Meixner Processes in Finance[*]

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Abstract

In the Black-Scholes option price model Brownian motion and the underlying Normal distribution play a fundamental role. Empirical evidence however shows that the normal distribution is a very poor model to fit real-life data. In order to achieve a better fit we replace the Brownian motion by a special Lévy process: the Meixner process. We show that the underlying Meixner distribution allows an almost perfect fit to the data by performing a number of statistical tests. We discuss properties of the driving Meixner process. Next, we give a valuation formula for derivative securities, state the analogue of the Black-Scholes differential equation, and compare the obtained prices with the classical Black-Scholes prices. Throughout the text the method is illustrated by the modeling of the Nikkei-225 Index. Similar analysis for other indices are given in the appendix.

1 Introduction

To price and hedge derivative securities it is crucial to have a good modeling of the probability distribution of the underlying product. The most famous continuous-time model is the celebrated Black-Scholes model [3]. It uses the Normal distribution to fit the log-returns of the underlying: the price process of the underlying is given by the geometric Brownian Motion

$$S_t = S_0 \exp\left(\left(\mu - \frac{\sigma^2}{2}\right)t + \sigma B_t\right),$$

where $\{B_t, t \geq 0\}$ is standard Brownian motion, i.e. B_t follows a normal distribution with mean 0 and variance t. Its key property is that it is complete, i.e. a perfect hedge is in an idealized market in theory possible. It is well known

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[*]This is a corrected version of the original EURANDOM Report 2001-002

however that the log-returns of most financial assets have an actual kurtosis that is higher than that of the normal distribution. In this paper we therefore propose another model which is based on the Meixner distribution.

The Meixner distribution belongs to the class of the infinitely divisible distributions and as such give rise to a Lévy process: The Meixner process. The Meixner process is very flexible, has a simple structure and leads to analytically and numerically tractable formulas. It was introduced in [17] and originates from the theory of orthogonal polynomials and was proposed to serve as a model of financial data in [9].

In the late 1980s and in the 1990s several other non-Brownian Lévy process models where proposed. Masan and Seneta [12] have proposed a Lévy process with variance gamma distributed increments. We mention also the Hyperbolic Model [6] proposed by Eberlein and Keller. In the same year Barndorff-Nielsen proposed the normal inverse Gaussian Lévy process [1]. Recently the CMGY model was introduced [4]. All models give a much better fit to the data and lead to an improvement with respect to the Black-Scholes model. In this paper we provide statistical evidence that the Meixner model performs also significantly better then the Black-Scholes Model. Moreover, the advantage of the Meixner model over the other Lévy models is that all crucial formulas are explicitly given, so that it is not depending on computationally demanding numerical inversion procedures. This numerical advantage can be important, when a big number op prices or related quantities has to be computed simultaneously.

Throughout this paper we illustrate the method by modeling the price process of the Nikkei-225 Index in the period from 01-01-1997 until 31-12-1999. The data set consists of the 737 daily log-returns of the index during the mentioned period. The mean of this data set is equal to 0.00036180, while its standard deviation equals 0.01599747. In the appendix one can find similar analysis for other indices.

This paper is organized as follows: we first introduce the Meixner distribution and the Meixner Process in Section 2. Next, in Section 3 we fit the Meixner distribution to our data set and we perform a number of statistical test in order to proof the high accuracy of the fit. In Section 4, we give the analogue of the Black-Scholes partial differential equation, we compute option prices in our new model, and compare them with the classical Black-Scholes prices. In the appendix we summarize the analysis for other indices: the German Dax Index, the FTSE-100 Index, the Swiss SMI, the Nasdaq Composite Index, and the French CAC-40 Index.

2 Meixner Distributions

The density of the Meixner distribution (Meixner(a, b, d, m)) is given by

$$f(x; a, b, m, d) = \frac{(2\cos(b/2))^{2d}}{2a\pi\Gamma(2d)} \exp\left(\frac{b(x-m)}{a}\right) \left|\Gamma\left(d + \frac{i(x-m)}{a}\right)\right|^2,$$

where $a > 0, -\pi < b < \pi, d > 0$, and $m \in \mathbb{R}$.

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Moments of all order of this distribution exist. Next, we give some relevant quantities; similar, but more involved, expressions exist for the moments and the skewness.

	Meixner(a, b, d, m)	**Normal**(μ, σ^2)
mean	$m + ad \tan(b/2)$	μ
variance	$\frac{a^2 d}{2}(\cos^{-2}(b/2))$	σ^2
kurtosis	$3 + \frac{3 - 2\cos^2(b/2)}{d}$	3

One can clearly see that the kurtosis of the Meixner distribution is always greater than the normal kurtosis.

The characteristic function of the Meixner(a, b, d, m) distribution is given by

$$E\left[\exp(iuX)\right] = \left(\frac{\cos(b/2)}{\cosh \frac{au - ib}{2}}\right)^{2d} \exp(imu)$$

Suppose $\phi(u)$ is the characteristic function of a distribution. If moreover for every positive integer n, $\phi(u)$ is also the nth power of a characteristic function, we say that the distribution is infinitely divisible. One can define for every such an infinitely divisible distribution a stochastic process, $X = \{X_t, t \geq 0\}$, called Lévy process, which starts at zero, has independent and stationary increments and such that the distribution of an increment over $[s, s + t]$, $s, t \geq 0$, i.e. $X_{t+s} - X_s$, has $(\phi(u))^t$ as characteristic function.

Clearly, the Meixner(a, b, d, m) distribution is infinitely divisible and we can associate with it a Lévy process which we call the Meixner process. More precisely, a Meixner process $\{M_t, t \geq 0\}$ is a stochastic process which starts at zero, i.e. $M_0 = 0$, has independent and stationary increments, and where the distribution of M_t is given by the Meixner distribution Meixner(a, b, dt, mt).

In general a Lévy process consists of three independent parts: a linear deterministic part, a Brownian part, and a pure jump part. It is easy to show that our Meixner process $\{M_t, t \geq 0\}$ has no Brownian part and a pure jump part governed by the Lévy measure

$$\nu(\mathrm{d}x) = d\frac{\exp(bx/a)}{x \sinh(\pi x/a)}\mathrm{d}x.$$

The Lévy measure $\nu(\mathrm{d}x)$ dictates how the jumps occur. Jumps of sizes in the set A occur according to a Poisson Process with parameter $\int_A \nu(\mathrm{d}x)$. Because $\int_{-\infty}^{+\infty} |x|\nu(\mathrm{d}x) = \infty$ it follows from standard Lévy process theory [2] [16], that our process is of infinite variation.

