

Quantifying the computational security of multi-user systems

(Work with M. Christiansen, F. du Pin Calmon & M. Médard)

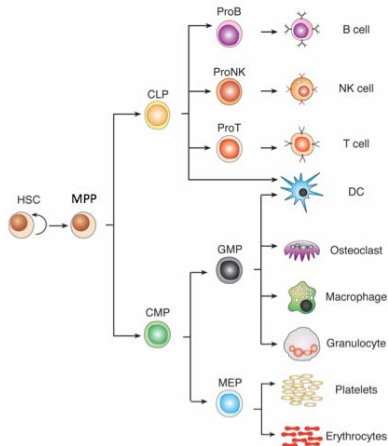
Ken Duffy

Hamilton Institute,
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Eurandom, July 2014



A stochastic network (?)



HUMAN FRONTIER SCIENCE PROGRAM
FUNDING FRONTIER RESEARCH INTO COMPLEX BIOLOGICAL SYSTEMS

S. Orkin & L. Zion *Cell*, 2008, S. Naik et al., *Nature*, 2013, L. Perié et al. *Cell Reports*, 2014.



Quantifying the computational security of multi-user systems



M. Christiansen



F. du Pin Calmon (MIT)



M. Médard (MIT)

M. Christiansen, K. Duffy, F. du Pin Calmon & M. Médard, <http://arxiv.org/abs/1405.5024>



A model of computational security

Computationally secure:

- User selects X , a string, from a collection of possibilities.
- Inquisitor knows the collection of all objects and can query each in turn.
- Computationally secure if collection of keys is large.



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Computationally secure:

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- Computationally secure if collection of keys is large.

Probability:

- What if X is picked probabilistically with a distribution known to the inquisitor?



Why non uniform?

Investigating the Distribution of Password Choices

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D. Malone & K. Maher, *Proc. WWW*, 2012



Why non uniform?

Rank	Cyphertext	indicitive hint	inferred password	#users
1	EQ7fIpT7i/Q=	One to six in numeral form	123456	1905308
2	j9p+HwtWWT86aMjgZFLzYg==	1234567890 ohne 0	123456789	445971
3	L8qbAD3j13jioxG6CatHBw==	answer is password	password	343956
4	BB4e6X+b2xLioxG6CatHBw==	adbeandonetwothree	adobe123	210932
5	j9p+HwtWWT/ioxG6CatHBw==	123456789 minus last number	12345678	201150
6	5djv7ZCI2ws=	1st 123456 letters	qwerty	130401
7	dQi0asWPYvQ=	1234567 is the password	1234567	124177
8	7LqYzKVe98I=	6 number 1s	111111	113684
9	PMDTbPOLZxu03SvrFUvYGA==	adobe photo editing software	photoshop	83269
10	e6MPXQ5G6a8=	one two three one two three	123123	82606

Table III: Top 10 Adobe passwords.

Why non uniform?



Rank	Cyphertext	indicitive hint	inferred password	#users
1	EQ7fIpt7i/Q=	One to six in numeral form	123456	1905308
2	j9p+HwtWWT86aMjgZFLzYg==	1234567890 ohne 0	123456789	445971
3	L8qbAD3j13jioxG6CatHBw==	answer is password	password	343956
4	BB4e6X+b2xLioxG6CatHBw==	adbeandonetwothree	adobe123	210932
5	j9p+HwtWWT/ioxG6CatHBw==	123456789 minus last number	12345678	201150
6	5djv7ZCI2ws=	1st 123456 letters	qwerty	130401
7	dQi0asWPYvQ=	1234567 is the password	1234567	124177
8	7LqYzKVe98I=	6 number 1s	111111	113684
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10	e6MPXQ5G6a8=	one two three one two three	123123	82606

Table III: Top 10 Adobe passwords.

