Performance analysis of zone picking systems

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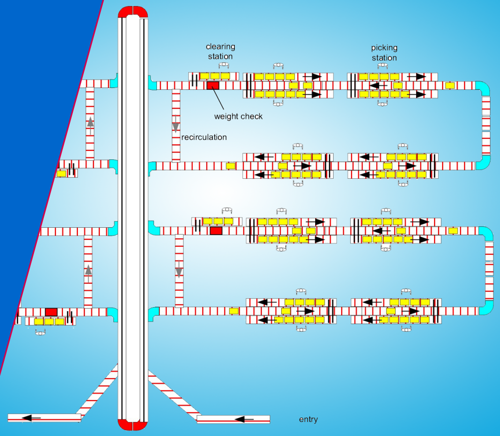

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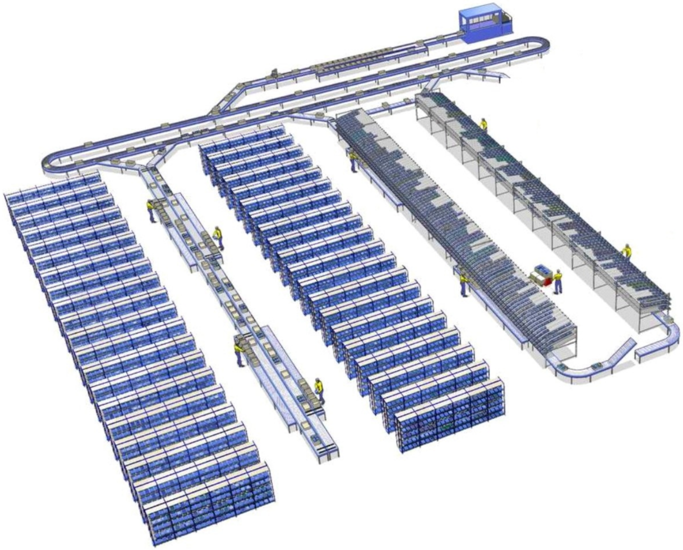
Thursday 29 November

Zone-picking systems



- Popular order-picking system
- Storage area divided in order-picking zones
- Reduction of walking distances and congestion in aisles
- Flexible capacity and high-throughput ability
- Fit-for-use for a wide range of products and order profiles

Motivation

- Develop fast and accurate method to predict performance:
  - utilization of pick stations
  - system throughput
  - order lead time
- Method to support design decisions:
  - layout of the network
  - size of zones
  - location of items
  - number of pickers and zones
  - WIP level

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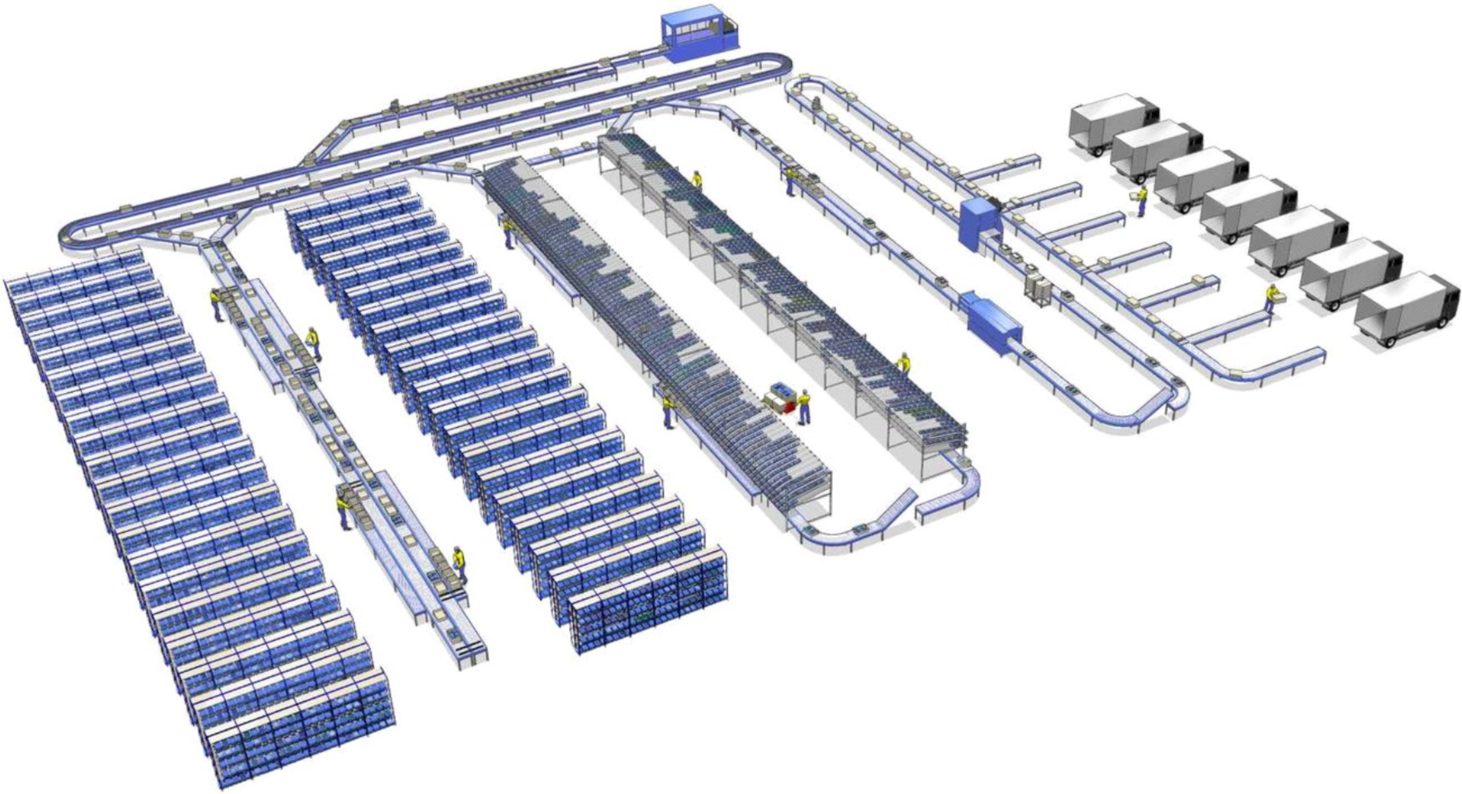
Zone-picking systems