Our Meixner(a, b, d, m) distribution has semiheavy tails [10]. This means that the tails of the density function behave as

$$f(x, a, b, d, m) \sim C_- |x|^{\rho_-} \exp(-\sigma_- |x|) \quad \text{as} \quad x \to -\infty$$
$$f(x, a, b, d, m) \sim C_+ |x|^{\rho_+} \exp(-\sigma_+ |x|) \quad \text{as} \quad x \to +\infty,$$

for some $\rho_-, \rho_+ \in \mathbb{R}$ and $C_-, C_+, \sigma_-, \sigma_+ \geq 0$. In case of the Meixner$(a, b, d, m)$,

$$\rho_- = \rho_+ = 2d - 1, \quad \sigma_- = (\pi - b)/a, \quad \sigma_+ = (\pi + b)/a.$$

3   Fitting and Statistical Justification

To estimate the Meixner distribution we assume independent observations and use moments estimators. In the particular case of the Nikkei-225 Index, the result of the estimation procedure is given by

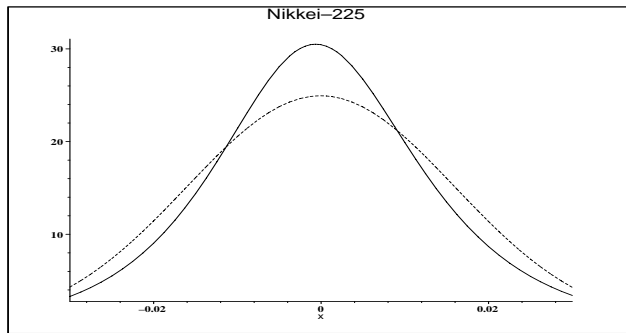$$\hat{a} = 0.02982825, \quad \hat{b} = 0.12716244, \quad \hat{d} = 0.57295483, \quad \hat{m} = -0.00112426$$

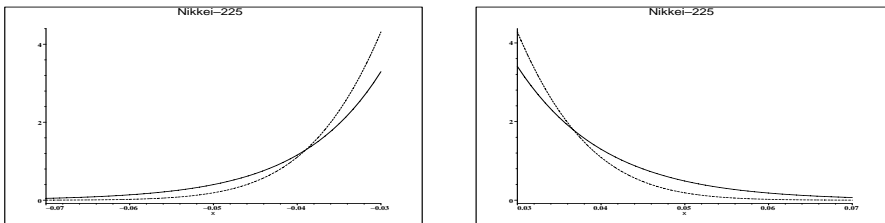Figure 1: Meixner density (solid) versus Normal density (dashed)

Figure 2: Meixner density tails (solid) versus Normal density tails (dashed)

From Figure 1, it is clear that there is considerably more mass around the center than the normal distribution can provide. Figure 2 zooms in at the tails. As can be expected from the semiheavyness of the tails, the Meixner distribution has significant fatter tails than the Normal distribution. This is in correspondence with empirical observations, see e.g. [6].

We use different tools for testing the goodness of fit: QQ-plots and χ^2-tests. It will be shown that we obtain an almost perfect fit. So we arrive at the conclusion that the daily stock returns of the stock can be modeled very accurately by the Meixner distribution.

3.1   QQ-plots

The first evidence is provide by a graphical method: the quantile-quantile plot
(QQ-plot). It is a qualitative yet very powerful method for testing the goodness
of fit. A QQ-plot of a sample of n points plots for every $j = 1, \ldots, n$ the
empirical $(j - (1/2))/n$-quantile of the data against the $(j - (1/2))/n$-quantile
of the fitted distribution. The plotted points should not deviated to much from
a straight line.

For the classical model based on the normal distribution, the deviation from
the straight line and thus the normal density is clearly seen from the next QQ-
plot in Figure 3.
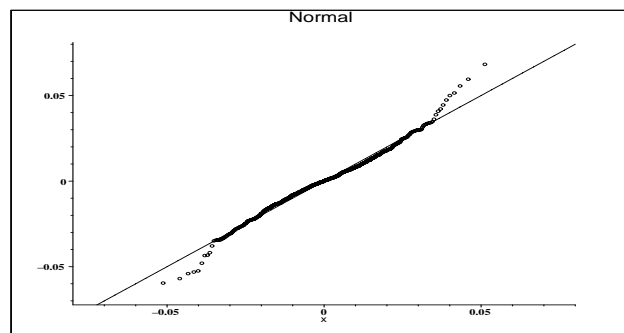
Figure 3: Normal QQ-plot

It can be seen that there is a severe problem in the tails if we try to fit the
data with the normal distribution. This problem almost completely disappears
when we use the Meixner distribution to fit the data, as can be seen in Figure
4.
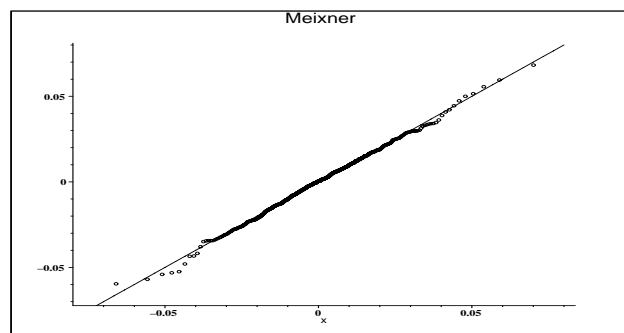
Figure 4: Meixner QQ-plot

The Meixner density shows an excellent fit. It indicates a strong preference

for the Meixner model over the classical normal one.

3.2 χ^2-tests

The χ^2-test counts the number of sample points falling into certain intervals and compares them with the expected number under the null hypothesis. We consider classes of equal width as well of equal probability. We take $N = 32$ classes of equal width. If necessary we collapse outer cells, such that the expected value of observations becomes greater than five. In our Nikkei-225 Index-example, we choose $-0.0225 + (j - 1) * (0.0015)$, $j = 1, \ldots, N - 1$, as the boundary points of the classes.

We consider also the case with $N = 28$ classes of equal probability, the class boundaries are now given by the i/N-quantiles $i = 1, \ldots, N - 1$ of the fitting distribution.

