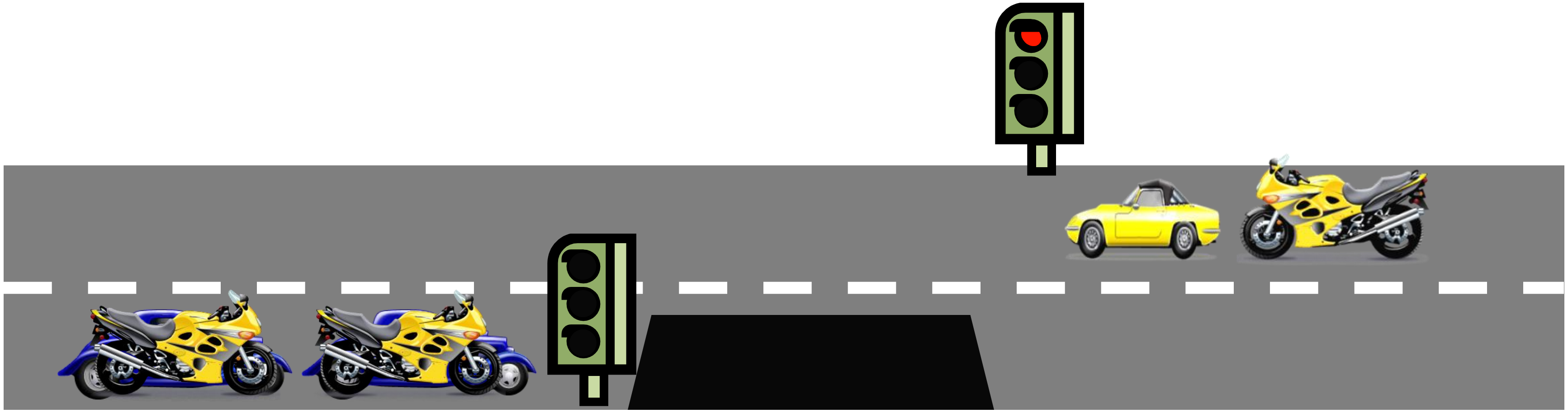
$$E(C) = \frac{r_1 + r_2}{1 - \rho} = \frac{16}{1/2} = 32$$

Roadblock – Model 2

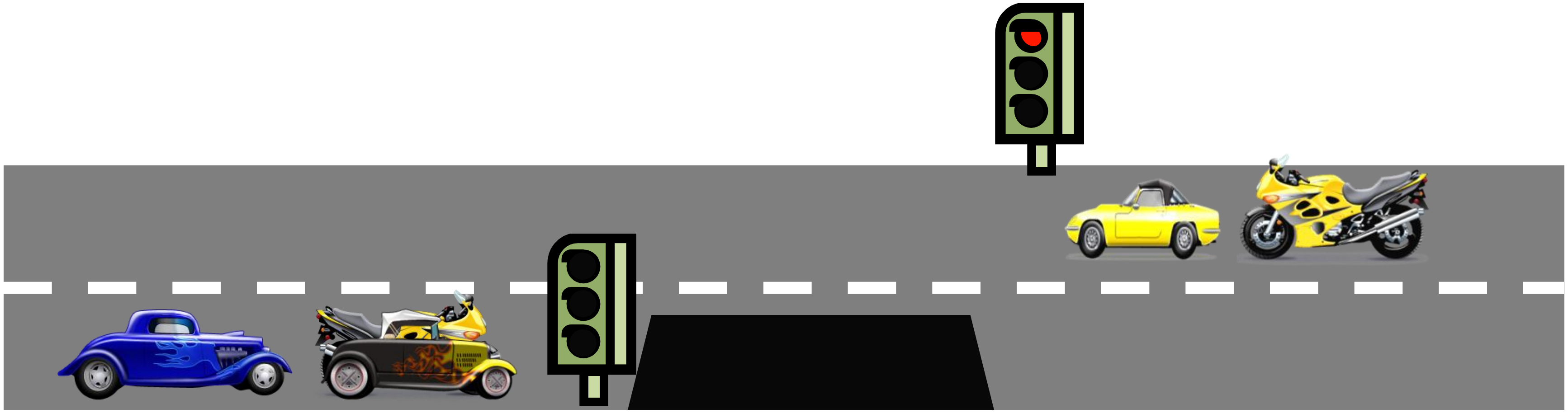


- Step 1: determine *probability distribution* of the cycle time
- Analysis using theory on branching processes
- Cycle time does not depend on the order in which cars in a queue are served

