

# Designing realised kernels to measure the ex-post variation of equity prices in the presence of noise\*

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## Abstract

This paper shows how to use realised kernels to carry out efficient feasible inference on the ex-post variation of underlying equity prices in the presence of simple models of market frictions. The analysis encompasses the case where subsampling is employed. The issue is subtle with only estimators which have symmetric weights delivering consistent estimators with mixed Gaussian limit theorems. The weights can be chosen to achieve the best possible rate of convergence and to have an asymptotic variance which is close to that of the maximum likelihood estimator in the parametric version of this problem. It parallels the independent and concurrent multiscale approach of Zhang (2004). This type of kernel can also be made to be robust to temporally dependent noise and endogenous sampling times. The finite sample performance of our estimators is studied using simulation, while empirical work illustrates their use in practice.

Keywords: Bipower variation; Long run variance estimator; Market frictions; Quadratic variation; Realised variance; Subsampling.

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