Because we have to estimate for the normal distribution two parameters we taken in this case $N - 3$ degrees of freedom. In the Meixner case, there has to be estimated 4 parameters, so we take in this case $N - 5$ degrees of freedom.

Table 1 shows the values of the χ^2-test statistic with equal width for the normal null hypotheses and the Meixner null hypotheses and different quantiles of the χ^2_{29} and χ^2_{27} distributions.

Table 2 shows the values of the χ^2-test statistic with equal probability for the normal null hypotheses and the Meixner null hypotheses and different quantiles of the χ^2_{23} and χ^2_{25} distributions.

In Tables 1 and 2 we also give the so-called P-values of the test-statistics. The P-value is the probability that values are even more extreme (more in the tail) than our test-statistic. It is clear that very small P-values lead to a rejection of the null hypotheses, because they are themselves extreme. P-values not close to zero indicate that the test statistic is not extreme and lead to acceptance of the hypothesis. To be precise we reject if the P-value is less than our level of significance, which we take 0.05, and accept otherwise.

χ^2_{Normal}	$\chi^2_{29,0.95}$	$\chi^2_{29,0.99}$	P_{Normal}-value
47.45527092	42.55696780	49.58788447	0.01672773

$\chi^2_{Meixner}$	$\chi^2_{27,0.95}$	$\chi^2_{27,0.99}$	$P_{Meixner}$-value
29.21660289	40.11327207	46.96294212	0.35047500

Table 1: χ^2_1 test-statistics and P-values (equal width)

We see that the Normal hypotheses is in both cases clearly rejected, whereas the Meixner hypotheses is accepted and yields a very high P-value.

4 Pricing of Derivatives

Throughout the text we will denote by r the daily interest rate, in our computations we will take $r = 0.0002$. We assume our market consist of one riskless

χ^2_{Normal}	$\chi^2_{25,0.95}$	$\chi^2_{25,0.99}$	P_{Normal}-value
47.87381276	37.65248413	44.31410490	0.00386153

$\chi^2_{Meixner}$	$\chi^2_{23,0.95}$	$\chi^2_{23,0.99}$	$P_{Meixner}$-value
20.44369064	35.17246163	41.63839812	0.61502001

Table 2: χ^2_2 test-statistics and P-values (equal probability)

asset (the bond) with price process given by $B_t = e^{rt}$ and one risky asset (the stock). The model which produces exactly Meixner(a, b, d, m) daily log-returns for the stock is given by

$$S_t = S_0 \exp(M_t).$$

Given our market model, let $G(S_T)$ denote the payoff of the derivative at its time of expiry T. In case of the European call with strike price K, we have $G(S_T) = (S_T - K)^+$. According to the fundamental theorem of asset pricing (see [5]) the arbitrage free price V_t of the derivative at time $t \in [0, T]$ is given by

$$V_t = E_Q[e^{-r(T-t)}G(S_T)|\mathcal{F}_t],$$

where the expectation is taken with respect to an equivalent martingale measure Q and $\mathcal{F} = \{\mathcal{F}_t, 0 \le t \le T\}$ is the natural filtration of $M = \{M_t, 0 \le t \le T\}$. An equivalent martingale measure is a probability measure which is equivalent (it has the same null-sets) to the given (historical) probability measure and under which the discounted process $\{e^{-rt}S_t\}$ is a martingale. Unfortunately for most models, in particular the more realistic ones, the class of equivalent measures is rather large and often covers the full no-arbitrage interval. In this perspective the Black-Scholes model, where there is an unique equivalent martingale measure, is very exceptional. Models with more than one equivalent measures are called incomplete.

Our Meixner model is such an incomplete model. Following Gerber and Shiu ([7] and [8]) we can by using the so-called Esscher transform easily find at least one equivalent martingale measure, which we will use in the sequel for the valuation of derivative securities. The choice of the Esscher measure may be justified by a utility maximizing argument (see [8]).

4.1 Option Pricing Formula

With the Esscher transform our equivalent martingale measure Q follows a Meixner$(a, a\theta + b, d, m)$ distribution (see also [9]), where θ is given by

$$\theta = \frac{-1}{a} \left(b + 2 \arctan \left(\frac{-\cos(a/2) + \exp((m-r)/(2d))}{\sin(a/2)} \right) \right)$$

For the Nikkei-225 Index, $\theta = 0.42190524$. Note the fact that the risk-neutral-measure only differs in the b-parameter. This parameter changes from $b_{real} =$

0.12716244 in the real world to $b_{riskneutral} = 0.13974713$ in the risk-neutral world.

For an European call option with strike price K and time to expiration T, the value at time 0 is therefore given by the expectation of the payoff under the martingale measure:

$$E_Q[e^{-rT} \max\{S_T - K, 0\}]$$

This expectation can be written as

$$S_0 \int_c^\infty f(x; a, a(\theta + 1) + b, dT, mT) \mathrm{d}x - e^{-rT} K \int_c^\infty f(x; a, a\theta + b, dT, mT) \mathrm{d}x, \tag{1}$$

where $c = \ln(K/S_0)$.

Similar formulas can be derived for other derivatives with a payoff function, $G(S_T) = G(S_0 \exp(M_T)) = F(M_T)$ which is only depending on the terminal value at time $t = T$.

If the price $V(t, M_t)$ at time t of the such a derivative satisfies some regularity conditions (i.e. $V(t, x) \in C^{(1,2)}$ (see [14])) it can also be obtained by solving a partial differential integral equation (PDIE) with a boundary condition:

$$
\begin{aligned}
rV(t, x) &= \gamma \frac{\partial}{\partial x} V(t, x) + \frac{\partial}{\partial t} V(t, x) \\
&\quad + \int_{-\infty}^{+\infty} \left(V(t, x + y) - V(t, x) - y \frac{\partial}{\partial x} V(t, x) \right) \nu_Q(\mathrm{d}y) \\
V(T, x) &= F(x),
\end{aligned}
$$

where $\nu_Q(\mathrm{d}y)$ is the Lévy measure of the risk-neutral distribution, i.e.

$$\nu_Q(\mathrm{d}x) = d \frac{\exp((a\theta + b)x/a)}{x \sinh(\pi x/a)} \mathrm{d}x,$$

and

$$\gamma = m + ad + \tan((a\theta + b)/2) - 2d \int_1^\infty \sinh((a\theta + b)x/a)/\sinh(\pi x/a) \mathrm{d}x.$$

This PDIE is the analogue of the Black-Scholes partial differential equation and follows from the Feynman-Kac formula for Lévy Processes [14].

