Out of Sample Forecasts of Quadratic Variation

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Introduction

High frequency data:

**Observed** log-price = **latent** efficient log-price + microstructure noise

\[ Y = X + \varepsilon \]

where \( \varepsilon \) summarizes bid-ask bounces, price discreteness, differences in trade sizes, gradual response of prices to a block trade, . . .

e.g., Roll (1984), Easley and O’Hara (1987)
Object of interest

Daily integrated variance

\[ IV = \int_0^T \sigma_t^2 \, dt \]

of the latent price process

\[ dX_t = \mu_t \, dt + \sigma_t \, dW_t \]

estimated using discretely sample data on the observed price process \( Y \) at times: \( 0 = t_0, t_1, \ldots, t_n = T \)
Object of interest

Daily integrated variance

\[ IV = \int_{0}^{T} \sigma_{t}^{2} dt \]

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estimated using discretely sample data on the observed price process \( Y \) at times: \( 0 = t_0, t_1, \ldots, t_n = T \)

**Goal:** forecast the integrated variance
Estimate of integrated variance

- **realized volatility (RV)**
  
  e.g., Andersen, Bollerslev, Diebold and Labys (2001)
  Barndorff-Nielsen and Shephard (2002)

- **two scales realized volatility (TSRV)**
  
  Zhang, Mykland and Aït-Sahalia (2005)
Our objective

Investigate out-of-sample performance of RV and TSRV as estimators of integrated variance
Our objective

Investigate out-of-sample performance of RV and TSRV as estimators of integrated variance.

Does the better performance of TSRV extend to out-of-sample forecasts when volatility has jumps, long memory, ...?
Outline

• RV and TSRV estimators

• Forecasts of integrated variance
  – Monte Carlo experiments
  – Empirical application
Realized volatility estimator

Let $\mathcal{H} = \{t_0, t_h, t_{2h}, \ldots, t_n\}$

$$RV = \sum_{t_i \in \mathcal{H}} (Y_{t_i+h} - Y_{t_i})^2$$
Realized volatility estimator

Let $\mathcal{H} = \{t_0, t_h, t_{2h}, \ldots, t_n\}$

$$RV = \sum_{t_i \in \mathcal{H}} (Y_{t_i+h} - Y_{t_i})^2$$

Biased and inefficient estimator of integrated variance:

- **Bias** = $2n_\mathcal{H} \mathbb{E}\varepsilon^2$

- **Discard large amount of data** $\rightarrow$ inefficient estimator
  
  Example: one NYSE day and transactions every second: $n = 23,400$
  
  RV at 5 minutes uses only $n_\mathcal{H} = 78$ observations
RV asymptotic distribution

\[ \text{RV} \approx \mathcal{L} \left[ \int_0^T \sigma_t^2 \, dt + \frac{2n_H E \varepsilon^2}{n_H} \right] + \left[ 4n_H E \varepsilon^4 + \frac{2T}{n_H} \int_0^T \sigma_t^4 \, dt \right]^{1/2} Z_{\text{total}} \]

- \( \int_0^T \sigma_t^2 \, dt \): object of interest
- \( \frac{2n_H E \varepsilon^2}{n_H} \): bias due to noise
- \( 4n_H E \varepsilon^4 \): due to noise
- \( \frac{2T}{n_H} \int_0^T \sigma_t^4 \, dt \): due to discretization
- Total variance

\( Z_{\text{total}} \)
Two Scales Realized Volatility estimator

- Subsampling: Partition the original grid \( G \) into \( K \) subsamples, \( G^{(k)} \), \( k = 1, \ldots, K \), e.g.

\[
G = \{ t_0, t_1, \ldots, t_K, t_{K+1}, \ldots, t_n \} \\
G^{(1)} = \{ t_0, t_K, \ldots \} \\
G^{(2)} = \{ t_1, t_{K+1}, \ldots \} \\
\vdots \\
G^{(K)} = \{ t_{K-1}, t_{2K-1}, \ldots \}
\]

and compute \( [Y, Y]^{(k)} = \sum_{t_i \in G^{(k)}} (Y_{t_i+K} - Y_t)^2 \)
Two Scales Realized Volatility estimator

- **Subsampling**: Partition the original grid $G$ into $K$ subsamples, $G^{(k)}$, $k = 1, \ldots, K$, e.g.