Roadblock – Model 2



Roadblock – Model 2



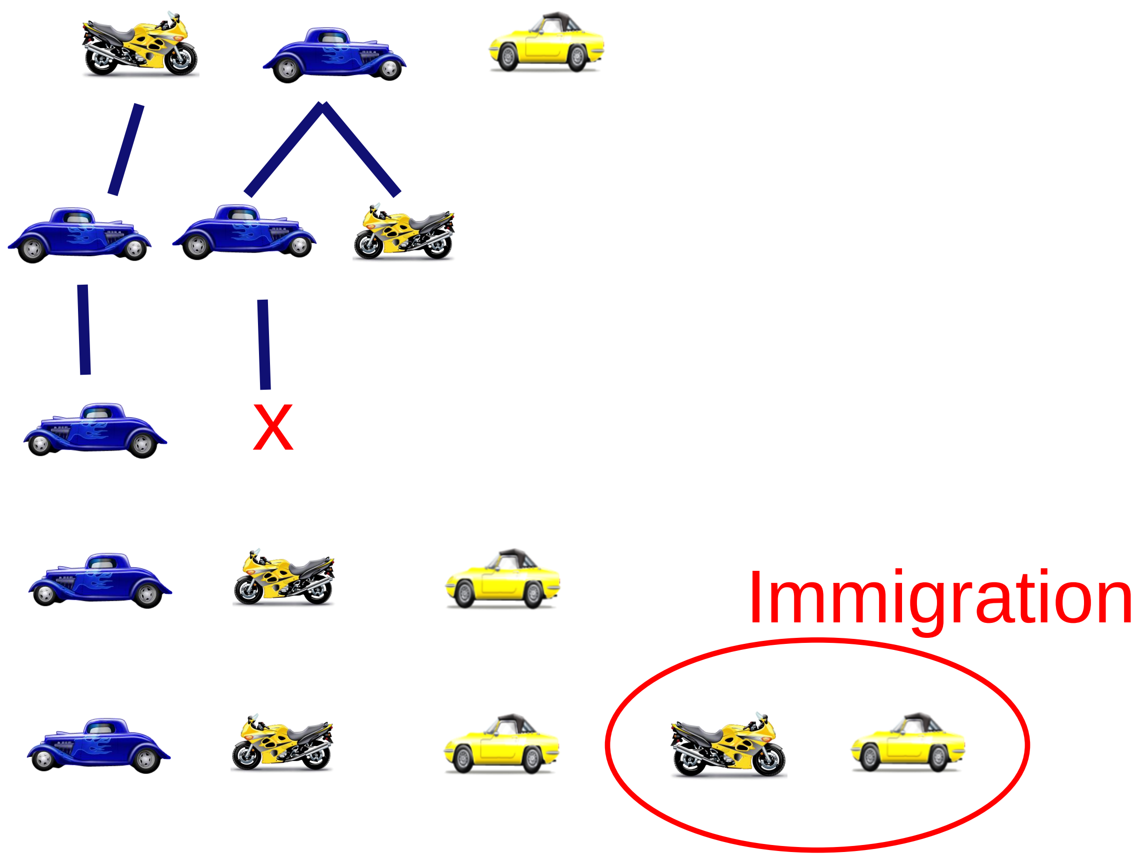
Roadblock – Model 2

- **Branching process with immigration**

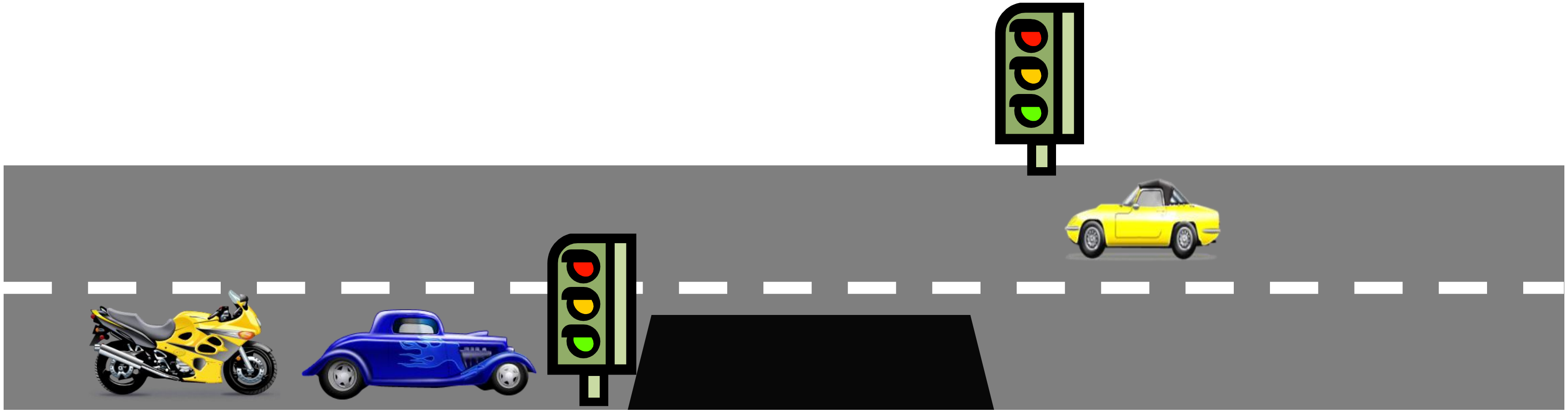
Start green 1

Start red 2

Start green 2



Roadblock – Model 2

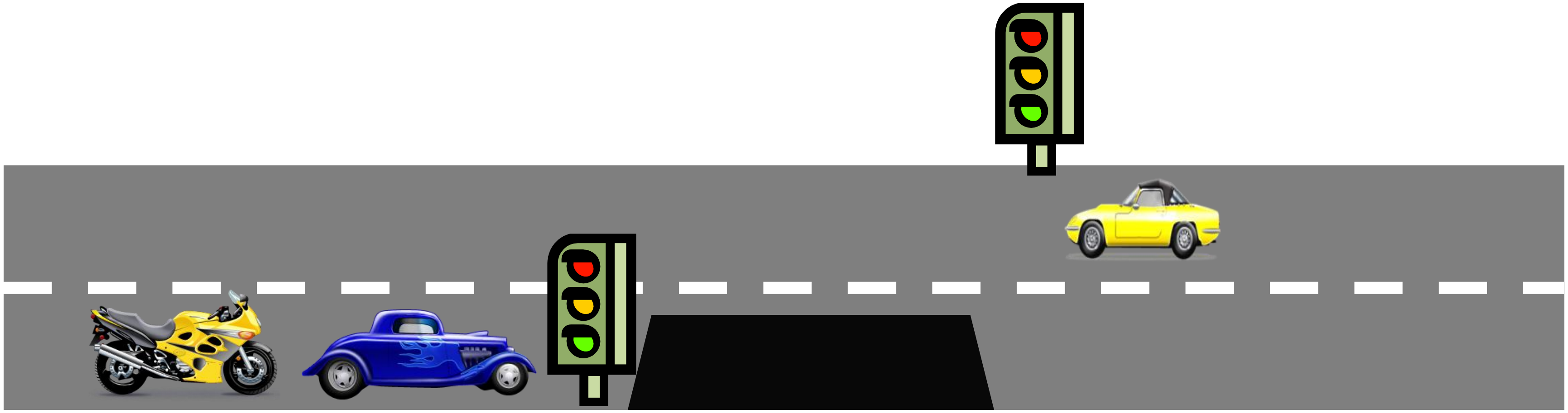


Mean waiting time:

$$E(W_i) = \left(1 - \rho_i\right) E(C_i^{res}), \qquad \text{for } i = 1, 2$$

$$E(W_1) = 14.5, \;\; E(W_2) = 11.6$$

Roadblock – Model 3

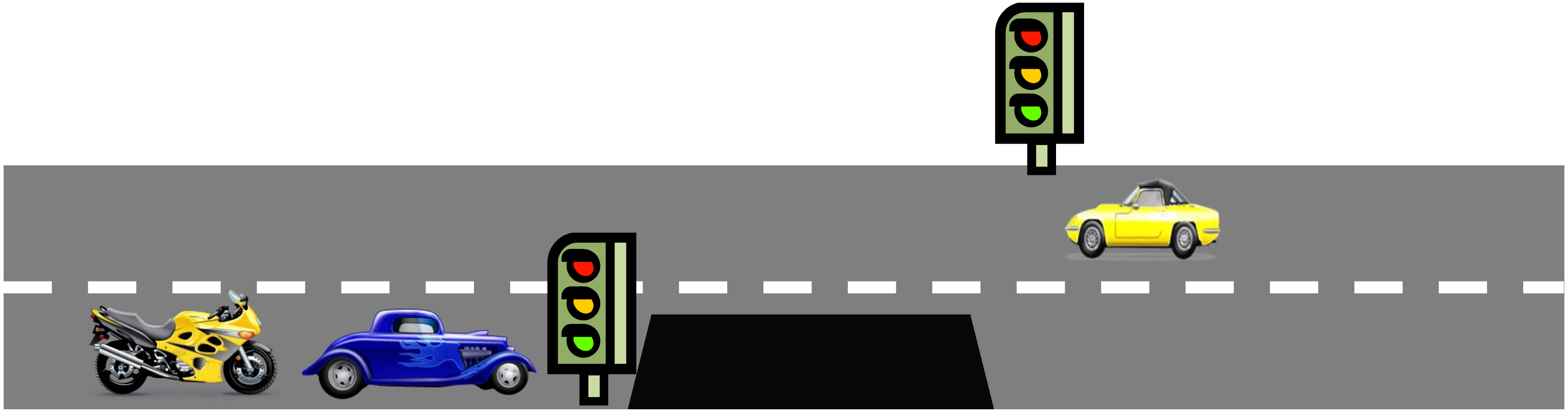


In practice: time limited

Our model: limited number of vehicles may cross

Roadblock – Model 3

Known:



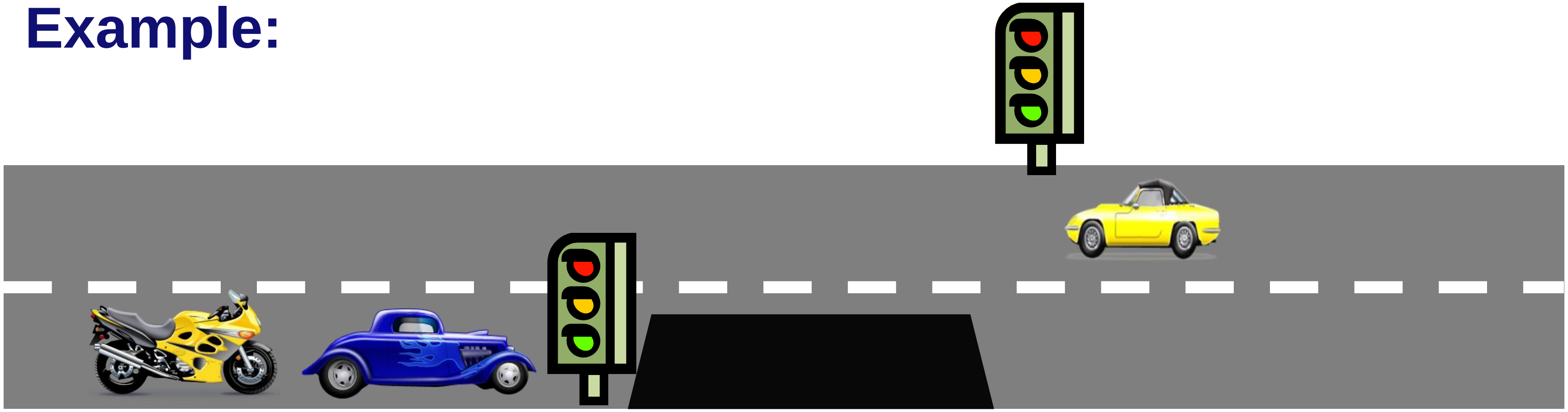
time limited **fixed time**

↑

“vacation model”

Roadblock – Model 3

Example:



λ_1 = 300 vehicles per hour
μ_1 = 1800 vehicles per hour
r_1 = 8 seconds red time
k_1 = max. 4 vehicles per green period

λ_2 = 600 vehicles per hour
μ_2 = 1800 vehicles per hour
r_2 = 8 seconds red time
k_2 = max. 12 vehicles per green period

Roadblock – Model 3

Mean cycle time:

$$E(C) = \frac{r_1 + r_2}{1 - \rho}$$

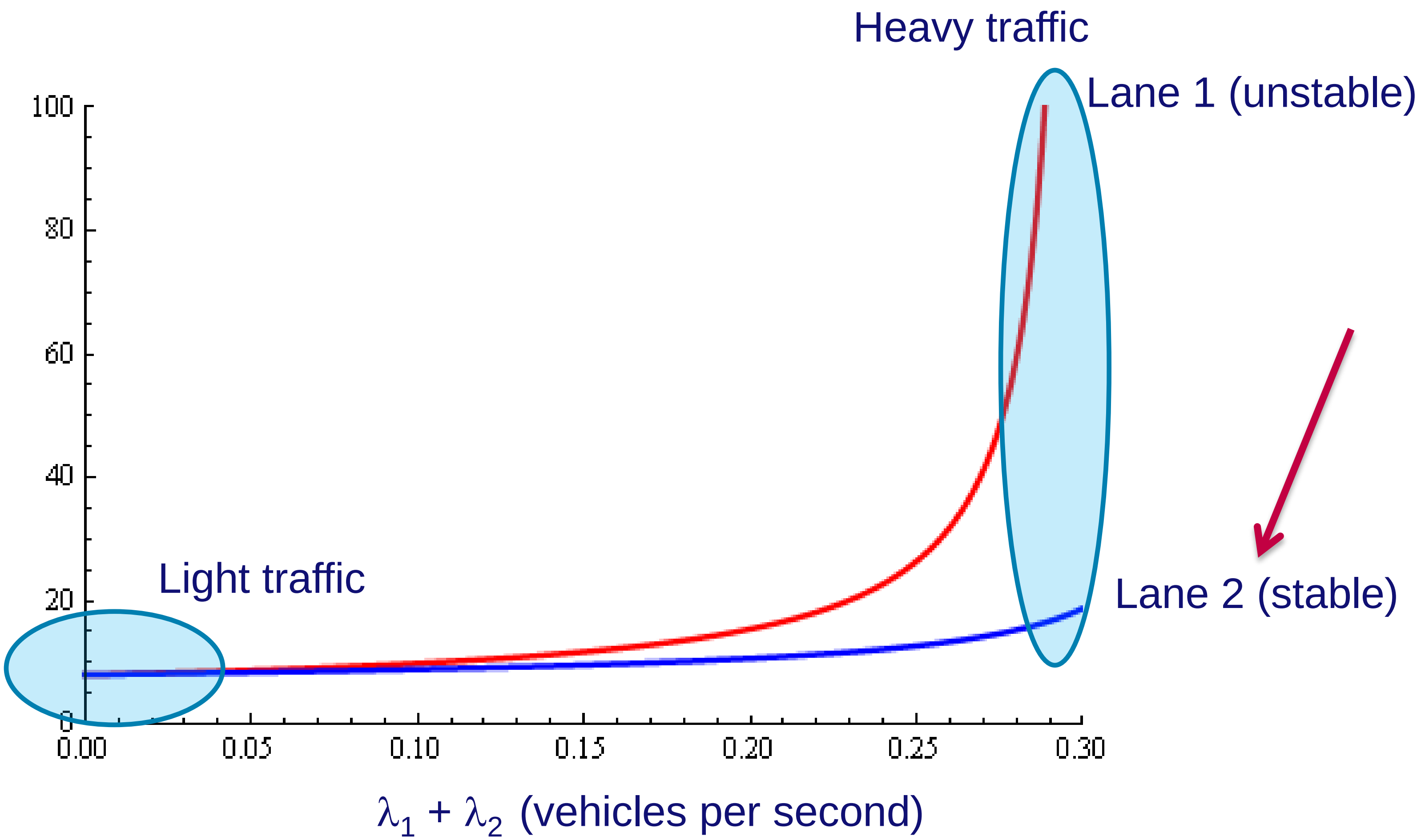
Stability condition:

$$\lambda_i E(C) < k_i$$

for $i = 1, 2$

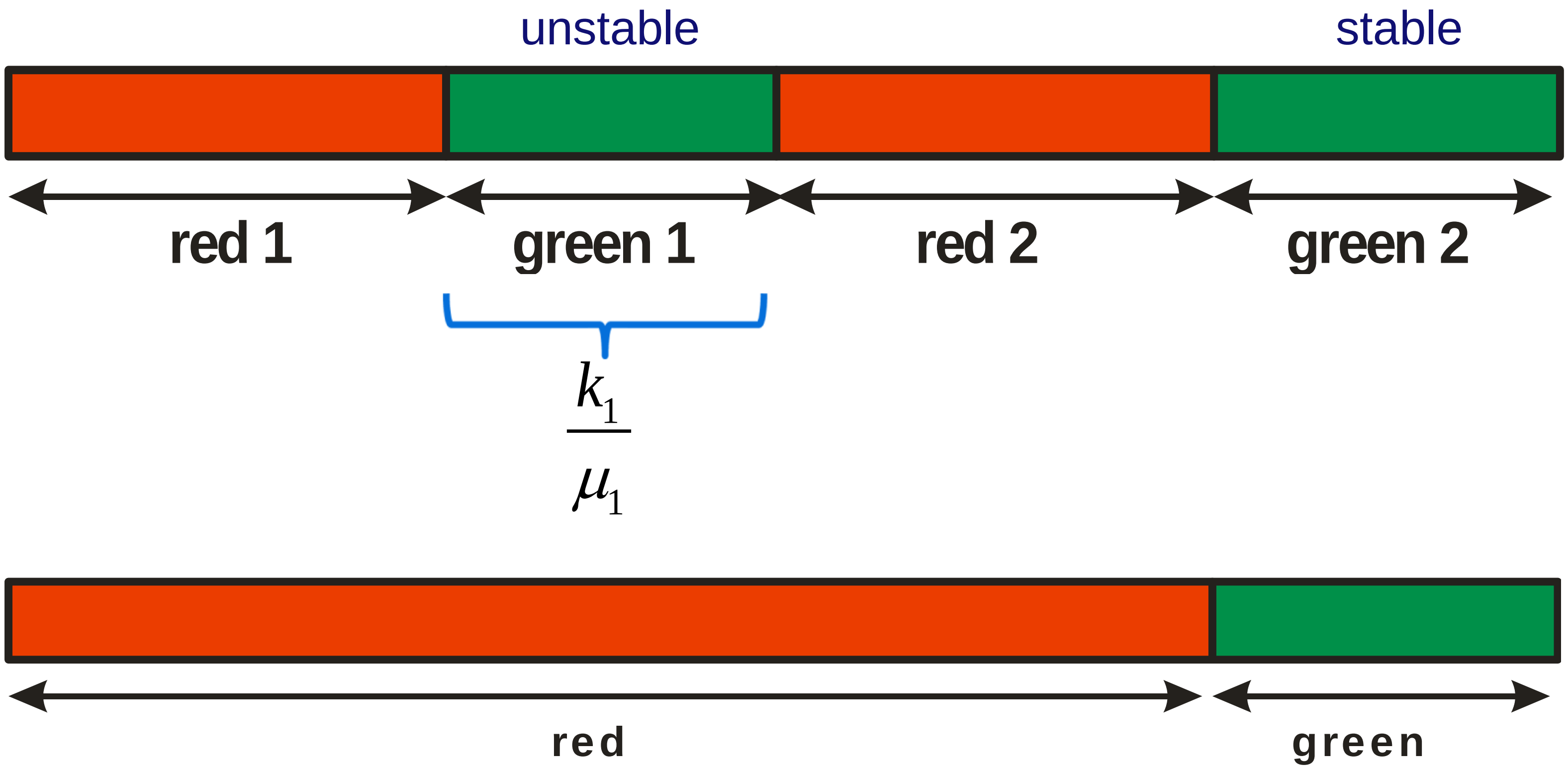
Roadblock – Model 3

Mean waiting times



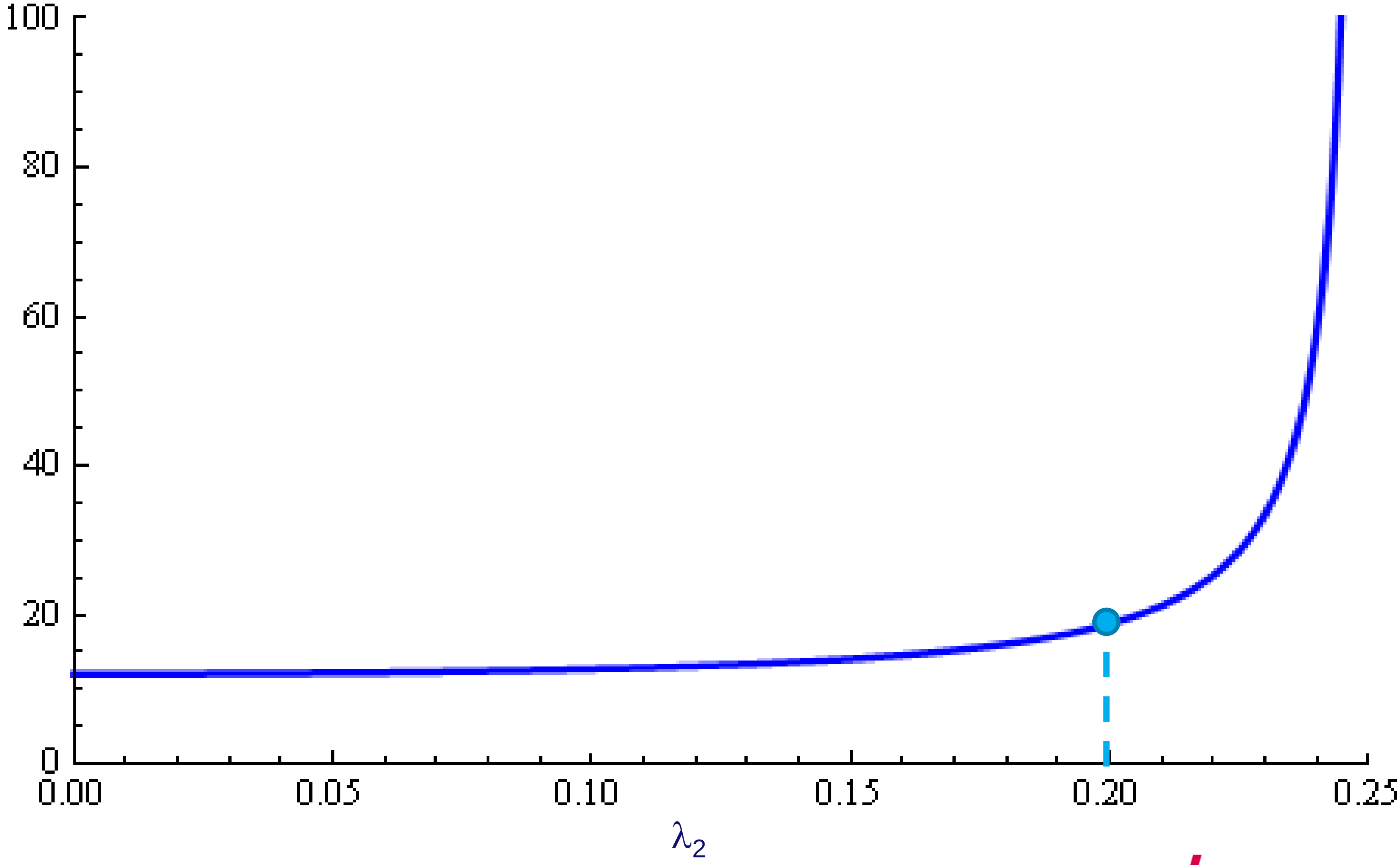
The stable queue

Cycle:

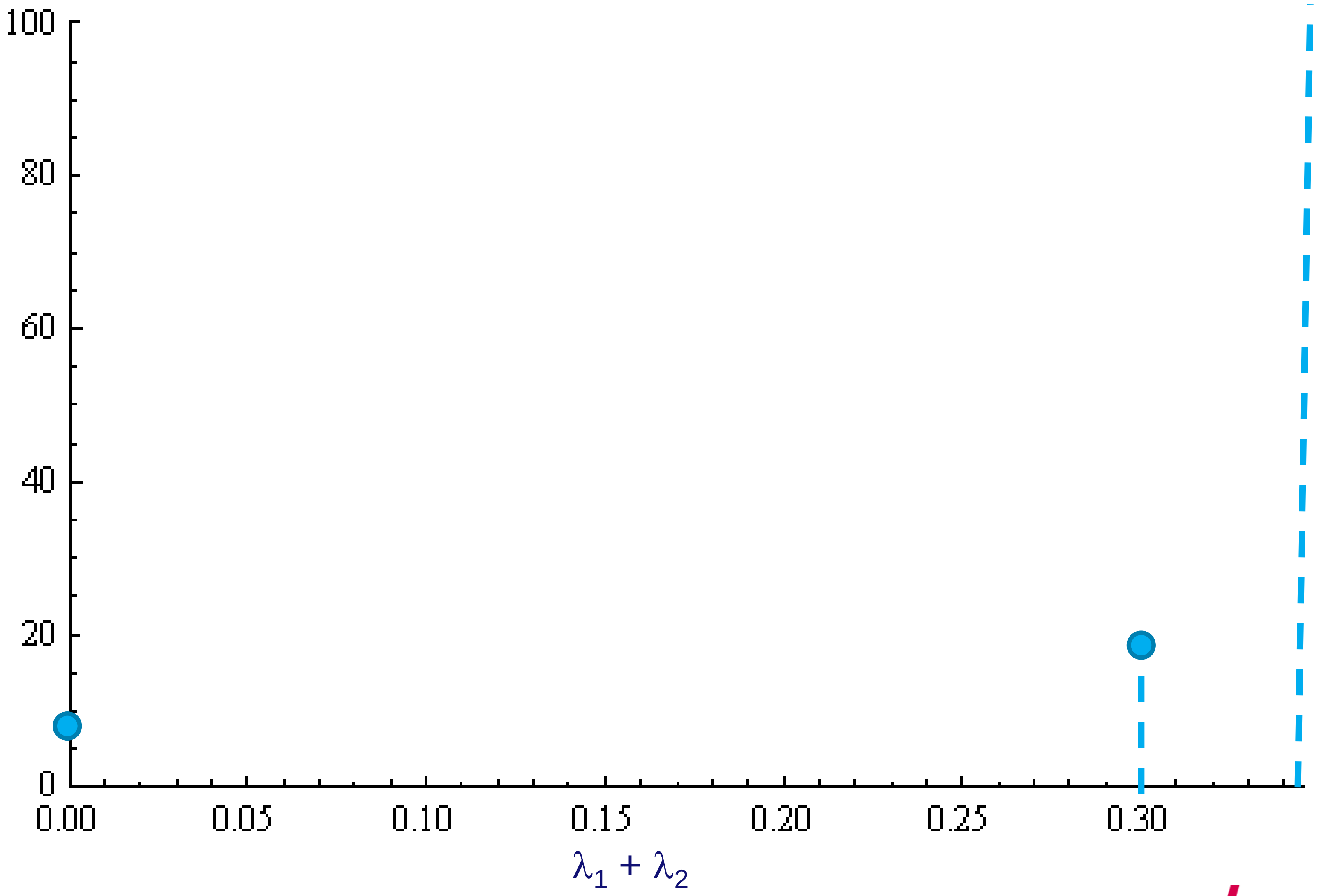


vacation model!

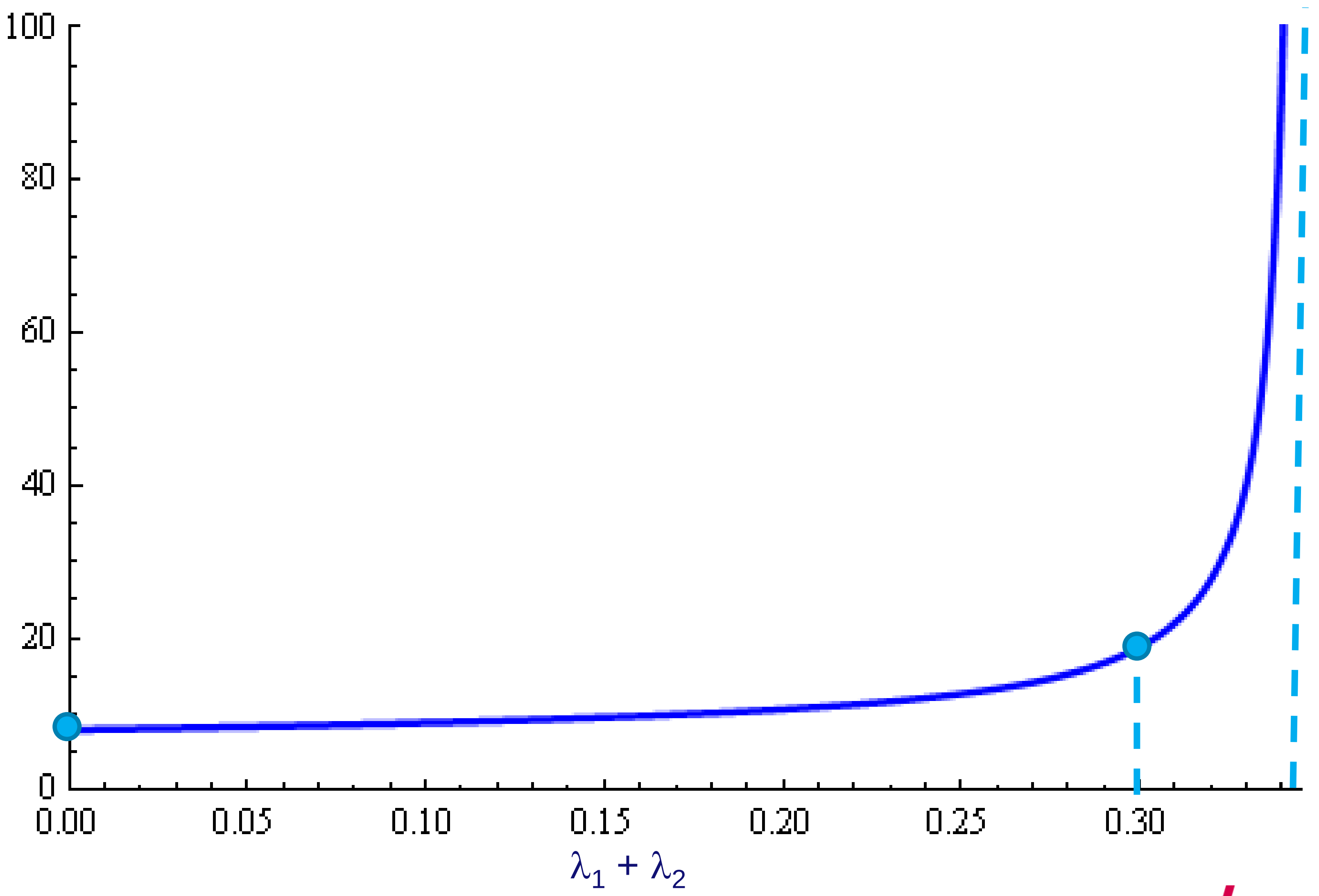
The stable queue: vacation model



The stable queue: approximation



The stable queue: approximation



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The unstable queue

Pseudo-conservation law:

$$\sum_{i=1}^{N} A_i E(W_i) = B + \sum_{i=1}^{N} C_i g_i$$

For two lanes:

$$A_1 E(W_1) + A_2 E(W_2) = B + C_1 g_1 + C_2 g_2$$

get from vacation model

The unstable queue

Pseudo-conservation law:

$$\sum_{i=1}^{N} A_i E(W_i) = B + \sum_{i=1}^{N} C_i g_i$$

For two lanes:

$$A_1 E(W_1) + A_2 E(W_2) = B + C_1 g_1 + C_2 g_2$$

In Heavy Traffic: known

The unstable queue

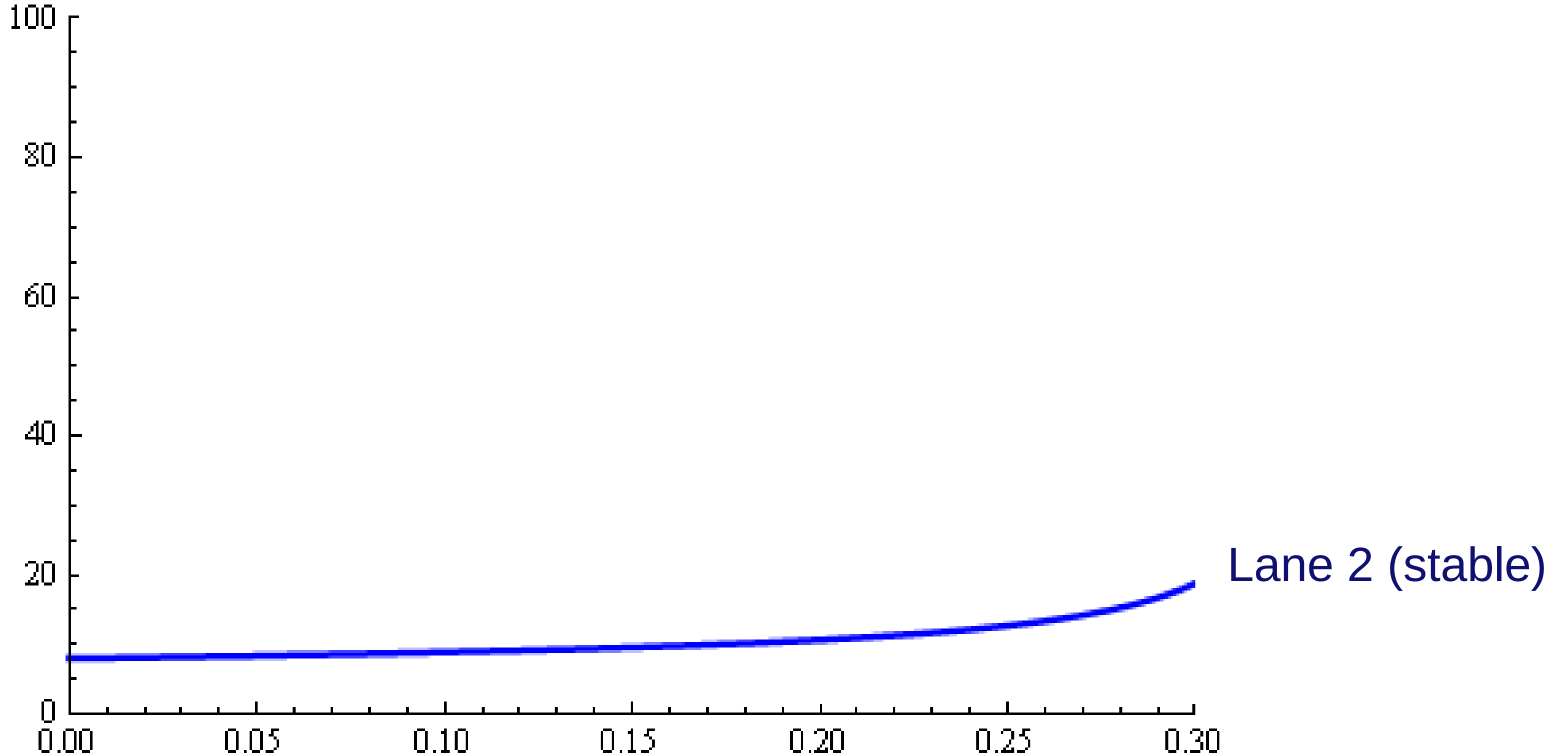
Pseudo-conservation law:

$$\sum_{i=1}^{N} A_i E(W_i) = B + \sum_{i=1}^{N} C_i g_i$$

For two lanes:

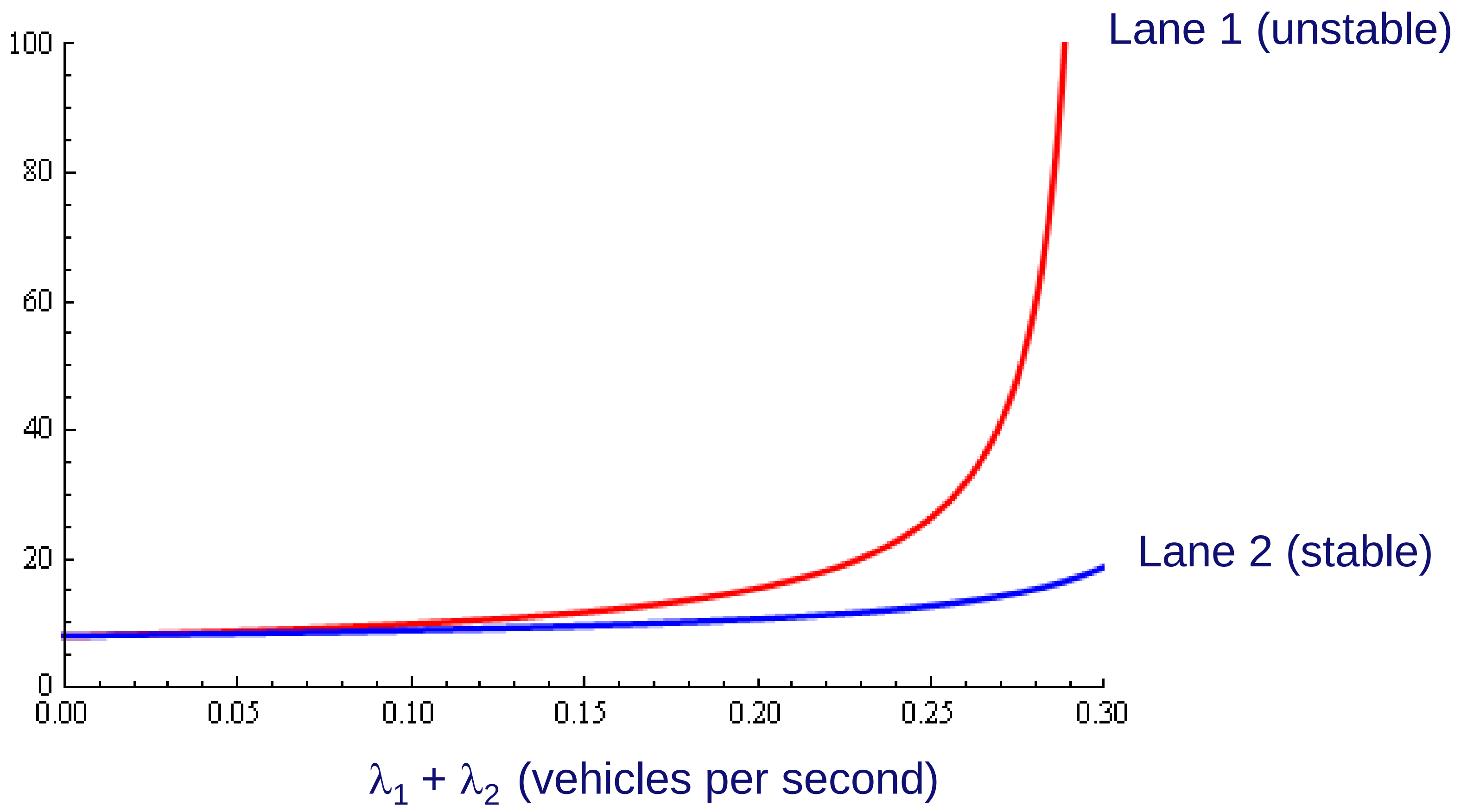
$$A_1 E(W_1) + A_2 E(W_2) = B + C_1 g_1 + C_2 g_2$$