4.2 Volatility Smile

In Figure 5 we compare the difference between the Meixner prices and the Black-Scholes prices for various maturities (3 days ($T = 3$), 3 weeks ($T = 15$) and 3 months ($T = 60$)) and different strike prices ($0.70 \leq K \leq 1.30$, $S_0 = 1$). Note how the shape and the difference changes as time to expiration increases.

In real markets traders are well aware that the future probability distribution of the underlying asset may not be lognormal and they use a volatility smile
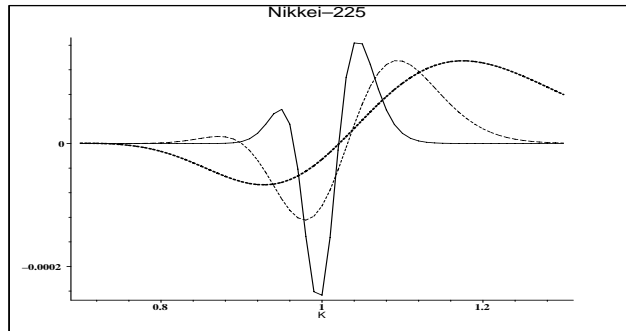
Figure 5: Difference between Meixner prices and Black-Scholes prices ($S_0 = 1$, $T = 3$ (solid), 15 (dot), 60 (thick))

adjustment. Typically the implicit volatility is higher in the money and out of the money. This smile-effect is decreasing with time to maturity. Moreover, smiles are frequently asymmetric.

We compute the prices of an European call option for different strike prices in our Meixner model. Next, we compute the implied Meixner volatility s, i.e. we look in the Black-Scholes model for the volatility parameter which give rise to the same option price as in the Meixner case. Figure 6 plots these implied volatility versus the strike price ratio. The dotted line is the volatility parameter in the Black-Scholes model.

We see how the Meixner pricing model incorporates a smile effect, and as such, because the Meixner model is closer to reality, justifies the smile effect in real markets. One can raise the question whether the smile effect in real markets is completely determined by an inaccurate modelling of the returns, or not. One can expect that a better model reduces the effect, but in real markets other additional risks, like e.g. illiquidity of not at the money options and bid/ask spreads, need also to be priced in.

Acknowledgments
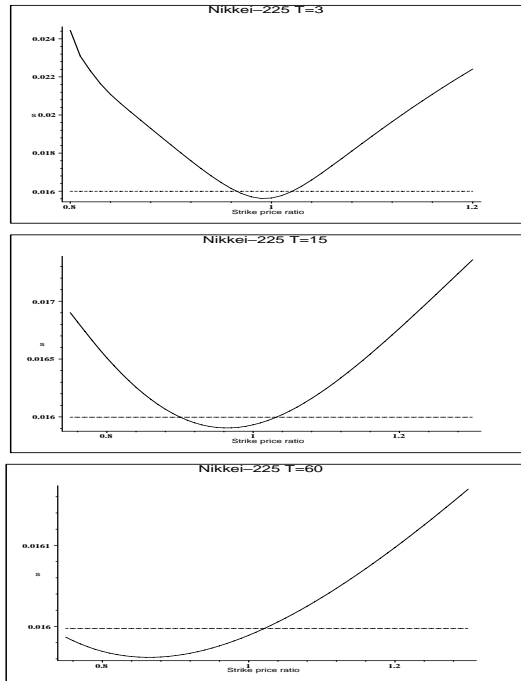
Figure 6: Volatility smile, $T = 3, 15, 60$ days

Appendix

In this appendix we summarizes the analysis of other indices. The data sets contain the log-returns over the period 1-1-1997 until 31-12-1999. We start by given the mean, μ, of the dataset, its standard deviation, σ, and the number of data points, n. For all indices we estimate the parameters a, b, d and m. We calculate θ with an assumed daily interest rate of $r = 0.0002$. We give the density plots, the Normal and the Meixner-QQ-plots, and the relevant values of the Pearson tests. We furthermore look as the difference between the Meixner price and the Black Scholes price of an European call option for various maturities. Finally, we plot the implied volatilities which result from the Meixner prices with respect to the Black-Scholes prices.

DAX Index

Parameters

$$\mu = 0.00118752, \quad \sigma = 0.01566708, \quad n = 752$$

$$\hat{a} = 0.02612297, \quad \hat{b} = -0.50801886, \quad \hat{d} = 0.67395917, \quad \hat{m} = 0.00575829$$

$$\theta = -4.46513538$$

Density

Meixner density (solid) versus Normal density (dashed):
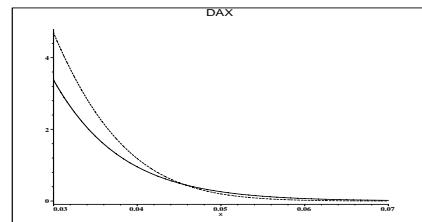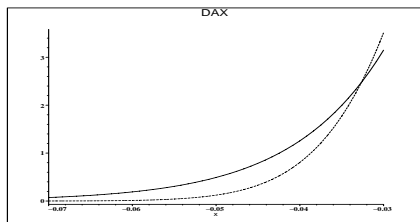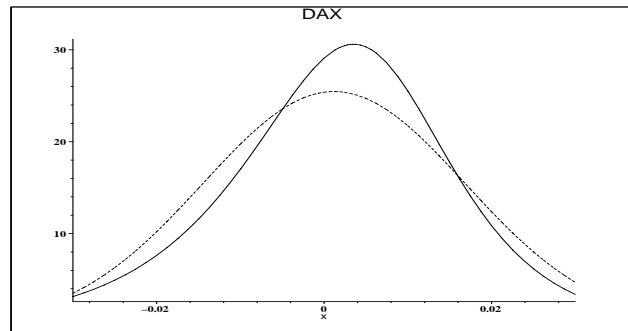
QQ-plots
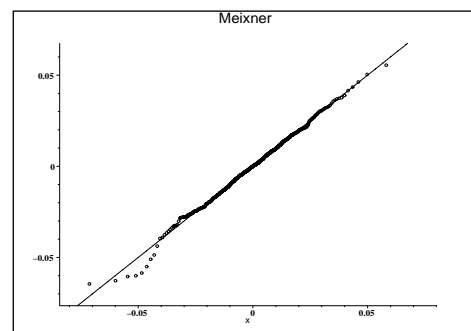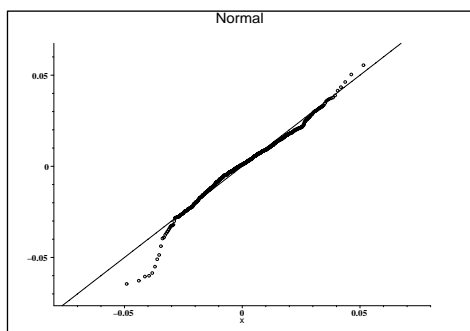
Pearson χ^2 test

$N = 32$ classes with class boundary points $-0.0225 + (j - 1) * (0.0015)$, $j = 1, \ldots, 31$.