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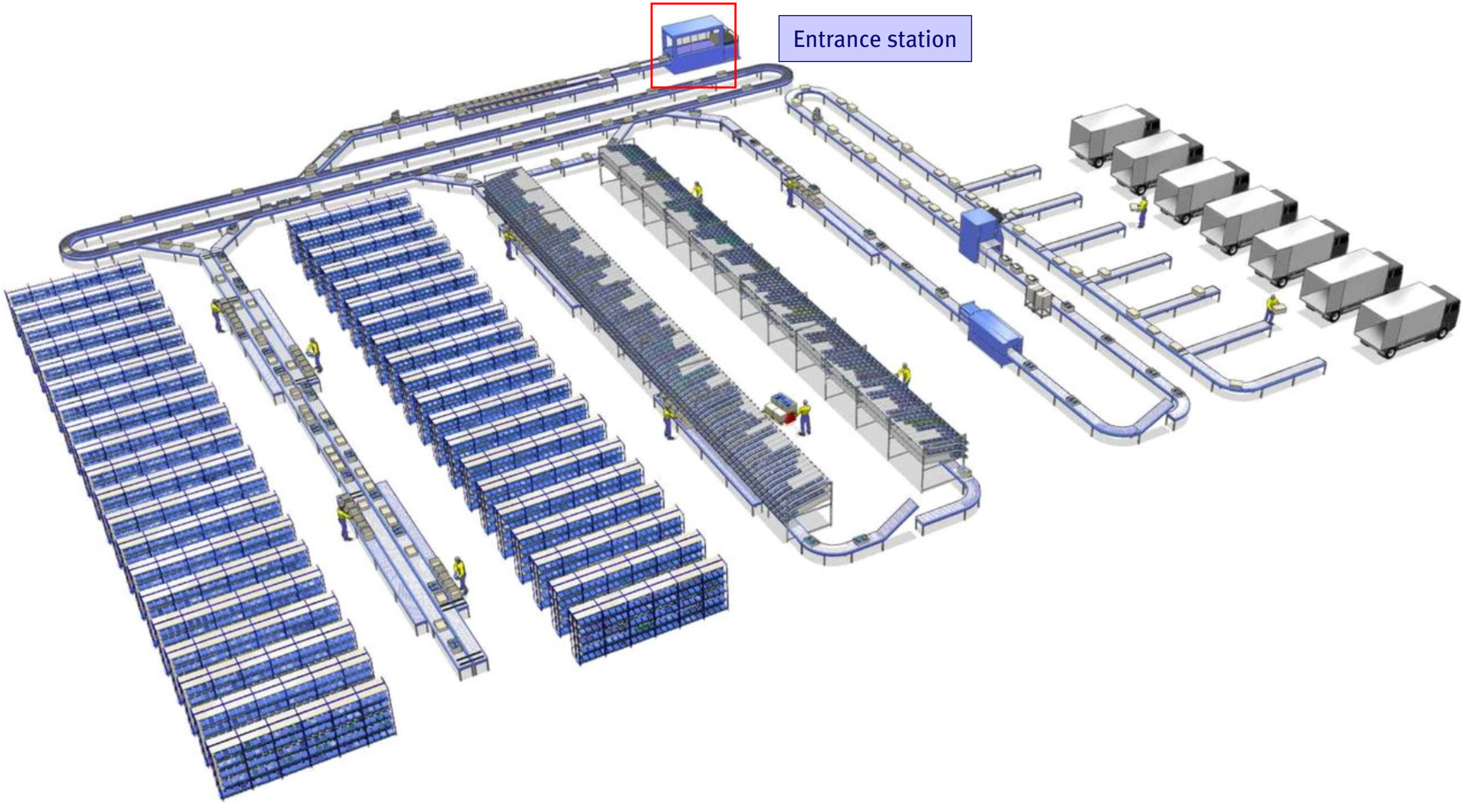
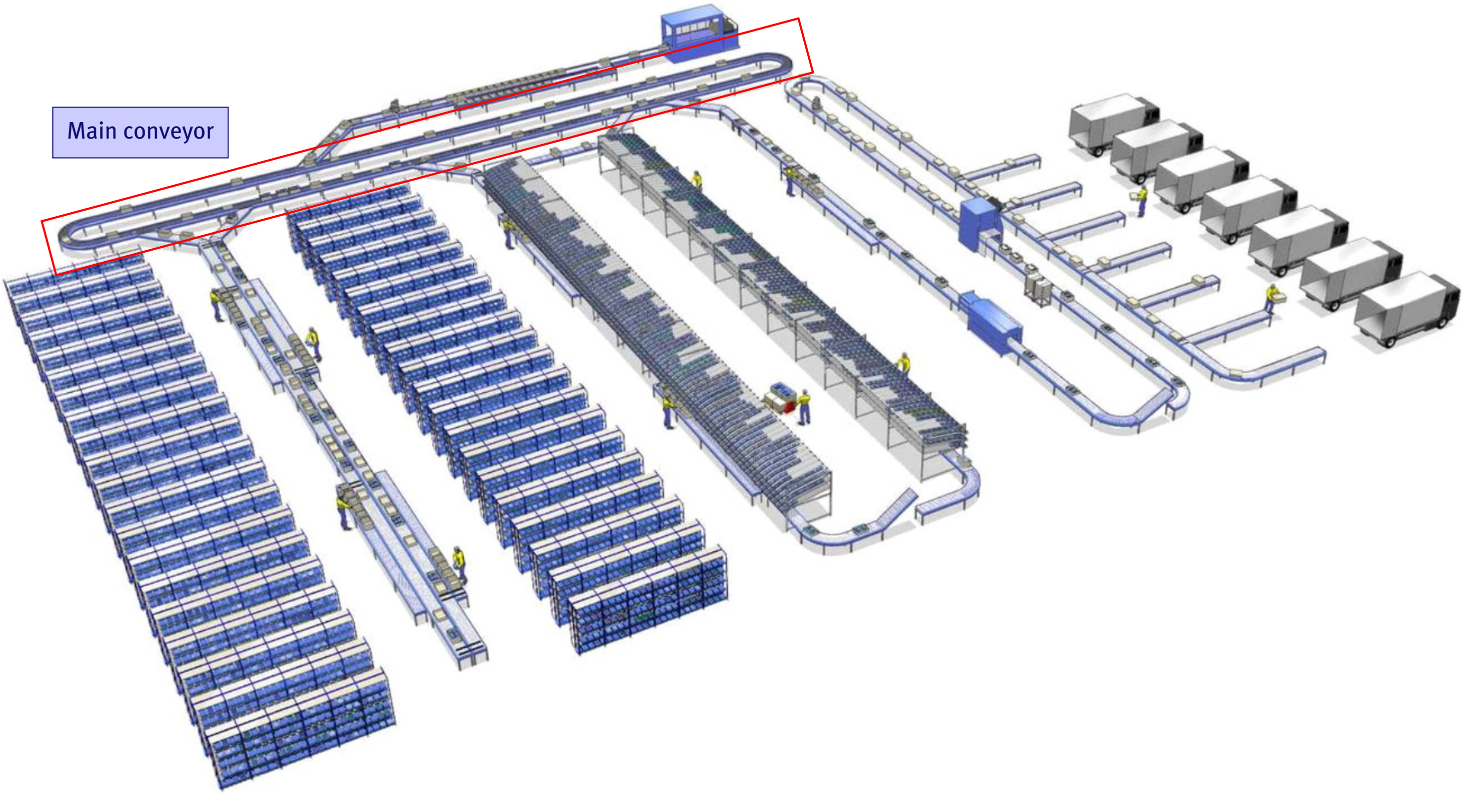
Zone-picking systems