The screenshot shows a list of passwords with redacted usernames and email addresses. The passwords are:



- 104544054-|---|@tue.nl|-k04E0Ij32H25n2auThm2+Q==|-Mockstraat|---
- 85681613-|---|@student.tue.nl|-f530chU7jFKSrr7+3wLJ0==|-band|---
- 116152651-|---|@student.tue.nl|-4cdiIQ49L5vbMw5FvUlykg==|-always|---
- 102588026-|---|@student.tue.nl|-czY0m424HNQ=-|-lekker|---
- 102532155-|---|@student.tue.nl|-1Y/UUZ5028=-|-wer|---
- 102832797-|---|@student.tue.nl|-dVhge4UQm66EtZxhuv+e=-|-whats my style|---
- 103129231-|---|@student.tue.nl|-8fr4yvaAFh7ioxG6CatHBw=-|-huh?|---
- 103950186-|---|@student.tue.nl|-KkajmR884g=-|-team|---
- 86311281-|---|@student.tue.nl|-urPuioT3a0LjwIhdplg=-|-moeilijk he!|---
- 99966113-|---|@tue.nl|-4mMPXgpz1hj6MaxEGshQIo=-|-Onge luk|---
- 100011401-|---|@student.tue.nl|-sgv75qDJ5yPioxG6CatHBw=-|-Ca|---
- 100113445-|---|@student.tue.nl|-ULGoDNR0wdaBkccrysXUKw=-|-TUE CODE|---
- 100302411-|---|@tue.nl|-nd9qkGw3tFC6TMMR0odTo/A=-|-tja|---
- 71282004-|---|@tue.nl|-0w0PFLMBKjioxG6CatHBw=-|-paardje|---
- 75818292-|---|@tue.nl|-eR52uEde5evioxG6CatHBw=-|-call-0|---
- 80990272-|---|@tue.nl|-joc17Lahw0ioxG6CatHBw=-|-all|---
- 81381850-|---|@tue.nl|-XRP/04egwLioxG6CatHBw=-|-standaard|---
- 84845420-|---|@student.tue.nl|-Qava9Ny648F1Akd+sFpx+Q=-|-naam hond|---
- 84215074-|---|@student.tue.nl|-7MLK9CfRNg=-|-bsdgsdgljasdfsdgl|---
- 84384895-|---|@student.tue.nl|-HcIkTBkmErW=-|-codit|---
- 102818000-|---|@student.tue.nl|-QkzPv3iDNYU=-|-lekker|---

What makes a password good?

<p>UNCOMMON (NON-GIBBERISH) BASE WORD</p> <p>ORDER UNKNOWN</p> <p>Tr0ub4dor &3</p> <p>CAPS? COMMON SUBSTITUTIONS NUMERALS PUNCTUATION</p> <p>(YOU CAN ADD A FEW MORE BITS TO ACCOUNT FOR THE FACT THAT THIS IS ONLY ONE OF A FEW COMMON WORDS.)</p>	<p>~28 BITS OF ENTROPY</p> <p>$2^{28} = 3$ DAYS AT 1000 GUESSES/SEC</p> <p>(REASONABLE ATTACK ON A WEAK PASSWORD WITH SERIOUS FEELING, CARRYING IS PROBABLY NEARLY IMPOSSIBLE, BUT IT'S NOT WHAT THE PERSON USING SHOULD WORRY ABOUT.)</p> <p>DIFFICULTY TO GUESS: EASY</p>	<p>WAS IT TROMBONE? NO, TROUBADOR. AND ONE OF THE 0s WAS A ZERO?</p> <p>AND THERE WAS SOME SYMBOL...</p>  <p>DIFFICULTY TO REMEMBER: HARD</p>
<p>correct horse battery staple</p> <p>FOUR RANDOM COMMON WORDS</p>	<p>~44 BITS OF ENTROPY</p> <p>$2^{44} = 530$ YEARS AT 1000 GUESSES/SEC</p> <p>DIFFICULTY TO GUESS: HARD</p>	<p>THAT'S A BATTERY STAPLE.</p> <p>CORRECT!</p>  <p>DIFFICULTY TO REMEMBER: YOU'VE ALREADY MEMORIZED IT</p>

THROUGH 20 YEARS OF EFFORT, WE'VE SUCCESSFULLY TRAINED EVERYONE TO USE PASSWORDS THAT ARE HARD FOR HUMANS TO REMEMBER, BUT EASY FOR COMPUTERS TO GUESS.

What makes a password good?