  \[
  G = \{ t_0 \ t_1 \ \ldots \ t_K \ t_{K+1} \ \ldots \ t_n \}
  \]

  \[
  G^{(1)} = \{ t_0 \ t_K \ \ldots \ \}
  \]

  \[
  G^{(2)} = \{ t_1 \ t_{K+1} \ \ldots \ \}
  \]

  \[
  \vdots
  \]

  \[
  G^{(K)} = \{ t_{K-1} \ t_{2K-1} \ \ldots \ \}
  \]

  and compute $[Y, Y]^{(k)}(k) = \sum_{t_i \in G^{(k)}} (Y_{t_i+K} - Y_{t_i})^2$

- **Averaging**: $[Y, Y]^{(avg)}(avg) = \sum_{k=1}^{K} [Y, Y]^{(k)} / K$
Two Scales Realized Volatility estimator

- **Subsampling**: Partition the original grid $G$ into $K$ subsamples, $G^{(k)}$, $k = 1, \ldots, K$, e.g.

  \[
  G = \{ t_0 \ t_1 \ \ldots \ t_K \ t_{K+1} \ \ldots \ t_n \}
  \]
  \[
  G^{(1)} = \{ t_0 \ \ldots \ t_K \}
  \]
  \[
  G^{(2)} = \{ t_1 \ t_{K+1} \}
  \]
  \[
  \vdots
  \]
  \[
  G^{(K)} = \{ t_{K-1} \ t_{2K-1} \ \ldots \}
  \]
  
  and compute $[Y, Y]^{(k)} = \sum_{t_i \in G^{(k)}} (Y_{t_i+K} - Y_{t_i})^2$

- **Averaging**: $[Y, Y]^{(avg)} = \frac{\sum_{k=1}^{K} [Y, Y]^{(k)}}{K}$

- **Bias-correction**:

  \[
  \text{TSRV} = [Y, Y]^{(avg)} - \frac{n}{\bar{n}}[Y, Y]^{(all)} \]

  slow time scale

  fast time scale
TSRV asymptotic distribution

Number of subsamples $K = c n^{2/3}$

$$
\text{TSRV} \overset{c}{\approx} \int_0^T \sigma_t^2 \, dt + \frac{1}{n^{1/6}} \left[ \frac{8}{c^2} E[\varepsilon^2]^2 + c T \frac{4}{3} \int_0^T \sigma_t^4 \, dt \right]^{1/2} Z_{\text{total}}
$$

object of interest due to noise due to discretization total variance

$K^*$ minimizes $\text{Var}[\text{TSRV}]$, $K^* = c^* n^{2/3}$,

$$
c^* = \left( \frac{T}{12 E[\varepsilon^2]^2} \int_0^T \sigma_t^4 \, dt \right)^{-1/3}
$$
Monte Carlo simulation: Setup

**Latent log-price:**  
\[ dX_t = (0.05 - \sigma_t^2/2) \, dt + \sigma_t \, dW_t \]

**Four different dynamics for \( \sigma_t \), time step = 1 second**
Monte Carlo simulation: Setup

**Latent log-price:**
\[ dX_t = (0.05 - \sigma_t^2/2) \, dt + \sigma_t \, dW_t \]

**Four different dynamics for \( \sigma_t \), time step = 1 second**

**Observed log-price:**
\[ Y_\tau = X_\tau + \varepsilon_\tau, \quad \varepsilon_\tau \sim NID\left(0, (0.10/100)^2\right) \]
Monte Carlo simulation: Setup

Latent log-price: \[ dX_t = (0.05 - \sigma_t^2/2) \, dt + \sigma_t \, dW_t \]

Four different dynamics for \( \sigma_t \), time step = 1 second

Observed log-price: \[ Y_T = X_T + \varepsilon_T, \quad \varepsilon_T \sim NID\left(0, (0.10/100)^2\right) \]

Each sample path: 101 trading days \( \rightarrow \) 100 estimates of daily IV

Each experiment: 10,000 simulated sample paths
Estimate of daily integrated variance

- RV:
  - 5, 10, 15, 30 minutes, raw log-returns
  - de-meaned MA(1) filtered 5 minutes log-returns

- TSRV: slow time scale 5, 10, 15, 30 minutes, and minimum variance
**Experiment 1: Heston volatility model**

Heston (1993) model: \( d\sigma^2 = 5(0.04 - \sigma^2)\, dt + 0.5\, \sigma dW \)