Zone-picking systems



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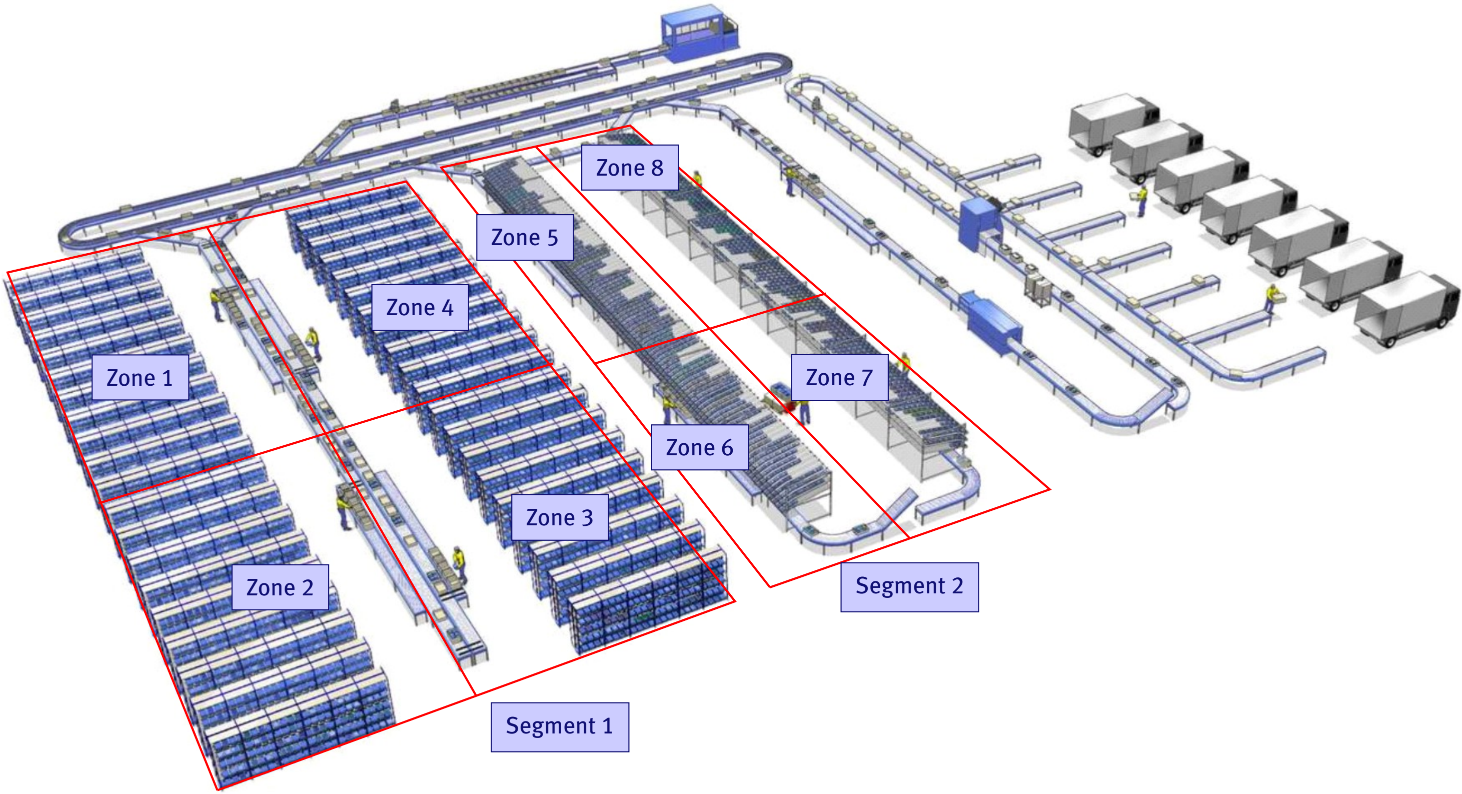
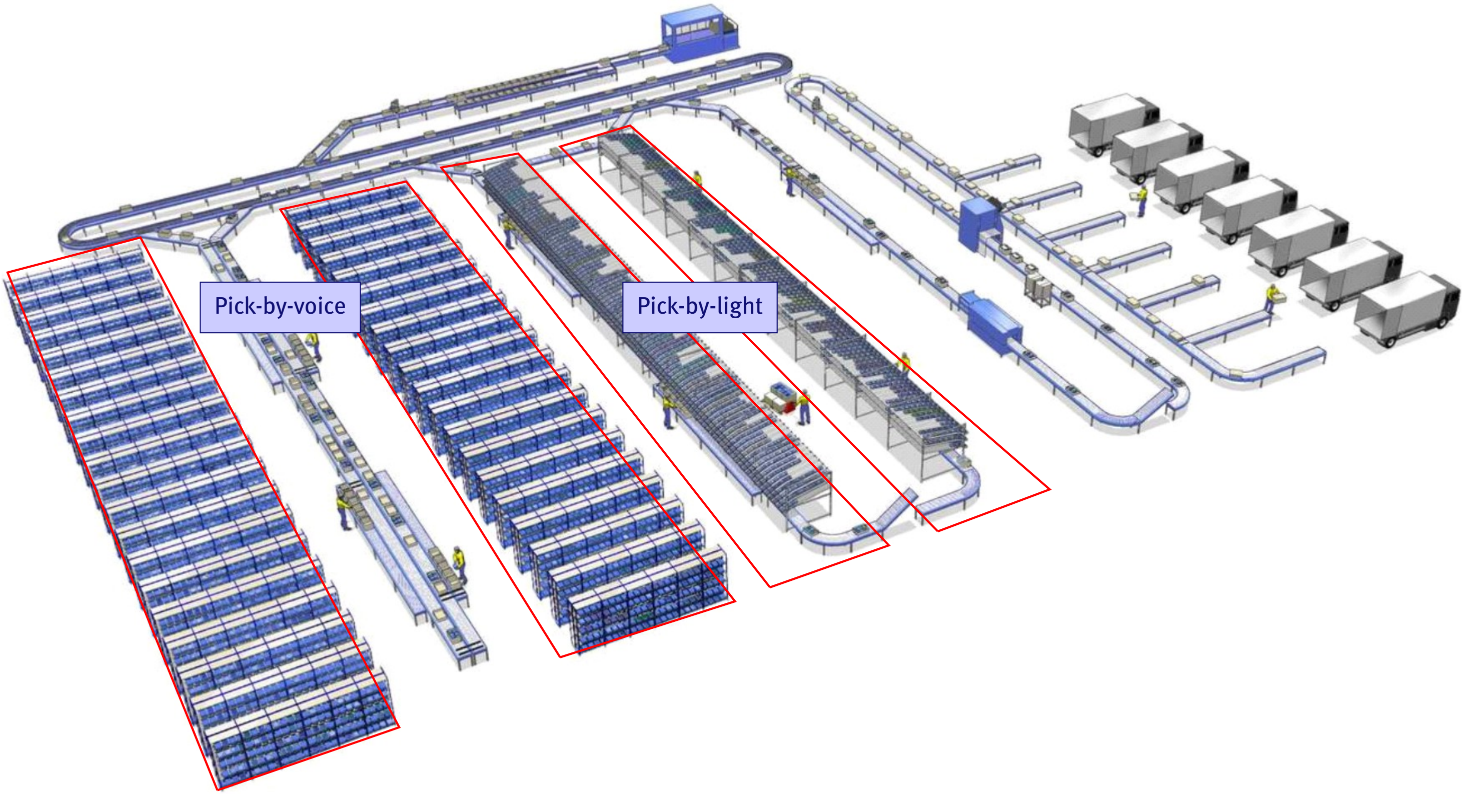
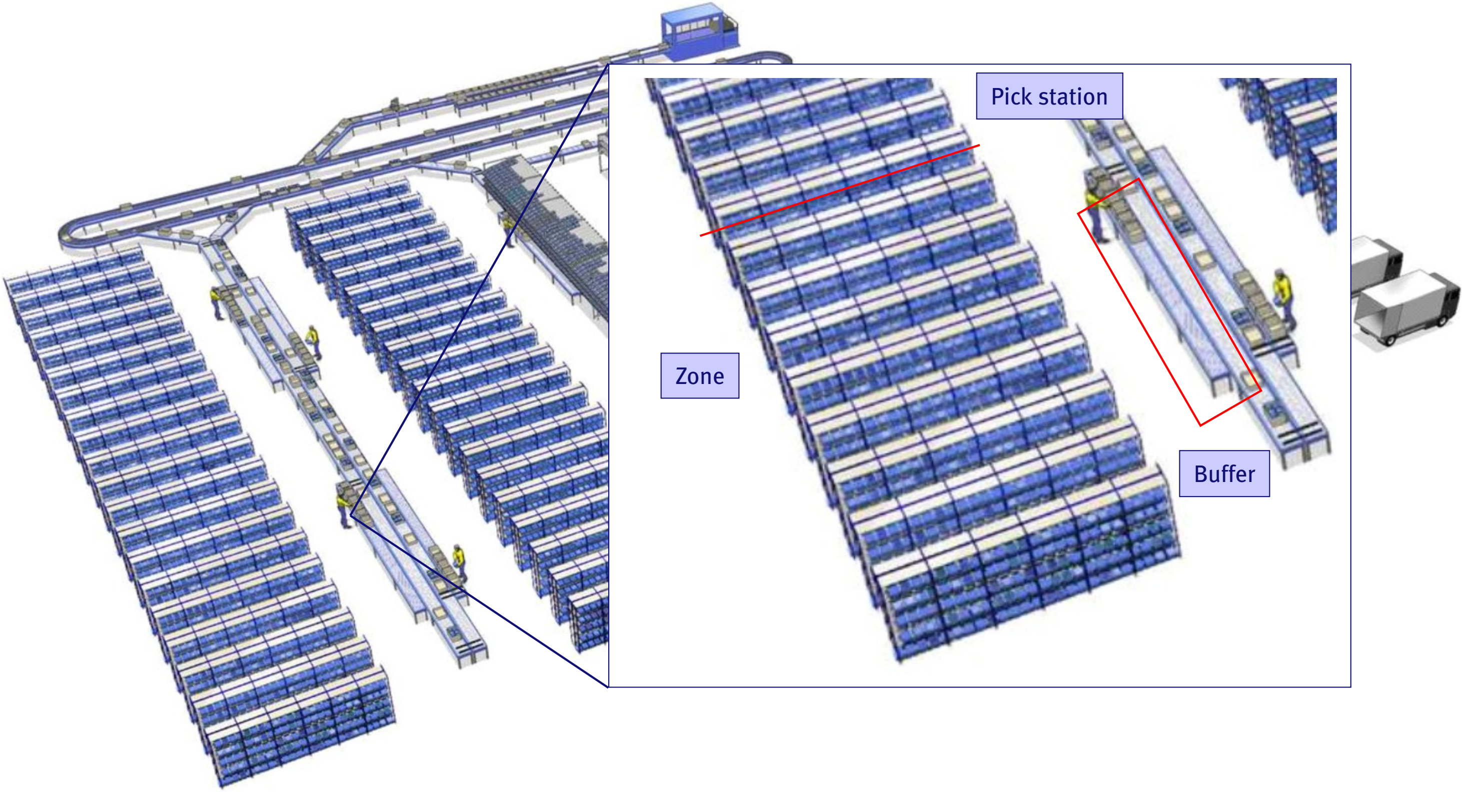