<p>UNCOMMON (NON-GIBBERISH) BASE WORD</p> <p>ORDER UNKNOWN</p> <p>Tr0ub4dor &3</p> <p>CAPS? COMMON SUBSTITUTIONS</p> <p>NUMERAL PUNCTUATION</p> <p>(YOU CAN ADD A FEW MORE BITS TO ACCOUNT FOR THE FACT THAT THIS IS ONLY ONE OF A FEW COMMON WORDS.)</p>	<p>~ 28 BITS OF ENTROPY</p> <p>$2^{28} = 3 \text{ DAYS AT } 1000 \text{ GUESSES/SEC}$</p> <p>DIFFICULTY TO GUESS: EASY</p>	<p>WAS IT TROMBONE? NO, TROUBADOR. AND ONE OF THE O'S WAS A ZERO?</p> <p>AND THERE WAS SOME SYMBOL...</p>  <p>DIFFICULTY TO REMEMBER: HARD</p>
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scriptsizexkcd.com/936/



Is Shannon Entropy the right measure?

Guessing and Entropy

James L. Massey

Signal & Info. Proc. Lab., Swiss Federal Inst. Tech, CH-8092 Zurich, Switzerland

J. L. Massey, *Proc. IEEE ISIT*, 1994.



Is Shannon Entropy the right measure?

- A word, W , picked from $\mathbb{A} = \{1, \dots, m\}$, has Shannon entropy

$$H = - \sum_{i \in \mathbb{A}} P(W = i) \log P(W = i).$$

- How should the inquisitor guess W ?



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& guess in order:



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& guess in order: the i^{th} most likely word on the i^{th} guess,
 $G : \mathbb{A} \mapsto \mathbb{N}$ such that $G(i) = i$ and

$$E(G(W)) = \sum_{i \in \mathbb{A}} i P(W = i).$$

J. L. Massey, *Proc. IEEE ISIT*, 1994.



What's the right measure of Guesswork?

An Inequality on Guessing and its Application to Sequential Decoding

Erdal Arikan, *Senior Member, IEEE*

E. Arikan, *IEEE Trans. Inf. Theory*, 1996.



What's the right measure of Guesswork?

A sequence $W_k \in \mathbb{A}^k$ made of i.i.d. letters. Define Rényi entropy

$$R_1(\beta) = \frac{1}{1-\beta} \log \sum_{w \in \mathbb{A}} P(W_1 = w)^\beta,$$

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Arikan's Proposition:

$$\lim_{k \rightarrow \infty} \frac{1}{k} \log \mathbb{E}(G(W_k)^\alpha) = \alpha R_1 \left(\frac{1}{1+\alpha} \right) \text{ for } \alpha > 0.$$

E. Arikan, *IEEE Trans. Inf. Theory*, 1996.



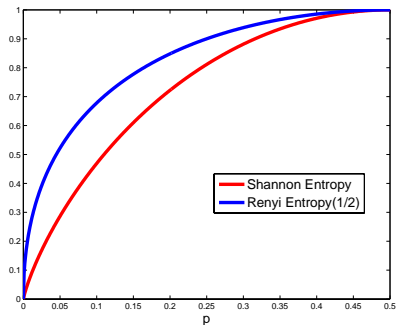
What's the right measure of Guesswork?

E.g. $\alpha = 1$, for large k

$$\mathbb{E}(G(W_k)) \approx \exp(kR_1(1/2))$$

where

$$R_1(1/2) = \log \left(\sum_{w \in \mathcal{A}} \sqrt{P(W_1 = w)} \right)^2.$$



E.g. Bernoulli Source, log base 2.

E. Arıkan, *IEEE Trans. Inf. Theory*, 1996.



Source generalization of Arikan's Proposition

With the Rényi entropy of W_k being

$$R_k(\beta) = \frac{1}{1-\beta} \log \sum_{w \in \mathbb{A}^k} P(W_k = w)^\beta,$$

and $R(\beta) = \lim_{k \rightarrow \infty} \frac{1}{k} R_k(\beta)$, generalizations prove

$$\lim_{k \rightarrow \infty} \frac{1}{k} \log \mathbb{E}(G(W_k)^\alpha) = \alpha R \left(\frac{1}{1+\alpha} \right) \text{ for } \alpha > -1.$$

D. Malone and W. G. Sullivan, *IEEE Trans. Inf. Theory*, 2004.

C.-E. Pfister and W. G. Sullivan, *IEEE Trans. Inf. Theory*, 2004.