χ^2_{Normal}	$\chi^2_{29,0.95}$	$\chi^2_{29,0.99}$	P_{Normal}-value
43.89175884	42.55696780	49.58788447	0.03757784

$\chi^2_{Meixner}$	$\chi^2_{27,0.95}$	$\chi^2_{27,0.99}$	$P_{Meixner}$-value
18.21277157	40.11327207	46.96294212	0.89688200

$N = 28$ equiprobable classes.

χ^2_{Normal}	$\chi^2_{25,0.95}$	$\chi^2_{23,0.99}$	P_{Normal}-value
41.53191489	37.65248413	44.31410490	0.02016812

$\chi^2_{Meixner}$	$\chi^2_{23,0.95}$	$\chi^2_{25,0.99}$	$P_{Meixner}$-value
27.45744679	35.17246163	41.63839812	0.23699573

Option prices comparison

Difference between Meixner prices and Black-Scholes prices ($S_0 = 1$, $T = 3$ (solid), 15 (thin dot), 60 (thick):
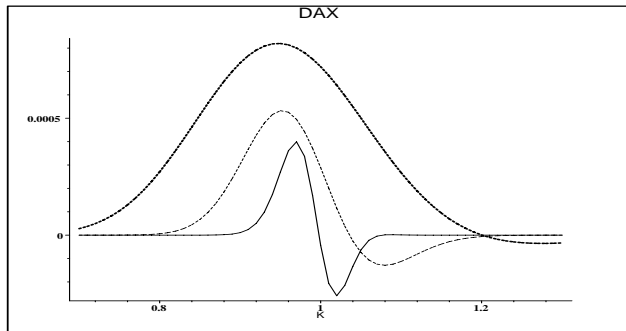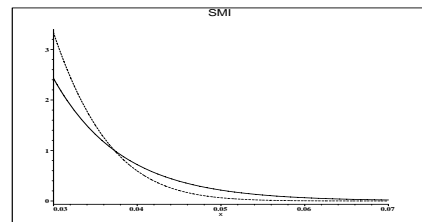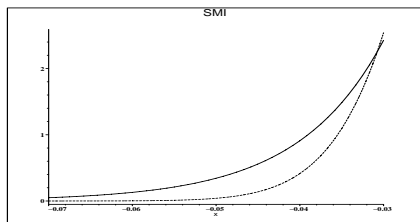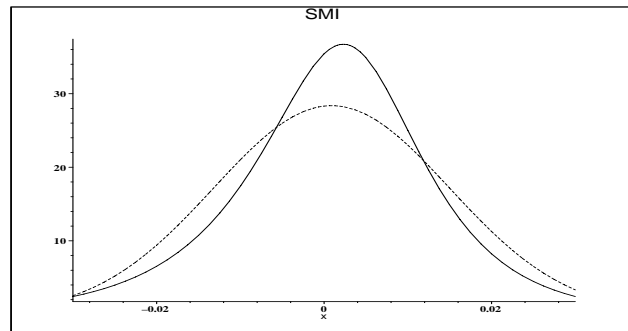

Volatility smile, $T = 3, 15, 60$ days:
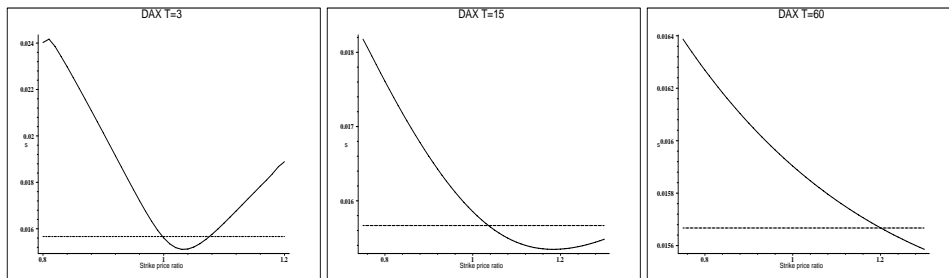
FTSE-100 Index

Parameters

$$\mu = 0.00070813, \quad \sigma = 0.01147848, \quad n = 756$$

$$\hat{a} = 0.01502403, \quad \hat{b} = -0.014336370, \quad \hat{d} = 1.16142851, \quad \hat{m} = 0.00196108$$

$$\theta = -4.34746821$$

Density

Meixner density (solid) versus Normal density (dashed):
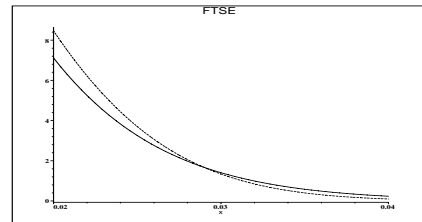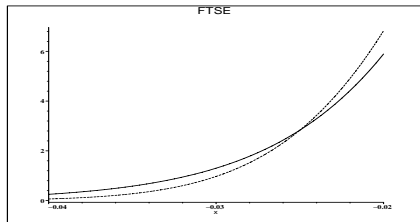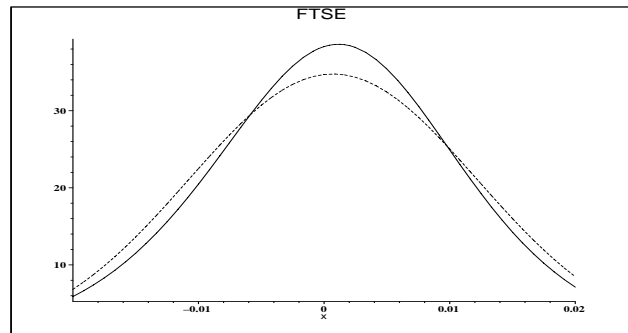
QQ-plots
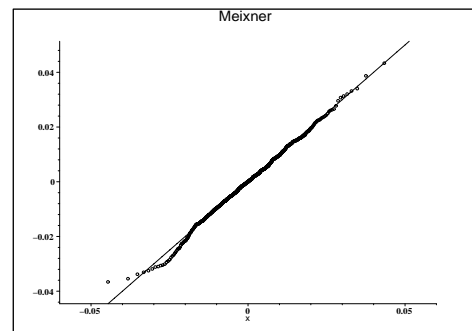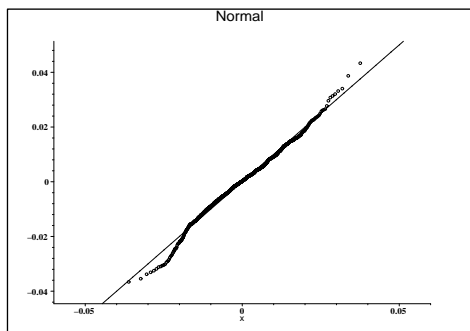
Pearson χ^2 test

