Brownian survival among Poissonian traps with random shapes at critical intensity

M. van den Berg *
E. Bolthausen †
F. den Hollander ‡

12th December 2003

Abstract

In this paper we consider a standard Brownian motion in $\mathbb{R}^d$, starting at 0 and observed until time t. The Brownian motion takes place in the presence of a Poisson random field of traps, whose centers have intensity ν_t and whose shapes are drawn randomly and independently according to a probability distribution Π, on the set of closed subsets of $\mathbb{R}^d$, subject to appropriate conditions. The Brownian motion is killed as soon as it hits one of the traps. With the help of a large deviation technique developed in an earlier paper, we find the tail of the probability S_t that the Brownian motion survives up to time t when

$$\nu_t = \begin{cases} ct^{-2/d}, & d \geq 3, \\ ct^{-1} \log^2 t, & d = 2, \end{cases}$$

where $c \in (0, \infty)$ is a parameter. This choice of intensity corresponds to a critical scaling. We give a detailed analysis of the rate constant in the tail of S_t as a function of c, including its limiting behaviour as $c \to \infty$ or $c \downarrow 0$. For $d \geq 3$, we find that there are two regimes, depending on the choice of Π. In one of the regimes there is a collapse transition at a critical value $c^* \in (0, \infty)$, where the optimal survival strategy changes from being diffusive to being subdiffusive. At c^*, the slope of the rate constant is discontinuous. For $d = 2$ there is again a collapse transition, but the rate constant is independent of Π and its slope is continuous.

AMS 2000 *subject classifications.* 60F10, 60G50, 35J20.
Key words and phrases. Brownian motion, Poisson random field of traps, random capacity, survival probability, large deviations, variational problems, Sobolev inequalities.

Acknowledgment. The research in this paper was supported by EPSRC (United Kingdom) grant GR/R37111 and by SNSF (Switzerland).

*School of Mathematics, University of Bristol, University Walk, Bristol BS8 1TW, United Kingdom, *m.vandenberg@bristol.ac.uk*

†Institut für Mathematik, Universität Zürich, Winterthurerstrasse 190, CH-8057 Zürich, Switzerland, *eb@amath.unizh.ch*

‡EURANDOM, P.O. Box 513, 5600 MB Eindhoven, The Netherlands, *denhollander@eurandom.tue.nl*