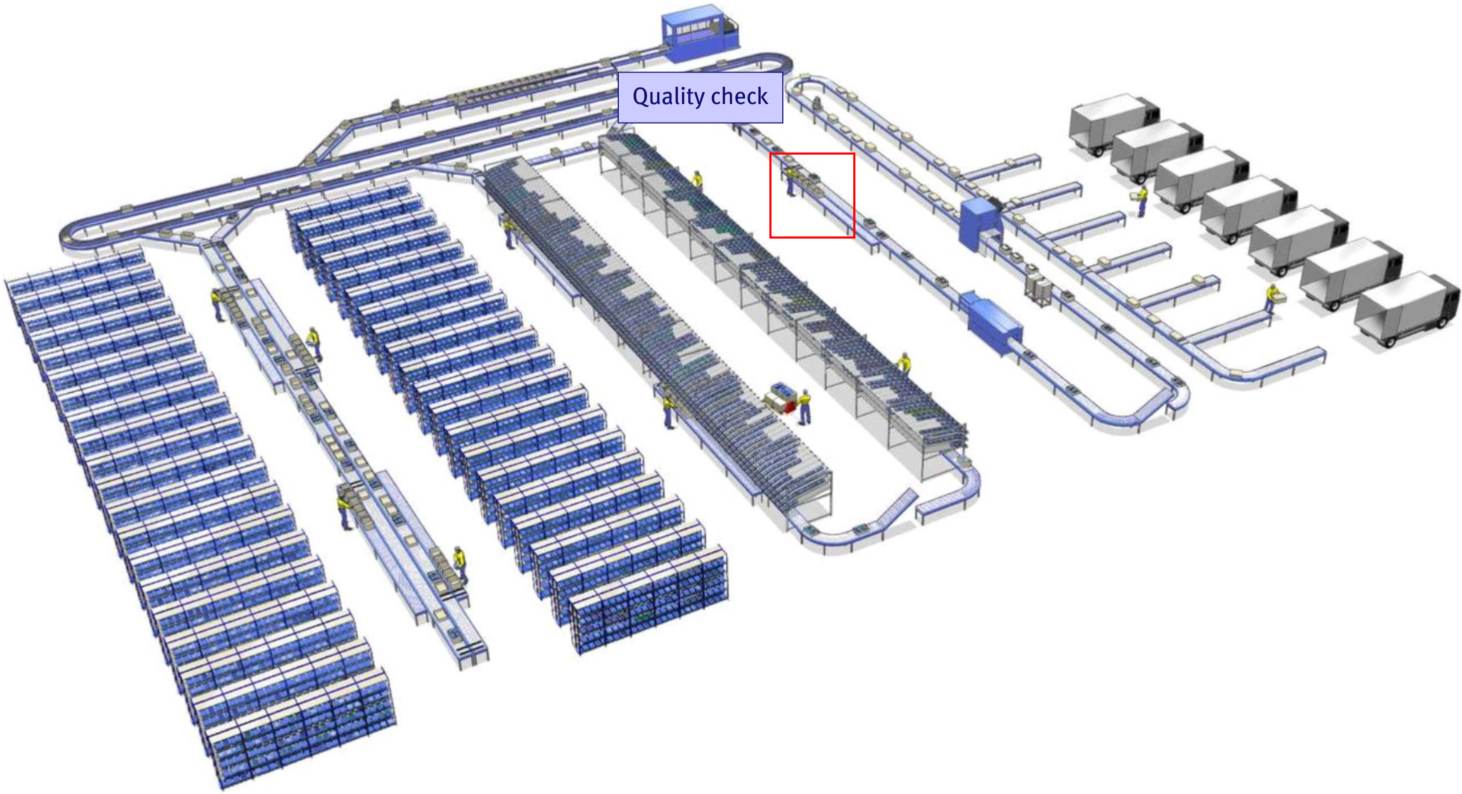
Zone-picking systems



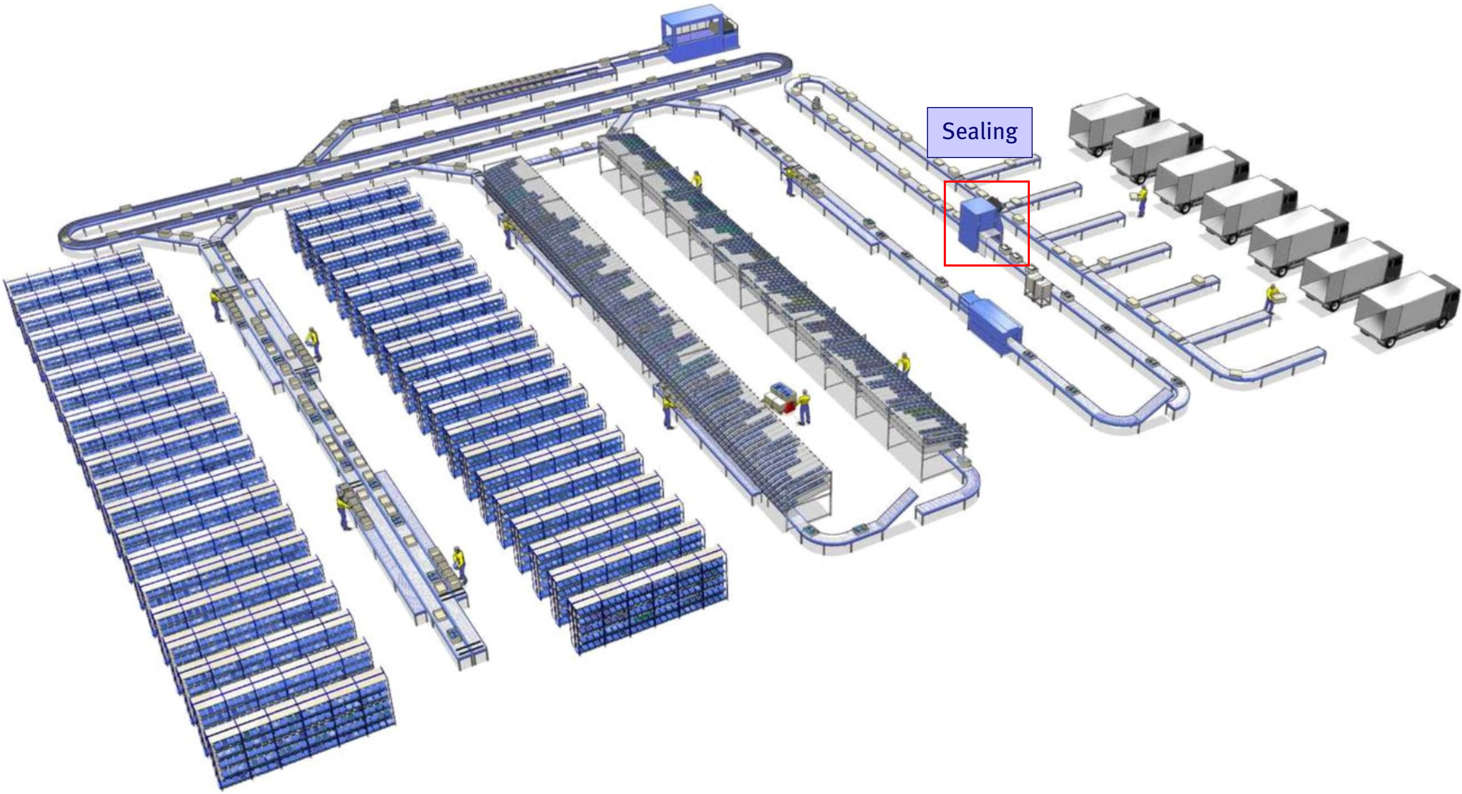
Zone-picking systems



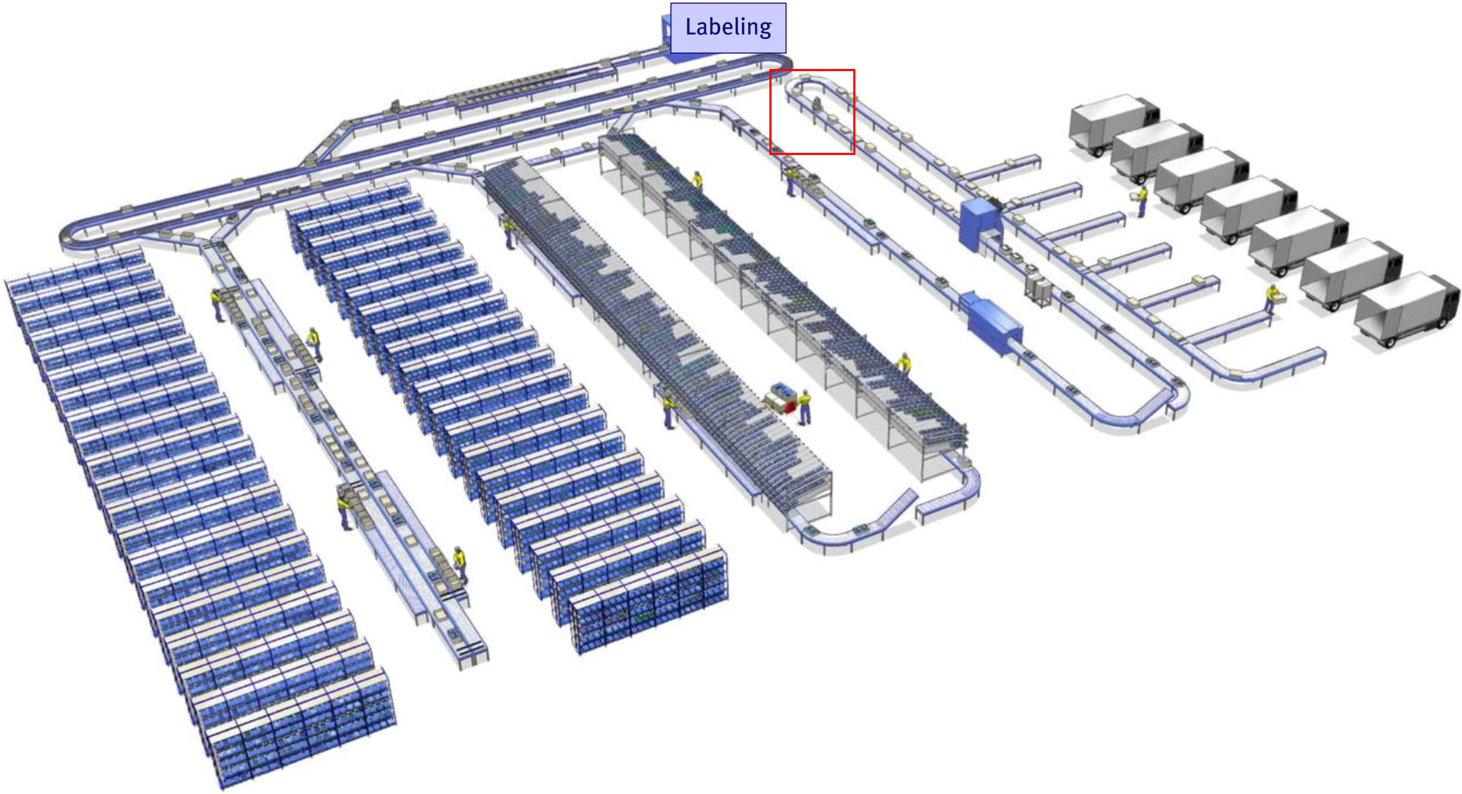
Zone-picking systems



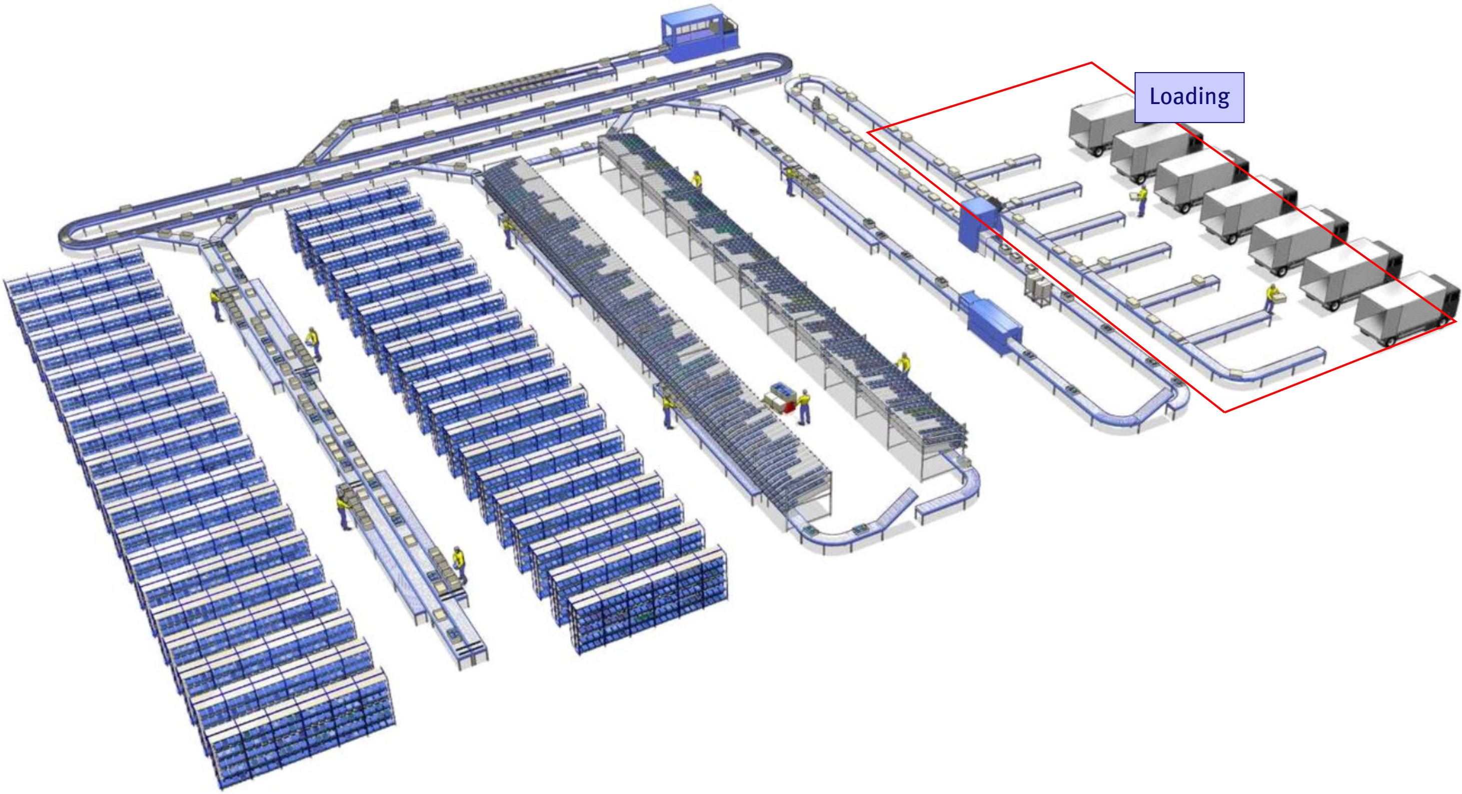
Zone-picking systems



Zone-picking systems



Zone-picking systems



Zone-picking systems

Disadvantage:

- congestion and blocking under heavy use
- leads to recirculation and long order lead times

Modeling:

- blocking is crucial aspect!
- describe elements (transport, zones) as network of queues