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Large deviations and guesswork distributions

Consider

$$\Lambda(\alpha) := \lim_{k \rightarrow \infty} \frac{1}{k} \log \mathbb{E}(G(W_k)^\alpha) = \lim_{k \rightarrow \infty} \frac{1}{k} \log \mathbb{E}(e^{\alpha \log(G(W_k))}) = \begin{cases} \alpha R \left(\frac{1}{1 + \alpha} \right) \\ -R(\infty) \end{cases}$$

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Large deviations and guesswork distributions

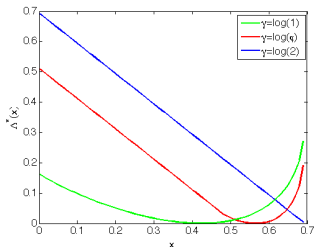
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Suggestive of

$$dP \left(\frac{1}{k} \log G(W_k) \approx x \right) \asymp \exp(-k\Lambda^*(x)) dx$$

$$\text{where } \Lambda_X^*(x) = \sup_{\alpha \in \mathbb{R}} (\alpha x - \Lambda_X(\alpha)).$$



For large k , some jiggery-pokery gives

$$P(G(W_k) = n) \approx \frac{1}{n} \exp \left(-k\Lambda^* \left(\frac{1}{k} \log n \right) \right).$$

M. Christiansen & K. Duffy, *IEEE Trans. Inf. Theory*, 2013.



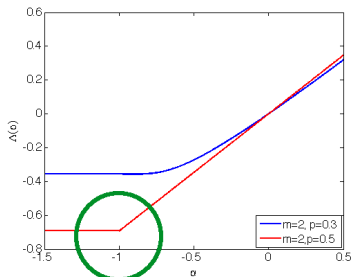
What's in a discontinuous derivative?

$$\Lambda(\alpha) = \begin{cases} \alpha R((1 + \alpha)^{-1}) & \text{if } \alpha \geq -1 \\ -R(\infty) & \text{if } \alpha \leq -1 \end{cases}$$

Define:

$$\begin{aligned} \gamma &= \lim_{\alpha \downarrow -1} \frac{d}{d\alpha} \Lambda(\alpha) \\ &= \lim_{\beta \rightarrow \infty} \left(R(\beta) - \frac{R'(\beta)}{\beta^2} \right). \end{aligned}$$

If i.i.d., then $\gamma = \log |\{w : P(W_1 = w) = P(G(W_1) = 1)\}|$.

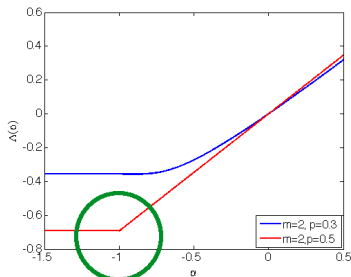


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If i.i.d., then $\gamma = \log |\{w : P(W_1 = w) = P(G(W_1) = 1)\}|$.

If not, then approximately $e^{k\gamma}$ "most likely words" of length k .



An aside on most likely words

Lemma: For $\{W_k\}$ constructed of Markovian letters with $\mathbb{A} = \{0, 1\}$,

$$\gamma = \lim_{\alpha \downarrow -1} \Lambda'(\alpha) \in \{0, \log(\phi), \log(2)\},$$

where $\phi = (1 + \sqrt{5})/2$ is the Golden Ratio, and no other values are possible.



Guessing a password over a wireless channel (on the effect of noise non-uniformity)

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Massachusetts Institute of Technology
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Brute force searching, the typical set and Guesswork

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M. Christiansen, K. Duffy, F. du Pin Calmon & M. Medard, *Proc. Allerton*, 2013
M. Christiansen, K. Duffy, F. du Pin Calmon & M. Medard, *Proc. ISIT*, 2013



Multiple users

$V \in \mathbb{N}$ users, independently picking strings

$$\vec{W}_k = \left(W_k^{(1)}, \dots, W_k^{(V)} \right) \in \mathbb{A}^{kV}.$$

Statistics of each user's selection known to an inquisitor who can query the veracity of (user, string) pair and we wish to identify $U \leq V$ of them.