**In-sample estimates of daily IV**

<table>
<thead>
<tr>
<th></th>
<th>Bias</th>
<th>Var</th>
<th>RMSE</th>
<th>Relative</th>
<th>Bias</th>
<th>Var</th>
<th>RMSE</th>
</tr>
</thead>
<tbody>
<tr>
<td>RV 5mn</td>
<td>1.560</td>
<td>0.318</td>
<td>1.659</td>
<td></td>
<td>2.508</td>
<td>47.701</td>
<td>7.348</td>
</tr>
<tr>
<td>RV 5mn MA(1)</td>
<td>1.348</td>
<td>0.274</td>
<td>1.446</td>
<td></td>
<td>2.026</td>
<td>26.985</td>
<td>5.576</td>
</tr>
<tr>
<td>TSRV 5mn</td>
<td>−0.014</td>
<td>0.071</td>
<td>0.266</td>
<td></td>
<td>−0.011</td>
<td>0.022</td>
<td>0.149</td>
</tr>
<tr>
<td>TSRV min var</td>
<td>−0.001</td>
<td>0.020</td>
<td>0.140</td>
<td></td>
<td>−0.004</td>
<td>0.010</td>
<td>0.099</td>
</tr>
</tbody>
</table>
Mincer-Zarnowitz regression:

\[ IV^{true} = b_0 + b_1 \text{TSRV}^{\text{forecast}} + b_2 \text{RV}^{\text{forecast}} + \text{error} \]

<table>
<thead>
<tr>
<th></th>
<th>( b_0 )</th>
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<tbody>
<tr>
<td>RV 5mn</td>
<td>-1.467 (0.016)</td>
<td></td>
<td>0.971 (0.005)</td>
<td>0.809</td>
</tr>
<tr>
<td>TSRV 5mn</td>
<td>0.018 (0.006)</td>
<td>0.991 (0.003)</td>
<td></td>
<td>0.928</td>
</tr>
<tr>
<td>TSRV 5mn vs.</td>
<td></td>
<td></td>
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<td></td>
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<tr>
<td>RV 5mn</td>
<td>0.146 (0.016)</td>
<td>1.057 (0.008)</td>
<td>-0.074 (0.009)</td>
<td>0.928</td>
</tr>
<tr>
<td>RV 10mn</td>
<td>0.159 (0.010)</td>
<td>1.120 (0.008)</td>
<td>-0.146 (0.008)</td>
<td>0.930</td>
</tr>
<tr>
<td>RV 15mn</td>
<td>0.119 (0.008)</td>
<td>1.107 (0.007)</td>
<td>-0.134 (0.007)</td>
<td>0.930</td>
</tr>
<tr>
<td>RV 30mn</td>
<td>0.089 (0.007)</td>
<td>1.085 (0.006)</td>
<td>-0.118 (0.006)</td>
<td>0.930</td>
</tr>
<tr>
<td>TSRV min var</td>
<td>0.005 (0.004)</td>
<td>0.993 (0.002)</td>
<td></td>
<td>0.961</td>
</tr>
</tbody>
</table>
Experiment 2: Jump-diffusion volatility model

Heston Jump-diffusion model (e.g., Eraker, Johannes and Polson, 2003)

\[ d\sigma^2 = 5 (0.035 - \sigma^2) \, dt + 0.5\sigma \, dW + J \, dq \]

Poisson process \( q \), jump size \( J \sim Exp \)

number of jumps \( \approx 2 \) per day; jump size \( \approx 2\% \) of unconditional variance
Out-of-sample forecasts: Heston jump-diffusion model

Model: \[ d\sigma^2 = 5 (0.035 - \sigma^2) dt + 0.5 \sigma dW + Jdq, \] number of jumps \( \approx 2 \) per day; jump size \( \approx 2\% \) of unconditional variance
Experiment 3:
Heterogeneous Autoregressive Realized Volatility model

Corsi (2004): Cascade model for the integrated variance

\[
IV^{(m)} \leftarrow RV^{(m)} \\
IV^{(w)} \leftarrow RV^{(m)}, RV^{(w)} \\
IV^{(d)} \leftarrow RV^{(m)}, RV^{(w)}, RV^{(d)}
\]
Experiment 3:
Heterogeneous Autoregressive Realized Volatility model