Method of analysis: queueing theory

Needed?

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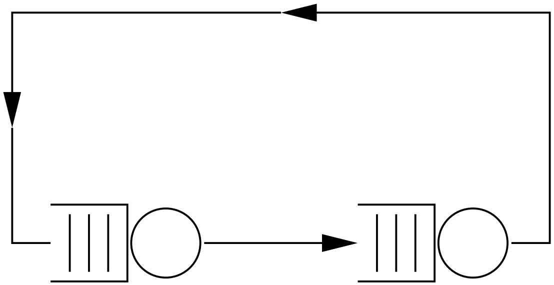
Some improvements?

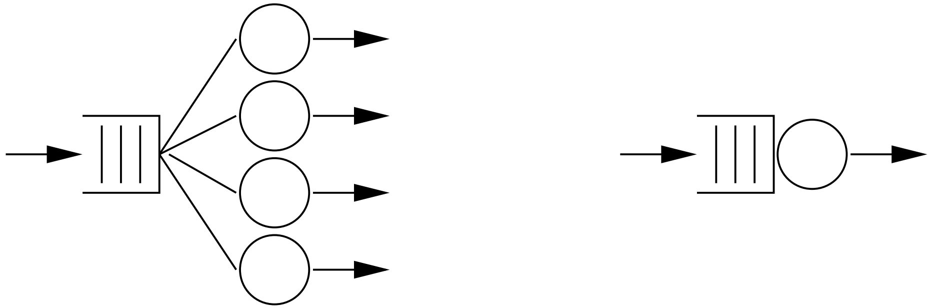


Pickers are equally fast, 10 circulating totes

Question: Replace one picker by a picker that is twice as fast.
How does this affect mean order lead time? Throughtput?

Question: Does your answer change in case of more totes? Less totes?

Multiple pickers or a single one?



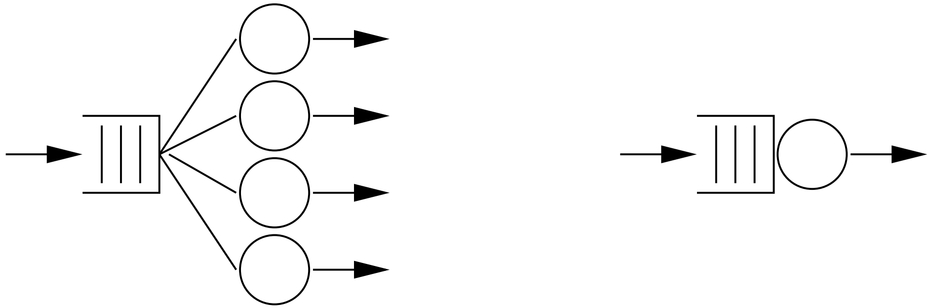
4 pickers, or one picker that is four times faster?