The Shannon Cipher System with a Guessing Wiretapper

Neri Merhav, *Fellow, IEEE*, and Erdal Arıkan, *Senior Member, IEEE*

N. Merhav & E. Arıkan, *IEEE Trans. Inf. Theory*, vol. 45, pp. 1860–1866, 1999.



The Shannon Cipher System with a Guessing Wiretapper

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Then, it is clear that the best guessing strategy (in any reasonable sense) is to first guess the most likely X given Y , then try the second most likely guess, and so on, until eventually, the correct message is found.



Optimal strategy?

G is optimal W_k if and only if

$P(G(W_k) \leq n) \geq P(S(W_k) \leq n)$ for all strategies S and all $n \in \{1, \dots, m^k\}$.



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Lemma

If $V = U$, the optimal strategies are those that guess from most likely to least likely.



If $U < V$, not guaranteed stochastic domination

Example: $V = 2$, $U = 1$ and $|\mathbb{A}| = 3$.



If $U < V$, not guaranteed stochastic domination

Example: $V = 2$, $U = 1$ and $|\mathbb{A}| = 3$.

User	Item probability		
Sem	0.6	0.2	0.2
Johan	0.6	0.2	0.2



If $U < V$, not guaranteed stochastic domination

Example: $V = 2$, $U = 1$ and $|\mathbb{A}| = 3$.

User	Item probability		
Sem	0.6	0.5	0.5
Johan	0.6	0.2	0.2



If $U < V$, not guaranteed stochastic domination

Example: $V = 2$, $U = 1$ and $|\mathbb{A}| = 3$.

Move to Johan			
User	Item	probability	
Sem	0.6	0.5	0.5
Johan	0.6	0.5	0.5

Stick with Sem			
User	Item	probability	
Sem	0.6	0.5	1
Johan	0.6	0.2	0.2



There exist asymptotically optimal strategies - Round-robin

For each $v \in \{1, \dots, V\}$ let $G^{(v)}$ denote its optimal strategy and define:

$$G_{\text{opt}}(U, V, \vec{W}_k) = \text{U-min} \left(G^{(1)}(W_k^{(1)}), \dots, G^{(V)}(W_k^{(V)}) \right),$$

where $\text{U-min} : \mathbb{R}^V \rightarrow \mathbb{R}$ gives the U^{th} smallest component.



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where $\text{U-min} : \mathbb{R}^V \rightarrow \mathbb{R}$ gives the U^{th} smallest component.

Then

$$G_{\text{opt}}(U, V, \vec{W}_k) \leq \text{real performance of round-robin} \leq V G_{\text{opt}}(U, V, \vec{W}_k)$$

and, as $k \rightarrow \infty$, these have the same asymptote.



Asymptotically optimal strategies satisfy a LDP

Theorem

$\{k^{-1} \log G_{\text{opt}}(U, V, \vec{W}_k)\}$ satisfies a large deviation principle. Defining

$$\delta^{(v)}(x) = \begin{cases} \Lambda_G^{(v)*}(x) & \text{if } x \leq H^{(v)} \\ 0 & \text{otherwise,} \end{cases} \quad \text{and} \quad \gamma^{(v)}(x) = \begin{cases} \Lambda_G^{(v)*}(x) & \text{if } x \geq H^{(v)} \\ 0 & \text{otherwise,} \end{cases}$$

the rate function is

$$I_{G_{\text{opt}}}(U, V, x) = \max_{v_1, \dots, v_V} \left(\Lambda_G^{(v_1)*}(x) + \sum_{i=2}^U \delta^{(v_i)}(x) + \sum_{i=U+1}^V \gamma^{(v_i)}(x) \right),$$

which may not be convex. The sCGF is

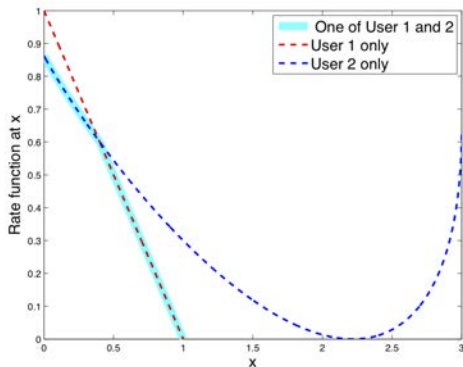
$$\begin{aligned} \Lambda_{G_{\text{opt}}}(U, V, \alpha) &= \lim_{k \rightarrow \infty} \frac{1}{k} \log E(\exp(\alpha \log G_{\text{opt}}(U, V, \vec{W}_k))) \\ &= \sup_{x \in [0, Vm]} (\alpha x - I_{G_{\text{opt}}}(U, V, x)). \end{aligned}$$



