

On the fine structure of price processes

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Abstract

In the last years starting from the Black-Scholes model many different models either based on semimartingales, purely continuous, pure jump and a mixture of both, or fractional Brownian motion have been proposed in an attempt to capture the empirical facts of heavy tails, skewness, excess kurtosis and long range dependence. So far, the main focus has been on analyzing the tail-behaviour of the log-returns.

We propose to include the fine structure into this analysis, which makes it possible to distinguish between semimartingales and fractional Brownian motion on the one hand and determine on the other hand the fine structure of either process in terms of the Blumenthal-Gettoor index or the Hurst exponent. From the point of view of modelling or risk analysis this gives insight if a pure jump process or a continuous process or even more specific a process based on a Brownian motion, a fractional Brownian motion, a generalized hyperbolic Lévy process or a CGMY process is suitable, depending on the time scale. Using tic-by-tic data the method may also be used to analyze the market microstructure. In addition, the Blumenthal-Gettoor index or the Hurst exponent is explicitly needed for power variation estimates for the integrated volatility, which is a measure of the volatility risk and the basic quantity for pricing variance and volatility swaps.

We provide a class of easily computable estimators for the Blumenthal-Gettoor index and the Hurst exponent respectively, based on high frequency data. Furthermore, we derive consistency and a distributional result for these estimators.