Question: What do you prefer, 4 pickers or one fast picker?

Question: What do you prefer if pick time variability is high?

Question: What do you prefer if the load is low?

Multiple pickers or a single one?

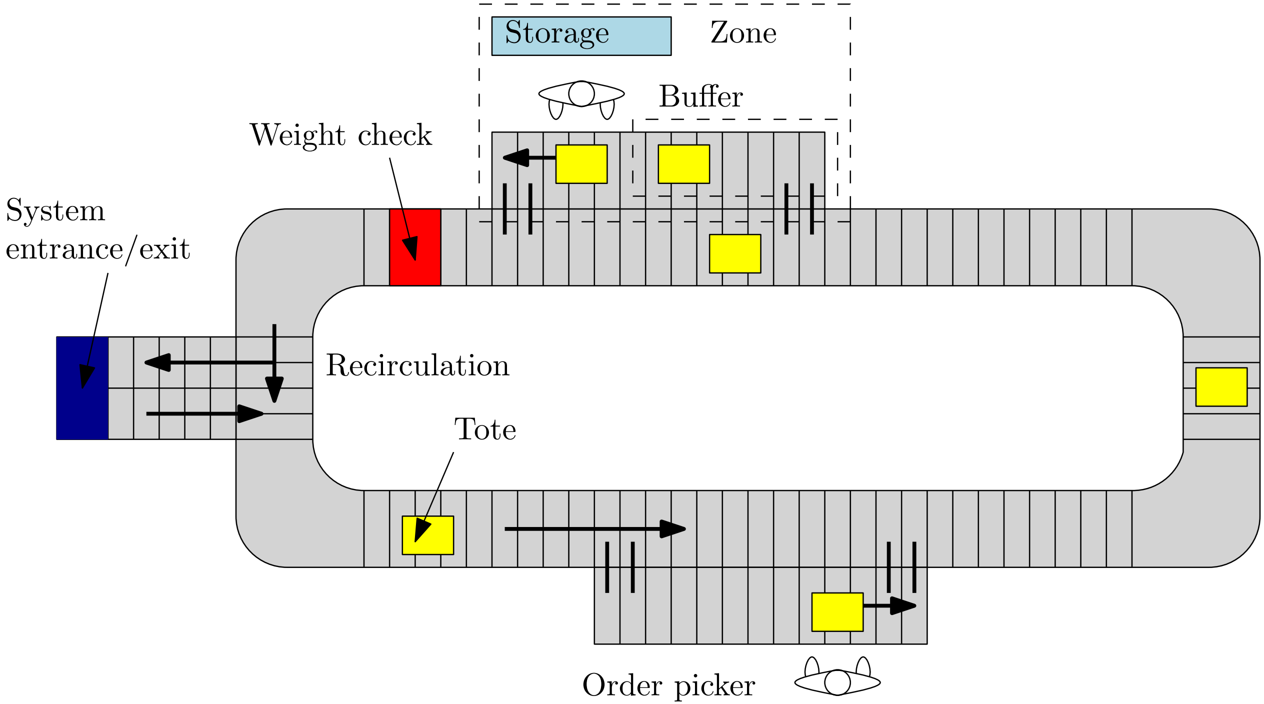


4 pickers, or one picker that is four times faster?

The mean order lead time can be predicted by...

$$E(S) \approx \frac{\Pi_W}{1-\rho}\frac{E(R)}{c} + E(B)$$

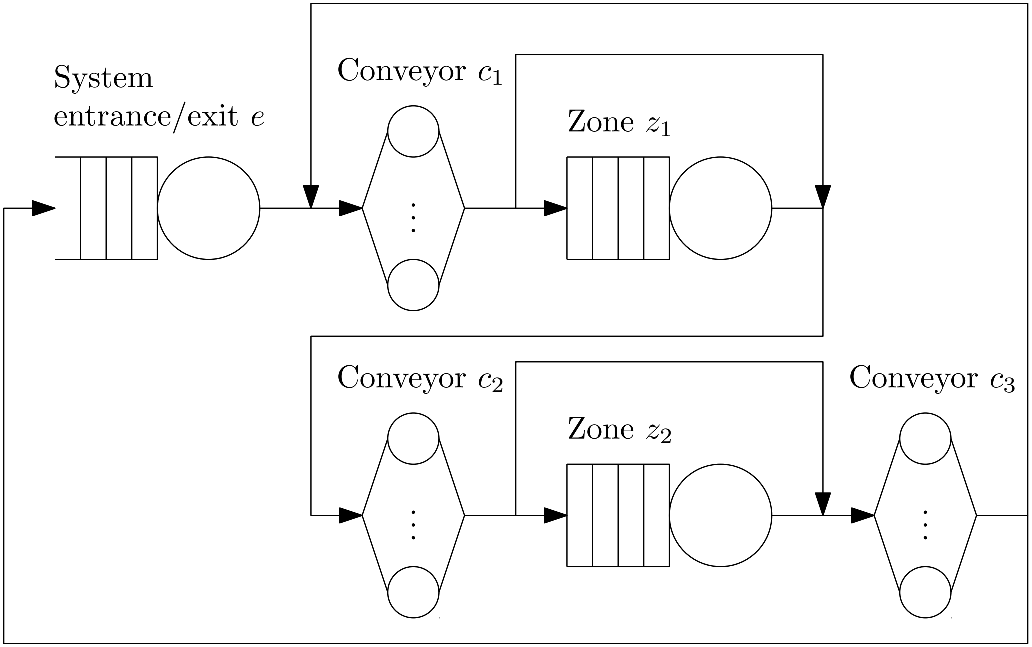
Layout of single-segment



Modeling of single-segment

- N is number of totes
- M is number of zones
- $\mathcal{S}$ is set of nodes; three types
  1. Entrance/exit, e
  2. Zones, $\mathcal{Z} = \{z_1, \ldots, z_M\}$
  3. Conveyors, $\mathcal{C} = \{c_1, \ldots, c_{M+1}\}$
- Each tote has class $\boldsymbol{r} \subseteq \mathcal{Z}$ of zones to be visited, for example, $\boldsymbol{r} = \{z_2, z_3\}$

Modeling of single-segment



Closed queueing network with $\mathcal{C} = \{c_1, c_2, c_3\}$ and $\mathcal{Z} = \{z_1, z_2\}$

Modeling of single-segment

- Entrance node releases new totes one-by-one of class $\boldsymbol{r}$ with probability $\psi_{\boldsymbol{r}}$ at exponential rate μ_e
- Conveyor nodes are delay nodes with a fixed delay of rate μ_i
- Zones have:
  - d_i (≥ 1) order pickers
  - Exponential pick times with rate μ_i
  - Finite buffers of size q_i

Analysis of single-segment

- Distribution of network is intractable: Approximate!
- tote jumps over full zone and proceeds as if zone has been visited...
- Jump-over network has product-form solution!

- Flows of jump-over network should match with block-and-recirculate:

  passing tote is labeled z_i not visited with probability b_{z_i} and labeled z_i visited otherwise, independent of whether the tote visited z_i or not

- b_{z_i} is blocking probability in block-and-recirculate network: Unknown!

Analysis of single-segment

Theorem:

Jump-over network has product-vorm stationary distribution:

$$\pi\left(\bar{x}\right) = \frac{1}{G} \prod_{i \in \mathcal{S}} \left(\frac{V_i}{\mu_i}\right)^{\bar{x}_i} \prod_{i \in \mathcal{C}} \frac{1}{\bar{x}_i!} \prod_{i \in \mathcal{Z}} \frac{1}{\gamma_i\left(\bar{x}_i\right)}$$

where

- $\bar{x}_i$ number of totes in node i
- G is normalizing constant
- V_i visiting frequency to node i
- γ_i is (queue dependent) service rate multiplier

Hence:

Jump-over netwerk can be exactly evaluated by Mean Value Analysis (MVA)

Analysis of single-segment

Arrival theorem for closed queueing network:

Blocking probrability b_{z_i} of zone z_i is equal to:

$$b_{z_i} = \pi_{z_i}\left(d_{z_i} + q_{z_i} | N - 1\right),$$

where $\pi_{z_i}(k|N)$ probability of k totes in zone z_i in network with N totes

Remark:
Probabilities $\pi_{z_i}(k|N)$ can be calculated recursively (over N) by MVA