A Merhav & Arikan example, $U = 1$, $V = 2$

$W_k^{(1)}$, Bernoulli on $\{0, 1\}$,

$$P(W_1^{(2)} = i) = \begin{cases} 0.55 & \text{if } i = 0 \\ 0.1 & \text{if } i \in \{1, 2\} \\ 0.05 & \text{if } i \in \{3, \dots, 7\} \end{cases}$$



All things being equal

Corollary

If users' statistics are all (asymptotically) the same, then

$$\Lambda_{G_{opt}}^*(U, V, x) = \begin{cases} U\Lambda_G^*(x) & \text{if } x \leq H \\ (V - U + 1)\Lambda_G^*(x) & \text{if } x \geq H \end{cases}$$

and

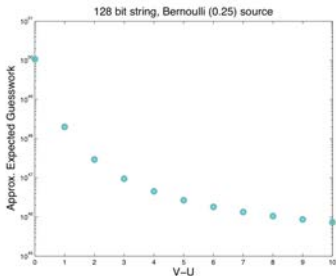
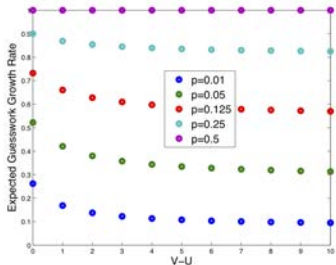
$$\Lambda_{G_{opt}}(U, V, \alpha) = \begin{cases} U\Lambda_G\left(\frac{\alpha}{U}\right) & \text{if } \alpha \leq 0 \\ (V - U + 1)\Lambda_G\left(\frac{\alpha}{V - U + 1}\right) & \text{if } \alpha \geq 0. \end{cases}$$



Multi-user guesswork growth rates

$n = V - U$, number of excess strings

$$\mathbb{E}(G_{\text{opt}}(U, V, \vec{W}_k)) \approx \exp\left(kR \left(\frac{n+1}{n+2}\right)\right), \text{ where } \frac{n+1}{n+2} \in \left\{\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \dots\right\}.$$



Concluding comments

- There's no "truly" optimal guessing strategy.
- Performance of asymptotically optimal strategies can be analysed.
- From an attacker's point of view, there's a law of diminishing returns in excess number of users.
- Shannon Entropy provides a universal lower bound on the guesswork growth rate of multi-user systems.



Concluding comments

- There's no "truly" optimal guessing strategy.
- Performance of asymptotically optimal strategies can be analysed.
- From an attacker's point of view, there's a law of diminishing returns in excess number of users.
- Shannon Entropy provides a universal lower bound on the guesswork growth rate of multi-user systems.
- If you had an Adobe password, change it everywhere.



Same as Facebook

```
108759368|--|...@mit.edu-|-RcjCaJFSHEvn1QXGvHdh5g==--|-same as facebook|--
```