The unstable queue: approximation

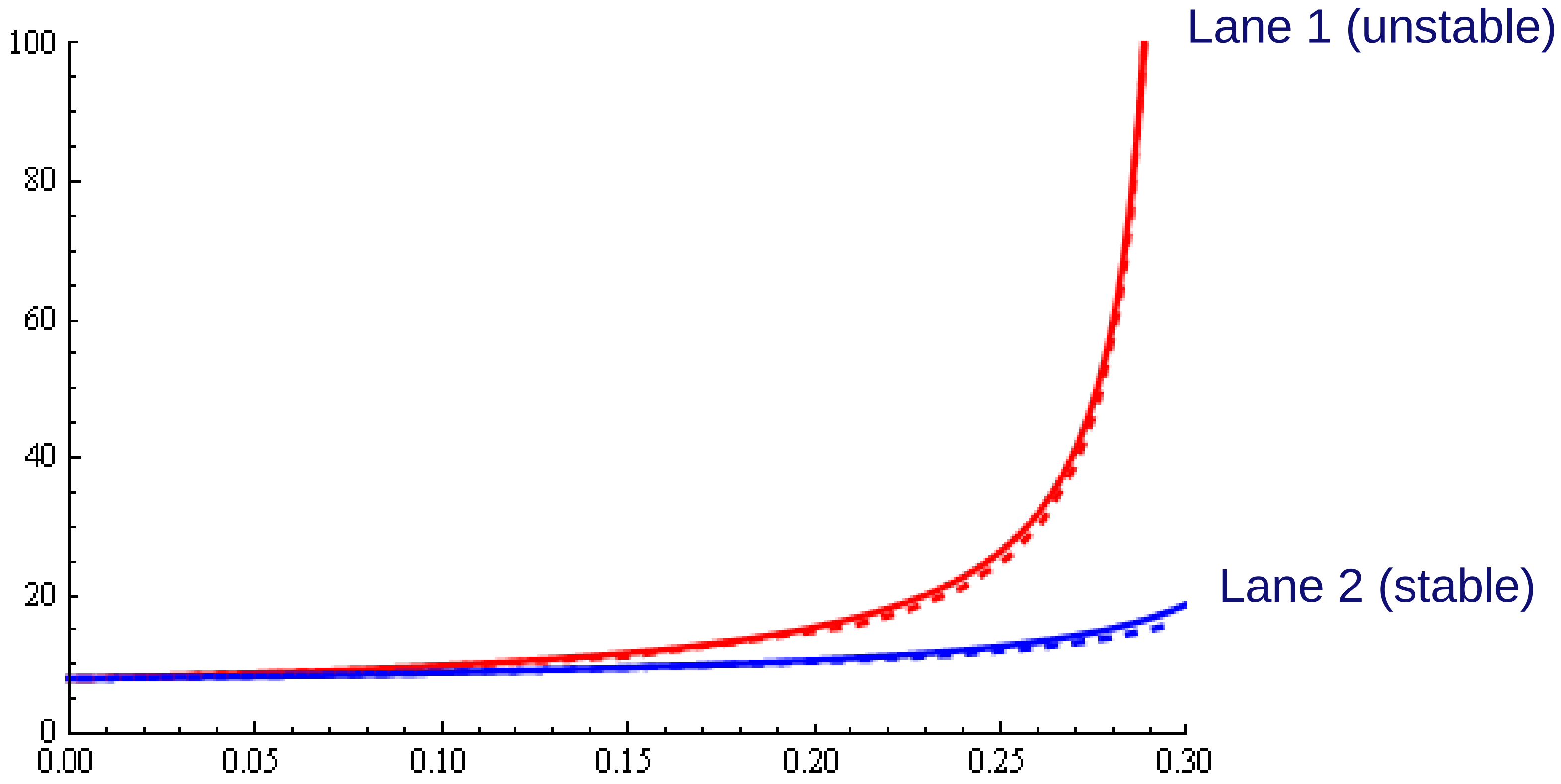


$\lambda_1 + \lambda_2$ (vehicles per second)

The unstable queue: approximation



Comparison with real waiting times



$\lambda_1 + \lambda_2$ (vehicles per second)

Mean waiting times in Example

- **Exhaustive:**

$$E(W_1) = 14.5,\ E(W_2) = 11.6$$

- ***k*-limited (simulated):**

$$E(W_1) = 25.6,\ E(W_2) = 12.1$$

- ***k*-limited (approximated):**

$$E(W_1) = 26.4,\ E(W_2) = 12.7$$

To be continued…

- Compare approximation with existing ones
- Prove HT limits for 2-lane model
(without and with all-red times)
- Improve performance of approximations if utilisation of both queues is almost the same
- Scaling maximum green times
if traffic intensity increases
- Models with more than 2 lanes
- Multiple lanes have green light simultaneously