Homogeneous nucleation for Glauber and Kawasaki dynamics in large volumes at low temperatures

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16 June 2008

Abstract: In this paper we study metastability in large volumes at low temperatures. We consider both Ising spins subject to Glauber spin-flip dynamics and lattice gas particles subject to Kawasaki hopping dynamics. Let β denote the inverse temperature and let $\Lambda_\beta \subset \mathbb{Z}^2$ be a square box with periodic boundary conditions such that $\lim_{\beta\to\infty} |\Lambda_\beta| = \infty$. We run the dynamics on Λ_β starting from a random initial configuration where all the droplets (= clusters of plus-spins, respectively, clusters of particles) are small. For large β, and for interaction parameters that correspond to the metastable regime, we investigate how the transition from the metastable state (with only small droplets) to the stable state (with one or more large droplets) takes place under the dynamics. This transition is triggered by the appearance of a single *critical droplet* somewhere in Λ_β. Using potential-theoretic methods, we compute the *average nucleation time* (= the first time a critical droplet appears and starts growing) up to a multiplicative factor that tends to one as $\beta \to \infty$. It turns out that this time grows as $Ke^{\Gamma\beta}/|\Lambda_\beta|$ for Glauber dynamics and $K\beta e^{\Gamma\beta}/|\Lambda_\beta|$ for Kawasaki dynamics, where Γ is the local canonical, respectively, grand-canonical energy to create a critical droplet and K is a constant reflecting the geometry of the critical droplet, provided these times tend to infinity (which puts a growth restriction on $|\Lambda_\beta|$). The fact that the average nucleation time is inversely proportional to $|\Lambda_\beta|$ is referred to as *homogeneous nucleation*, because it says that the critical droplet for the transition appears essentially independently in small boxes that partition Λ_β.
*Keywords:*Glauber dynamics, Kawasaki dynamics, critical droplet, metastable transition time, last-exit biased distribution, Dirichlet principle, Berman-Konsowa principle, capacity, flow, cluster expansion.

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