Same as Facebook

```
108759368|--| . ██████████@mit.edu|--RcjCaJFSHEvn1QXGvHdh5g==|--same as facebook|--
02833749|--| ██████████@uwo.ca|--RcjCaJFSHEvn1QXGvHdh5g==|--hotmail|--
99835647|--| ██████████@gmail.com|--RcjCaJFSHEvn1QXGvHdh5g==|--traktor|--
81179368|--| ██████████@hotmail.com|--RcjCaJFSHEvn1QXGvHdh5g==|--it's...|--
97533272|--| ██████████@aol.com|--RcjCaJFSHEvn1QXGvHdh5g==|--crazy|--
78507698|--| ██████████@cyleearchitect.com|--RcjCaJFSHEvn1QXGvHdh5g==|--pet|--
76699302|--| ██████████@earthlink.net|--RcjCaJFSHEvn1QXGvHdh5g==|--life is|--
93375144|--| ██████████@gmail.com|--RcjCaJFSHEvn1QXGvHdh5g==|--what|--
98443191|--| ██████████@yahoo.com|--RcjCaJFSHEvn1QXGvHdh5g==|--old xanga password no #s|--
94581897|--| ██████████@live.co|--RcjCaJFSHEvn1QXGvHdh5g==|--crazy|--
115987129|--| ██████████@roadrunner.com|--RcjCaJFSHEvn1QXGvHdh5g==|--how i feel|--
88136333|--| ██████████@students.harker.org|--RcjCaJFSHEvn1QXGvHdh5g==|--my life in a nutshell|--
91052978|--| ██████████@hotmail.com|--RcjCaJFSHEvn1QXGvHdh5g==|--blonde|--
98743970|--| ██████████@apla.org|--RcjCaJFSHEvn1QXGvHdh5g==|--loco|--
108713584|--| ██████████@gmail.com|--RcjCaJFSHEvn1QXGvHdh5g==|--life|--
115136071|--| ██████████@gmail.com|--RcjCaJFSHEvn1QXGvHdh5g==|--email password|--
100958121|--| ██████████@gmu.edu|--RcjCaJFSHEvn1QXGvHdh5g==|--parties|--
79703283|--| ██████████@rogers.com|--RcjCaJFSHEvn1QXGvHdh5g==|--No capitals and no numbers crazy|--
107014628|--| ██████████@hotmail.com|--RcjCaJFSHEvn1QXGvHdh5g==|--xanga password|--
106493153|--| ██████████@hotmail.com|--RcjCaJFSHEvn1QXGvHdh5g==|--crazy|--
1079083777|--| ██████████@cmecmortgages.com|--RcjCaJFSHEvn1QXGvHdh5g==|--life|--
107190304|--| ██████████@gmail.com|--RcjCaJFSHEvn1QXGvHdh5g==|--crazy|--
83966207|--| ██████████@bscglobal.net|--RcjCaJFSHEvn1QXGvHdh5g==|--usual|--
115328531|--| ██████████@gmail.com|--RcjCaJFSHEvn1QXGvHdh5g==|--insanity|--
117762012|--| ██████████@yahoo.com|--RcjCaJFSHEvn1QXGvHdh5g==|--teta|--
119656192|--| ██████████@gmail.com|--RcjCaJFSHEvn1QXGvHdh5g==|--email password|--
121222022|--| ██████████@gmail.com|--RcjCaJFSHEvn1QXGvHdh5g==|--|--
125637223|--| ██████████@alumni.ou.edu|--RcjCaJFSHEvn1QXGvHdh5g==|--cranham ...|--
139114270|--| ██████████@yahoo.com|--RcjCaJFSHEvn1QXGvHdh5g==|--crazines|--
140819982|--| ██████████@hunter78|--RcjCaJFSHEvn1QXGvHdh5g==|--duh|--
143617611|--| ██████████@gmail.com|--RcjCaJFSHEvn1QXGvHdh5g==|--crazy|--
144240340|--| ██████████@yahoo.com|--RcjCaJFSHEvn1QXGvHdh5g==|--crazy|--
144760809|--| ██████████@yahoo.com|--RcjCaJFSHEvn1QXGvHdh5g==|--|--
146339856|--| ██████████@hotmail.com|--RcjCaJFSHEvn1QXGvHdh5g==|--waleed|--
155683719|--| ██████████@yahoo.com|--RcjCaJFSHEvn1QXGvHdh5g==|--not crazy84|--
169246561|--| ██████████@vianet.ca|--RcjCaJFSHEvn1QXGvHdh5g==|--same as always|--
170354523|--| ██████████@shaw.ca|--RcjCaJFSHEvn1QXGvHdh5g==|--usual p|--
170751657|--| ██████████@gmail.com|--RcjCaJFSHEvn1QXGvHdh5g==|--reg|--
172569707|--| ██████████@gmail.com|--RcjCaJFSHEvn1QXGvHdh5g==|--Hotmail|--
187279970|--| ██████████@yahoo.com|--RcjCaJFSHEvn1QXGvHdh5g==|--what are you?|--
```