$N = 30$ classes with class boundary points $-0.0185 + (j - 1) * (0.0015)$, $j = 1, \ldots, 29$.

χ^2_{Normal}	$\chi^2_{27,0.95}$	$\chi^2_{27,0.99}$	P_{Normal}-value
42.42944787	40.11327207	46.96294212	0.02984292

$\chi^2_{Meixner}$	$\chi^2_{25,0.95}$	$\chi^2_{25,0.99}$	$P_{Meixner}$-value
32.79237172	37.65248413	44.31410490	0.13634104

$N = 28$ equiprobable classes.

χ^2_{Normal}	$\chi^2_{25,0.95}$	$\chi^2_{25,0.99}$	P_{Normal}-value
52.88888889	37.65248413	44.31410490	0.00092385

$\chi^2_{Meixner}$	$\chi^2_{23,0.95}$	$\chi^2_{23,0.99}$	$P_{Meixner}$-value
33.33333333	35.17246163	41.63839812	0.07543185

Option prices comparison

Difference between Meixner prices and Black-Scholes prices ($S_0 = 1$, $T = 3$ (solid), 15 (thin dot), 60 (thick):

Volatility smile, $T = 3, 15, 60$ days:

14
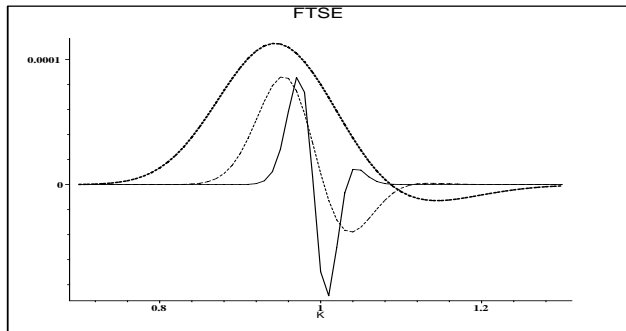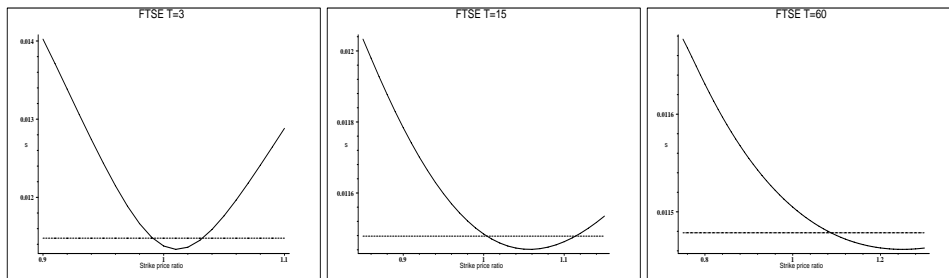
SMI Index

Parameters

$$\mu = 0.00089330, \quad \sigma = 0.01406170, \quad n = 731$$

$$\hat{a} = 0.02954687, \quad \hat{b} = -0.33888717, \quad \hat{d} = 0.44012011, \quad \hat{m} = 0.00311801$$

$$\theta = -3.97213216$$

Density

Meixner density (solid) versus Normal density (dashed):

QQ-plots

15
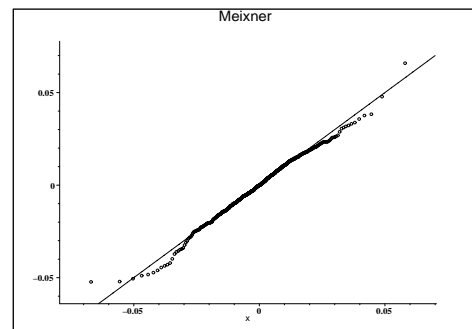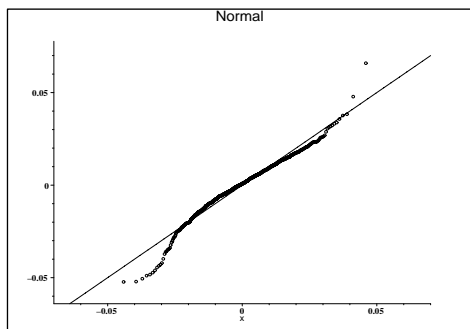
Pearson χ^2 test

$N = 32$ classes with class boundary points $-0.018 + (j - 1) * (0.0012)$, $j = 1, \ldots, 31$.

χ^2_{Normal}	$\chi^2_{29,0.95}$	$\chi^2_{29,0.99}$	P_{Normal}-value
42.92659121	42.55696780	49.58788447	0.04624445

$\chi^2_{Meixner}$	$\chi^2_{27,0.95}$	$\chi^2_{27,0.99}$	$P_{Meixner}$-value
24.88926666	40.11327207	46.96294212	0.58066769

$N = 28$ equiprobable classes.

