Quenched LDP for words in a letter sequence

Matthias Birkner[†]
Andreas Greven[‡]
Frank den Hollander[§]

July 16, 2008

Abstract: When we cut an i.i.d. sequence of letters into words according to an independent renewal process, we obtain an i.i.d. sequence of words. In the *annealed* large deviation principle (LDP) for the empirical process of words, the rate function is the specific relative entropy of the observed law of words w.r.t. the reference law of words. In the present paper we consider the *quenched* LDP, i.e., we condition on a typical letter sequence. We focus on the case where the renewal process has an *algebraic* tail. The rate function turns out to be a sum of two terms, one being the annealed rate function, the other being proportional to the specific relative entropy of the observed law of letters w.r.t. the reference law of letters, with the former being obtained by concatenating the words and randomising the location of the origin. The proportionality constant equals the tail exponent of the renewal process. Earlier work by Birkner considered the case where the renewal process has an exponential tail, in which case the rate function turns out to be the first term on the set where the second term vanishes and to be infinite elsewhere.

We apply our LDP to prove that the radius of convergence of the moment generating function of the collision local time of two strongly transient random walks on $\mathbb{Z}^d$, $d \geq 1$, strictly increases when we condition on one of the random walks, both in discrete time and in continuous time. The presence of these gaps implies the existence of an *intermediate phase* for the long-time behaviour of a class of coupled branching processes, interacting diffusions, respectively, directed polymers in random environments.

Keywords: Letters and words, renewal process, empirical process, annealed vs. quenched, large deviation principle, rate function, specific relative entropy, collision local time.

[†]Weierstraß-Institut für Angewandte Analysis und Stochastik, Mohrenstr. 39, 10117 Berlin, Germany; Institut für Angewandte Mathematik, Universität Bonn, Wegelerstr. 6, 53115 Bonn, Germany

[‡]Mathematisches Institut, Universität Erlangen-Nürnberg, Bismarckstrasse $1\frac{1}{2}$, 91054 Erlangen, Germany

[§]Mathematical Institute, Leiden University, P.O. Box 9512, 2300 RA Leiden, The Netherlands