Intermittency on catalysts: voter model

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Abstract: In this paper we study intermittency for the parabolic Anderson equation $\partial u/\partial t = \kappa\Delta u + \gamma\xi u$ with $u\colon \mathbb{Z}^d \times [0,\infty) \to \mathbb{R}$, where $\kappa \in [0,\infty)$ is the diffusion constant, Δ is the discrete Laplacian, $\gamma \in (0,\infty)$ is the coupling constant, and $\xi\colon \mathbb{Z}^d \times [0,\infty) \to \mathbb{R}$ is a space-time random medium. The solution of this equation describes the evolution of a "reactant" u under the influence of a "catalyst" ξ.

We focus on the case where ξ is the voter model with opinions 0 and 1 that are updated according to a random walk transition kernel, starting from either the Bernoulli measure ν_ρ or the equilibrium measure μ_ρ, where $\rho \in (0,1)$ is the density of 1's. We consider the annealed Lyapunov exponents, i.e., the exponential growth rates of the successive moments of u. We show that these exponents are trivial when the random walk is not strongly transient, but display an interesting dependence on the diffusion constant κ when the random walk is strongly transient, with qualitatively different behavior in different dimensions.

In earlier work we considered the case where ξ is a field of independent simple random walks in a Poisson equilibrium, respectively, a symmetric exclusion process in a Bernoulli equilibrium, which are both reversible dynamics. In the present work, a main obstacle is the non-reversibility of the voter model dynamics, since this precludes the application of spectral techniques. The duality with coalescing random walks is key to our analysis, and leads to a representation formula for the Lyapunov exponents that allows for the application of large deviation estimates.

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Key words and phrases. Parabolic Anderson equation, catalytic random medium, voter model, coalescing random walks, Lyapunov exponents, intermittency, large deviations.

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