Analysis of single-segment

- Step 0:
  Initialize $b_{z_i}^{(0)} = 0$ and $j = 0$
- Step 1:
  Calculate by means of MVA:
  1. Mean order lead times and throughput
  2. Distribution of totes per node
- Step 2:
  $j = j + 1$ and estimate new blocking probabilities
  $b_{z_i}^{(j)} = \pi_{z_i}\left(d_{z_i} + q_{z_i}|N - 1\right)$
- Step 3:
  Return to Step 1 until $\left|b_{z_i}^{(j)} - b_{z_i}^{(j-1)}\right| < \epsilon$

Results for single-segment

Parameters single-segment test set (9600 cases)

Name	Parameter
Number of zones	1,2,3,4,5,6,7,8
Number of totes	10,20,30,40,50,60,70,80
Transport mean of conveyors	20,30,40,50,60
Service mean of zones	10,15,20,25,30
Buffer size of zones	0,1
Number of order pickers	1,2,3

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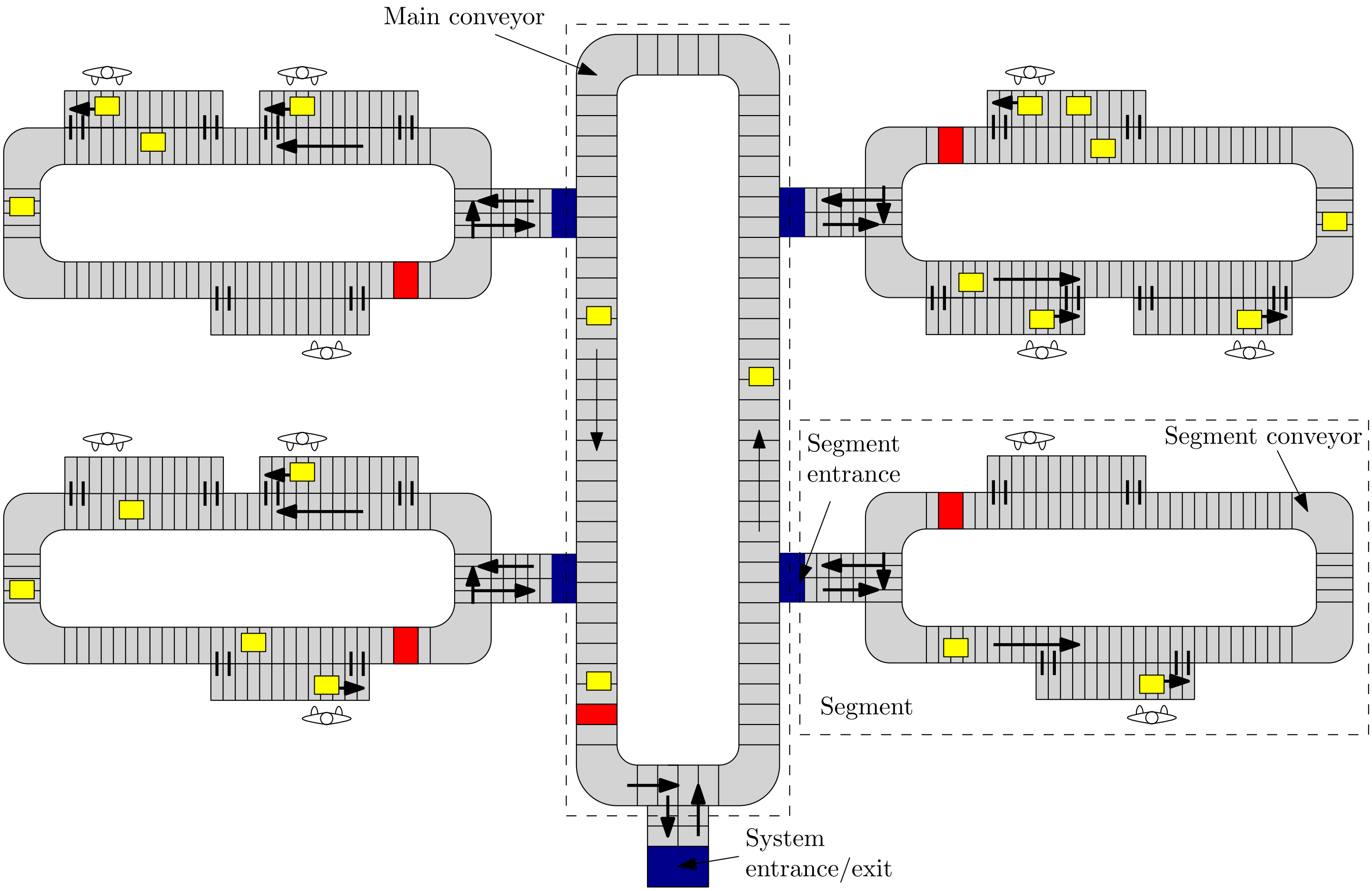
Results for single-segment

Zones	Error (%) in system throughput					Error (%) in circulation				
	Avg.	SD.	$0-1$	$1-5$	>5	Avg	SD	$0-1$	$1-5$	>5
1	0.08	0.08	100.0	0.0	0.0	0.08	0.11	100.0	0.0	0.0
2	0.67	0.84	70.0	29.8	0.2	0.78	1.23	69.0	29.8	1.3
3	0.78	1.03	68.2	31.7	0.2	0.94	1.48	67.2	30.3	2.5
4	0.73	1.05	71.9	27.8	0.3	0.90	1.52	71.3	25.9	2.8
5	0.64	1.00	76.6	23.3	0.2	0.80	1.45	75.0	22.4	2.6
6	0.54	0.91	80.4	19.5	0.1	0.68	1.33	78.6	18.9	2.5
7	0.45	0.81	83.8	16.2	0.0	0.57	1.20	82.4	15.8	1.8
8	0.38	0.71	86.7	13.3	0.0	0.48	1.07	85.2	13.5	1.3

Layout of multi-segment



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Modeling of multi-segment

- K is number of segments
- N is number of totes
- N^k is maximum number of totes in segment k
- M is number of zones
- $\mathcal{S}$ is set of nodes; three types
  1. Entrance/exit nodes, $\mathcal{E} = \{e_0, e_1, \ldots, e_K\}$
  2. Zones, $\mathcal{Z} = \cup_{k=1}^{K} \mathcal{Z}^k$, $\mathcal{Z}^k = \left\{z_1^k, \ldots, z_{m^k}^k\right\}$
  3. Conveyors, $\mathcal{C} = \cup_{k=0}^{K} \mathcal{C}^k$,
     $\mathcal{C}^0 = \left\{c_1^0, \ldots, c_{K+1}^0\right\}$ and $\mathcal{C}^k = \left\{c_1^k, \ldots, c_{m^k+1}^k\right\}$
- Each tote has class $\boldsymbol{r} \subseteq \mathcal{Z}$ of zones to be visited, for example, $\boldsymbol{r} = \left\{z_2^1, z_3^2\right\}$

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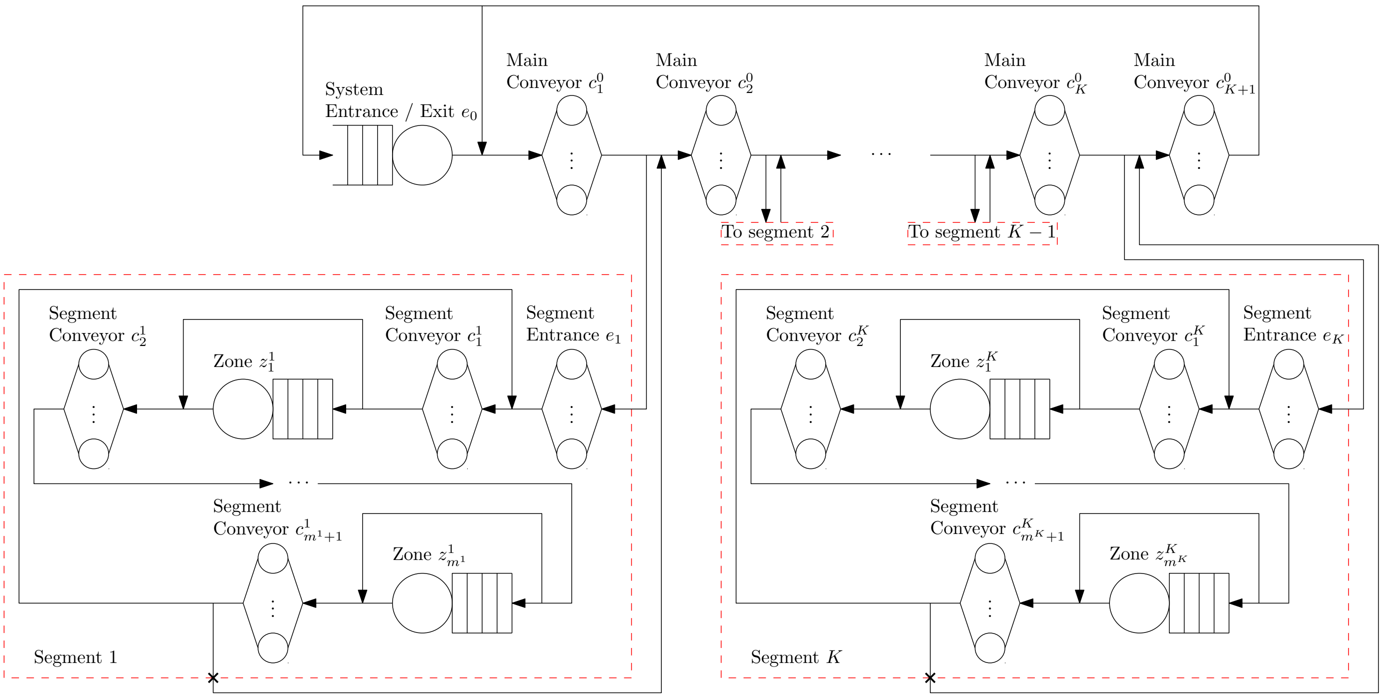
Modeling of multi-segment