χ^2_{Normal}	$\chi^2_{25,0.95}$	$\chi^2_{25,0.99}$	P_{Normal}-value
44.91792068	37.65248413	44.31410490	0.00854436

$\chi^2_{Meixner}$	$\chi^2_{23,0.95}$	$\chi^2_{23,0.99}$	$P_{Meixner}$-value
24.15731875	35.17246163	41.63839812	0.39514026

Option prices comparison

Difference between Meixner prices and Black-Scholes prices ($S_0 = 1$, $T = 3$ (solid), 15 (thin dot), 60 (thick):
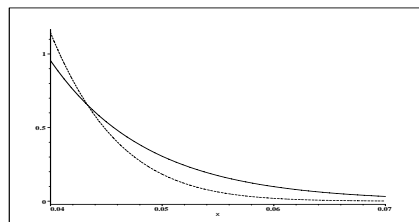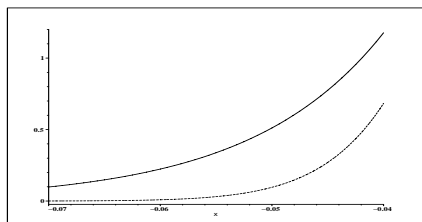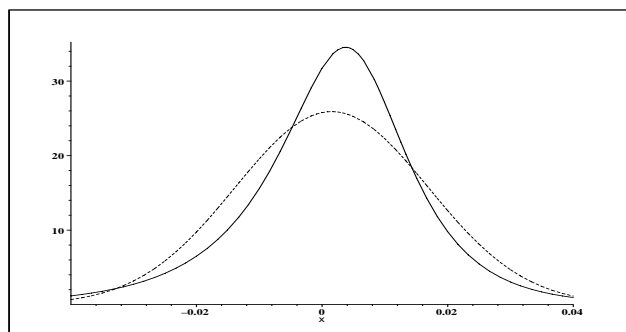

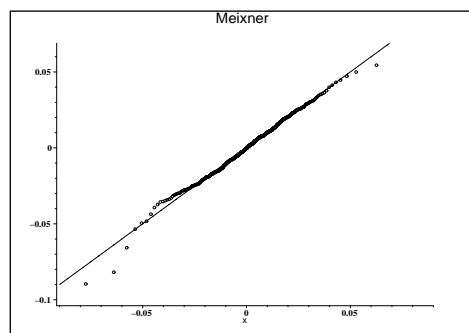Volatility smile, $T = 3, 15, 60$ days:

16
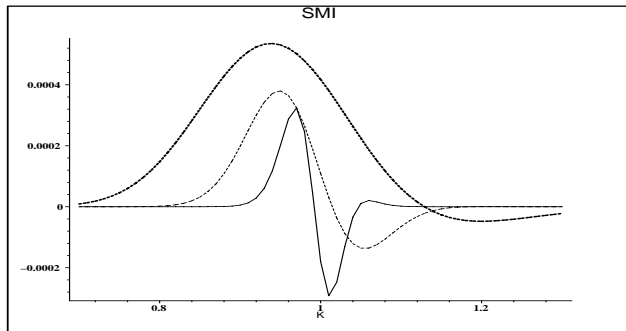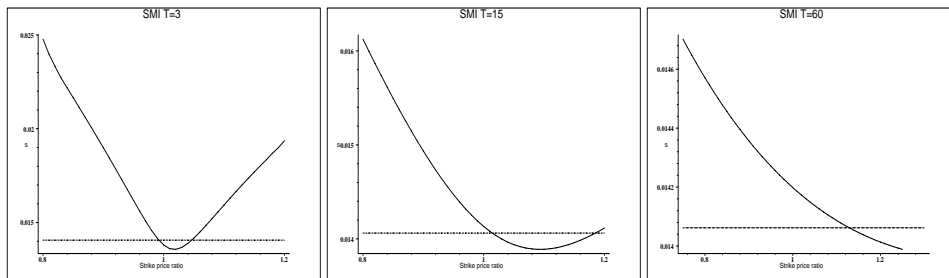
Nasdaq Composite Index

Parameters

$$\mu = 0.00152919, \quad \sigma = 0.01540092, \quad n = 756$$

$$\hat{a} = 0.03346698, \quad \hat{b} = -0.49356259, \quad \hat{d} = 0.39826126, \quad \hat{m} = 0.00488688$$

$$\theta = -5.95888693$$

Density

Meixner density (solid) versus Normal density (dashed):

QQ-plots

17
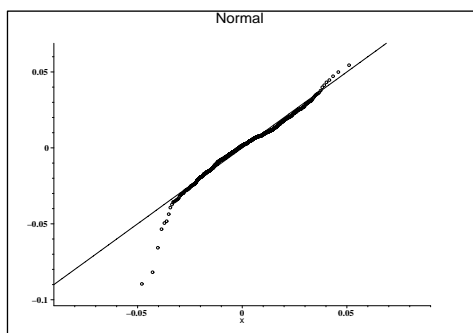
Pearson χ^2 test

$N = 32$ classes with class boundary points $-0.03 + (j-1) * (0.002)$, $j = 1, \ldots, 31$.

χ^2_{Normal}	$\chi^2_{29,0.95}$	$\chi^2_{29,0.99}$	P_{Normal}-value
52.4891763	42.55696780	49.58788447	0.00480544

$\chi^2_{Meixner}$	$\chi^2_{27,0.95}$	$\chi^2_{27,0.99}$	$P_{Meixner}$-value
27.40028797	40.11327207	46.96294212	0.44236623

$N = 28$ equiprobable classes.

χ^2_{Normal}	$\chi^2_{25,0.95}$	$\chi^2_{25,0.99}$	P_{Normal}-value
67.55555556	37.65248413	44.31410490	0.00000881

$\chi^2_{Meixner}$	$\chi^2_{23,0.95}$	$\chi^2_{23,0.99}$	$P_{Meixner}$-value
31.62962964	35.17246163	41.63839812	0.10809438