Corsi (2004): Cascade model for the integrated variance

\[
IV^{(m)} \leftarrow RV^{(m)} \\
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IV^{(d)} \leftarrow RV^{(m)}, RV^{(w)}, RV^{(d)}
\]

Reduced form:

\[
IV_{t+\Delta}^{(d)} = 0.002 + 0.45 RV_{t}^{(d)} + 0.30 RV_{t}^{(w)} + 0.20 RV_{t}^{(m)} + \text{error}
\]

\[
RV_{t}^{(d)} = \sum_{t_j \in \{\text{day } t\}} r_{t_j}^2 \\
\text{and similarly } RV_{t}^{(w)} \text{ and } RV_{t}^{(m)}
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Experiment 3:  
Heterogeneous Autoregressive Realized Volatility model

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Reduced form:

\[ IV_{t+\Delta} = 0.002 + 0.45 RV_t^{(d)} + 0.30 RV_t^{(w)} + 0.20 RV_t^{(m)} + \text{error} \]

\[ RV_t^{(d)} = \sum_{t_j \in \{\text{day } t\}} r_{t_j}^2 \quad \text{and similarly } RV_t^{(w)} \text{ and } RV_t^{(m)} \]

Intrady latent log-return

\[ r_t = X_t - X_{t-\Delta} = \sqrt{IV_t^{(d)}} z_t \]

and \( z_t \sim NID(0, 1) \), \( \Delta = 1 \) second
Simulated path of HAR-RV process, $IV^{(d)} = 0.004 + 0.35 RV^{(d)} + 0.30 RV^{(w)} + 0.25 RV^{(m)} + \omega^{(d)}$

$\omega^{(d)} \sim NID(0, 0.003^2)$. Efficient log-return $r_t = X_t - X_{t-\Delta}$, and $\Delta = 3$ hours
Out-of-sample forecasts of IV: HAR-RV model

Mincer-Zarnowitz regression:

\[ \text{IV}^{\text{true}} = b_0 + b_1 \text{TSRV}^{\text{forecast}} + b_2 \text{RV}^{\text{forecast}} + \text{error} \]

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<tr>
<td>RV 5mn</td>
<td>-1.499 (0.017)</td>
<td>0.981 (0.005)</td>
<td>0.762</td>
<td></td>
</tr>
<tr>
<td>TSRV 5mn</td>
<td>-0.005 (0.006)</td>
<td>1.017 (0.004)</td>
<td>0.891</td>
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TSRV 5mn vs.

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<tr>
<td>RV 10mn</td>
<td>-0.027 (0.011)</td>
<td>0.996 (0.009)</td>
<td>0.023 (0.009)</td>
<td>0.891</td>
</tr>
<tr>
<td>RV 15mn</td>
<td>-0.012 (0.009)</td>
<td>1.009 (0.008)</td>
<td>0.009 (0.009)</td>
<td>0.891</td>
</tr>
<tr>
<td>RV 30mn</td>
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<td>0.998 (0.007)</td>
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<td>0.891</td>
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<tr>
<td>TSRV min var</td>
<td>0.009 (0.005)</td>
<td>0.997 (0.003)</td>
<td>0.918</td>
<td></td>
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</table>
Experiment 4: Fractional Ornstein-Uhlenbeck model

Long memory model: $W_H$ fractional Brownian motion, $H = 0.7$

$$dZ = 20 (0.2 - Z) \, dt + 0.012 \, dW_H$$

e.g., Comte and Renault (1998), Cheridito, Kawaguchi and Maejima (2003)
Experiment 4: Fractional Ornstein-Uhlenbeck model

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$$dZ = 20(0.2 - Z)\, dt + 0.012\, dW_H$$

e.g., Comte and Renault (1998), Cheridito, Kawaguchi and Maejima (2003)

Set $\sigma = Z$ and forecast

$$I_{\sigma_m} = \int_{T_m}^{T_{m+1}} \sigma_t \, dt$$
Experiment 4: Fractional Ornstein-Uhlenbeck model