Corresponding closed queueing network

Analysis of multi-segment

- Blocking at two levels:
  1. when zone is full
  2. when segment is full
- Jump-over network:

  passing tote is labeled segment k not visited with prob B_k, and labeled segment k visited, independent of whether tote visited segment k or not

- B_k is blocking probability of segment k in block-and-recirculate network: Unknown!

Analysis of multi-segment

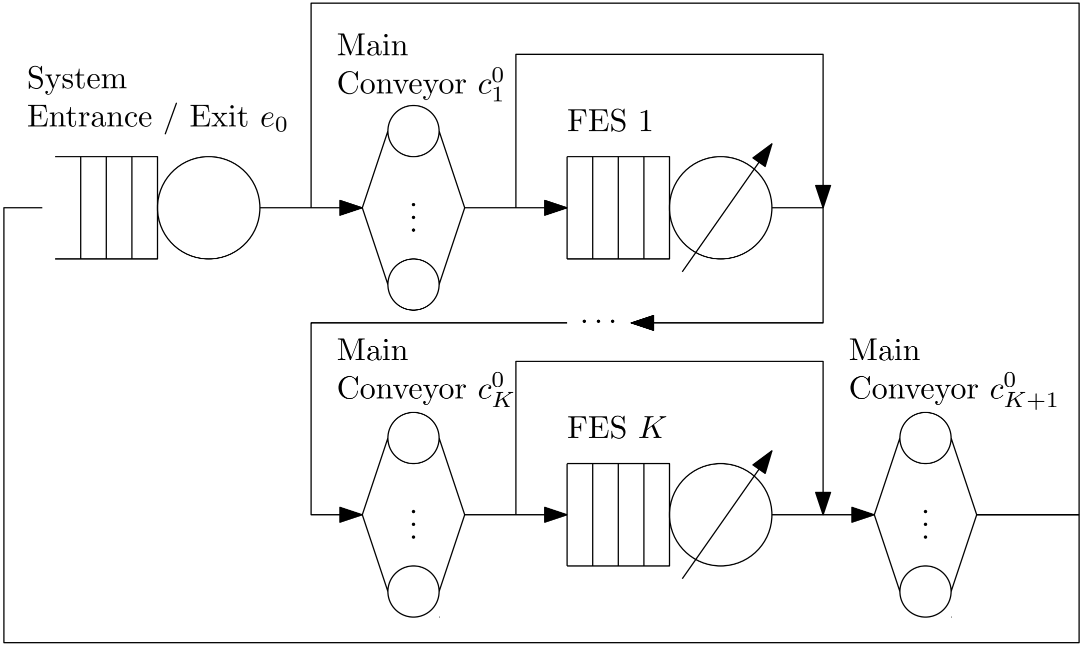
Aggregation:

Replace segments by flow equivalent servers, with rates

$$\mu_{FES_k}(n) = X^k(n), \ n = 1, \ldots, N^k, \ k = 1, \ldots, K$$

where $X^k(n)$ is throughput of segment k in isolation

Aggregation of multi-segment



Segments replaced by flow equivalent servers

Aggregation van multi-segment

- Norton's theorem:
  Aggregate network has same performance as jump-over network
- Analysis of aggregate network same as one of single-segment network!

- Arrival theorem:
  Blockings probability B_k of segment k is equal to:

  $$B_k = \Pi_k(N^k | N - 1),$$

  where $\Pi_k(n|N)$ is probability of n totes in segment k in network with N totes

Analysis of multi-segment

- Step 0:
  Initialize $b^{(0)}_{z^k_i} = B^{(0)}_k = 0$ and $j = 0$
- Step 1:
  Calculate for aggregate network and each segment by MVA:
  1. Mean order lead times and throughput
  2. Distribution of totes per node
- Step 2:
  $j = j + 1$ and estimate new blocking probabilities
  $b^{(j)}_{z^k_i} = \pi_{z^k_i}\left(d_{z^k_i} + q_{z^k_i} | N-1, N^k - 1\right), \quad B^{(j)}_k = \Pi_k(N^k | N-1)$
- Step 3:
  Return to Step 1 until $\left| b^{(j)}_{z^k_i} - b^{(j-1)}_{z^k_i} \right| < \epsilon$ en $\left| B^{(j)}_k - B^{(j-1)}_k \right| < \epsilon$

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Results for multi-segment

Example using real-life data of large Dutch wholesaler of non-food:

- 4 segments
- 3 pick-by-light segments with each 2×4 zones
- 1 pallet pick with 3 zones
- Total:
  1. 24 pick-by-light zones
  2. 3 pallet pick zones

Compare 3 storage strategies:

1. Minimize expected number of segments to be visited (Current).
2. Balance work-load over segments (Balanced).
3. Random storage (Random).

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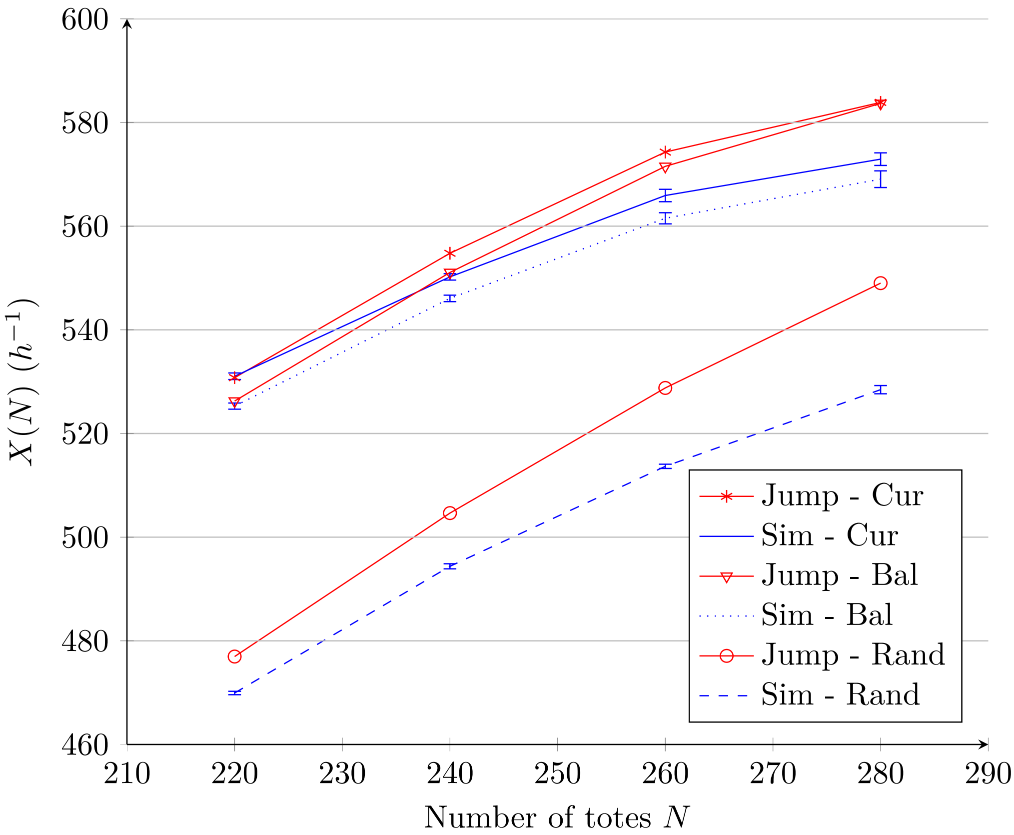
Results for multi-segment



Conclusions

- Zone-picking system can be described by:

  closed multi-class queueing network with block-and-recirculate blocking

- Network can be approximated by jump-over network

- Excellent and fast estimates of performance by MVA and Norton