# Intermittency on catalysts: symmetric exclusion

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**Abstract**

We continue our study of intermittency for the parabolic Anderson equation $\partial u/\partial t = \kappa\Delta u + \xi u$, where $u\colon \mathbb{Z}^d \times [0,\infty) \to \mathbb{R}$, $\kappa$ is the diffusion constant, $\Delta$ is the discrete Laplacian, and $\xi\colon \mathbb{Z}^d \times [0,\infty) \to \mathbb{R}$ is a space-time random medium. The solution of the equation describes the evolution of a "reactant" $u$ under the influence of a "catalyst" $\xi$.

In this paper we focus on the case where $\xi$ is exclusion with a symmetric random walk transition kernel, starting from equilibrium with density $\rho \in (0,1)$. We consider the annealed Lyapunov exponents, i.e., the exponential growth rates of the successive moments of $u$. We show that these exponents are trivial when the random walk is recurrent, but display an interesting dependence on the diffusion constant $\kappa$ when the random walk is transient, with qualitatively different behavior in different dimensions. Special attention is given to the asymptotics of the exponents for $\kappa \to \infty$, which is controlled by moderate deviations of $\xi$ requiring a delicate expansion argument.

In an earlier paper by Gärtner and den Hollander the case where $\xi$ is a Poisson field of independent (simple) random walks was studied. The two cases show interesting differences and similarities. Throughout the paper, a comparison of the two cases plays a crucial role.

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