Option prices comparison

Difference between Meixner prices and Black-Scholes prices ($S_0 = 1$, $T = 3$ (solid), 15 (thin dot), 60 (thick):
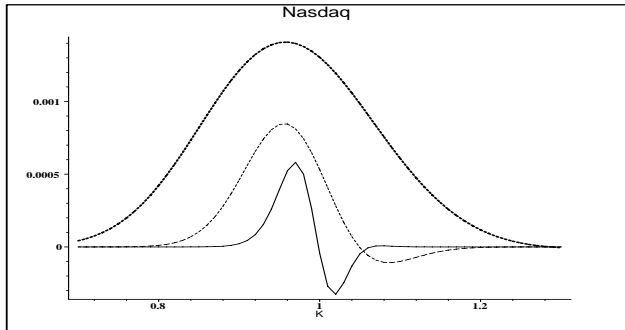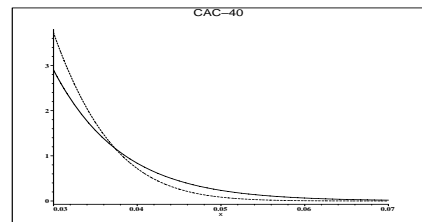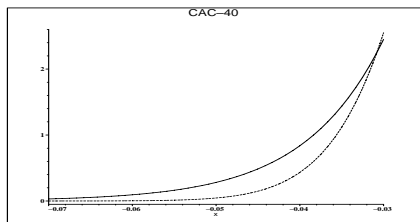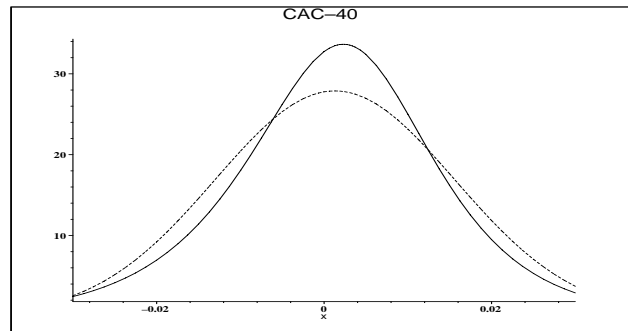

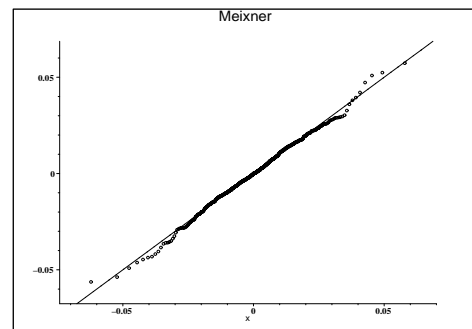Volatility smile, $T = 3, 15, 60$ days:
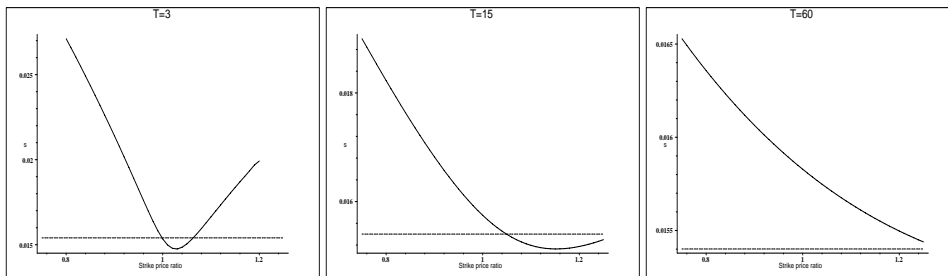
CAC-40 Index

Parameters

$$\mu = 0.00129089, \quad \sigma = 0.01430603, \quad n = 752$$

$$\hat{a} = 0.02539854, \quad \hat{b} = -0.23804755, \quad \hat{d} = 0.62558083, \quad \hat{m} = 0.00319102$$

$$\theta = -5.77928595$$

Density

Meixner density (solid) versus Normal density (dashed):

QQ-plots

19
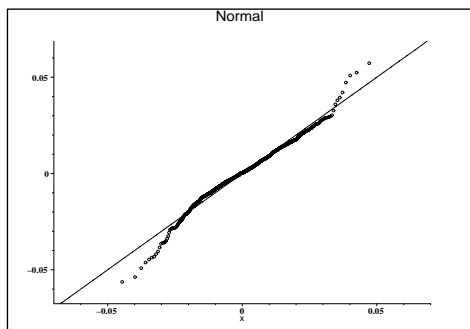
Pearson χ^2 test

$N = 32$ classes with class boundary points $-0.018 + (j-1) * (0.0012)$, $j = 1, \ldots, 31$.

χ^2_{Normal}	$\chi^2_{29,0.95}$	$\chi^2_{29,0.99}$	P_{Normal}-value
44.99274115	42.55696780	49.58788447	0.02947184

$\chi^2_{Meixner}$	$\chi^2_{27,0.95}$	$\chi^2_{27,0.99}$	$P_{Meixner}$-value
31.82751935	40.11327207	46.96294212	0.23853247

$N = 28$ equiprobable classes.

χ^2_{Normal}	$\chi^2_{25,0.95}$	$\chi^2_{25,0.99}$	P_{Normal}-value
42.50000000	37.65248413	44.31410490	0.01587090

$\chi^2_{Meixner}$	$\chi^2_{23,0.95}$	$\chi^2_{23,0.99}$	$P_{Meixner}$-value
24.62765955	35.17246163	41.63839812	0.36976456

Option prices comparison

Difference between Meixner prices and Black-Scholes prices ($S_0 = 1$, $T = 3$ (solid), 15 (thin dot), 60 (thick):
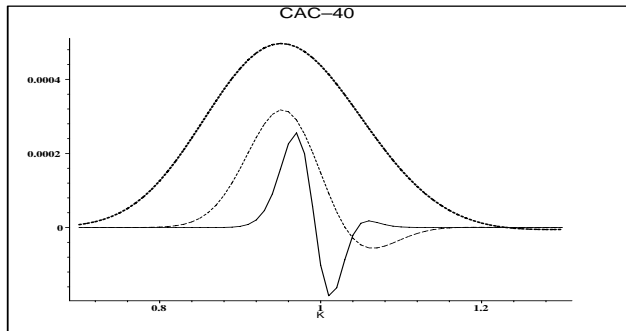
Volatility smile, $T = 3, 15, 60$ days:
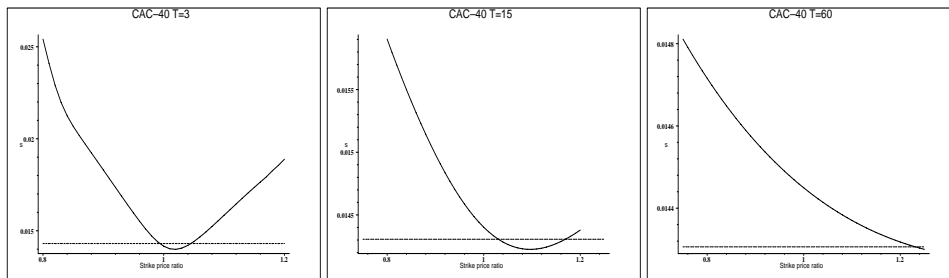
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