

On the localized phase of a copolymer in an emulsion: supercritical percolation regime

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14th September 2007

Abstract

In this paper we study a two-dimensional directed self-avoiding walk model of a random copolymer in a random emulsion. The copolymer is a random concatenation of monomers of two types, A and B , each occurring with density $\frac{1}{2}$. The emulsion is a random mixture of liquids of two types, A and B , organised in large square blocks occurring with density p and $1-p$, respectively, where $p \in (0, 1)$. The copolymer in the emulsion has an energy that is minus α times the number of AA -matches minus β times the number of BB -matches, where without loss of generality the interaction parameters can be taken from the cone $\{(\alpha, \beta) \in \mathbb{R}^2: \alpha \geq |\beta|\}$. To make the model mathematically tractable, we assume that the copolymer is directed and can only enter and exit a pair of neighbouring blocks at diagonally opposite corners.

In [5], a variational expression was derived for the quenched free energy per monomer in the limit as the length n of the copolymer tends to infinity and the blocks in the emulsion have size L_n such that $L_n \rightarrow \infty$ and $L_n/n \rightarrow 0$. Under this restriction, the free energy is self-averaging with respect to both types of randomness. It was found that in the supercritical percolation regime $p \geq p_c$, with p_c the critical probability for directed bond percolation on the square lattice, the free energy has a phase transition along a curve in the cone that is independent of p . At this critical curve, there is a transition from a phase where the copolymer is fully delocalized into the A -blocks to a phase where it is partially localized near the AB -interface. In the present paper we prove three theorems that complete the analysis of the phase diagram : (1) the critical curve is strictly increasing; (2) the phase transition is second order; (3) the free energy is infinitely differentiable throughout the partially localized phase.

In the subcritical percolation regime $p < p_c$, the phase diagram is much more complex. This regime will be treated in a forthcoming paper.

AMS 2000 subject classifications. 60F10, 60K37, 82B27.

Key words and phrases. Random copolymer, random emulsion, localization, delocalization, phase transition, percolation, large deviations.

Acknowledgment. NP is supported by a postdoctoral fellowship from the Netherlands Organization for Scientific Research (grant 613.000.438). FdH and NP are grateful to the Pacific Institute for the Mathematical Sciences and the Mathematics Department of the University of British Columbia, Vancouver, Canada, for hospitality: FdH from January to August 2006, NP from mid-March to mid-April 2006 when the work in this paper started.

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