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Set $\sigma = Z$ and forecast

$$I_{\sigma m} = \int_{T_m}^{T_{m+1}} \sigma_t \, dt$$

Exactly specified forecast regression:

$$E[I_{\sigma m} | I_{\sigma m-1}] = E[I_{\sigma m}] + \frac{\text{Cov}(I_{\sigma m}, I_{\sigma m-1})}{\text{Var}(I_{\sigma m-1})} (I_{\sigma m-1} - E[I_{\sigma m-1}])$$
Simulated path of classical and fractional Ornstein-Uhlenbeck process; Comte and Renault (1998)
\[ dZ = 10 (0.2 - Z) \, dt + 0.2 \, dW_H, \] FBM with Hurst exponent \( H = 0.7, \) \( dt = 1 \) day
## In-sample estimates of IV: Fractional OU model

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<tr>
<td>RV 5mn</td>
<td>1.558</td>
<td>0.288</td>
<td>1.647</td>
<td>1.036</td>
<td>0.219</td>
<td>1.137</td>
<td></td>
</tr>
<tr>
<td>RV 5mn MA(1)</td>
<td>1.360</td>
<td>0.229</td>
<td>1.442</td>
<td>0.900</td>
<td>0.155</td>
<td>0.982</td>
<td></td>
</tr>
<tr>
<td>TSRV 5mn</td>
<td>−0.018</td>
<td>0.048</td>
<td>0.220</td>
<td>−0.011</td>
<td>0.017</td>
<td>0.132</td>
<td></td>
</tr>
<tr>
<td>TSRV min var</td>
<td>−0.006</td>
<td>0.017</td>
<td>0.129</td>
<td>−0.004</td>
<td>0.006</td>
<td>0.080</td>
<td></td>
</tr>
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</table>
Distribution of TSRV: Fractional OU model

Asymptotic and small sample distributions of minimum variance TSRV
Out-of-sample forecasts of IV: Fractional OU model

Mincer-Zarnowitz regression:

$$IV^{true} = b_0 + b_1 \text{TSRV}^{\text{forecast}} + b_2 \text{RV}^{\text{forecast}} + error$$

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<tr>
<td>RV 5mn</td>
<td>−1.155 (0.032)</td>
<td>0.873 (0.010)</td>
<td>0.438</td>
<td></td>
</tr>
<tr>
<td>TSRV 5mn</td>
<td>0.006 (0.009)</td>
<td>1.007 (0.006)</td>
<td>0.757</td>
<td></td>
</tr>
<tr>
<td>TSRV 5mn vs.</td>
<td></td>
<td></td>
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<tr>
<td>RV 5mn</td>
<td>0.306 (0.024)</td>
<td>1.111 (0.010)</td>
<td>−0.147 (0.011)</td>
<td>0.762</td>
</tr>
<tr>
<td>RV 10mn</td>
<td>0.263 (0.017)</td>
<td>1.150 (0.009)</td>
<td>−0.202 (0.011)</td>
<td>0.766</td>
</tr>
<tr>
<td>RV 15mn</td>
<td>0.209 (0.014)</td>
<td>1.139 (0.009)</td>
<td>−0.194 (0.010)</td>
<td>0.766</td>
</tr>
<tr>
<td>RV 30mn</td>
<td>0.159 (0.012)</td>
<td>1.118 (0.008)</td>
<td>−0.176 (0.009)</td>
<td>0.766</td>
</tr>
<tr>
<td>TSRV min var</td>
<td>−0.009 (0.006)</td>
<td>1.010 (0.004)</td>
<td></td>
<td>0.877</td>
</tr>
</tbody>
</table>
Empirical application: IBM stock

- Transaction prices: TAQ database

- May 2, 2003 – May 2, 2005: from 9:30 until 16:00

- Pre-processing: eliminated
  - obvious errors (zero prices, out of order transaction times, ...)
  - bounce back outliers larger than 0.03
IBM stock from May 2003 to May 2005: daily log-return in %; daily $\sqrt{IV}$ on annual base
In-sample forecasts of $\sqrt{\text{IV}}$: IBM stock

Mincer-Zarnowitz regression:

$$\sqrt{\text{IV}}^{\text{true}} = b_0 + b_1 \text{TSRV}^{\text{forecast}} + b_2 \text{RV}^{\text{forecast}} + \text{error}$$

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<td>$1.056$</td>
<td>$0.371$</td>
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<tr>
<td>vs. RV 5mn</td>
<td>$-0.058$</td>
<td>$1.079$</td>
<td>$-0.036$</td>
<td>$0.371$</td>
</tr>
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One day ahead forecasts of $\sqrt{\text{IV}}$: AR(1) model for IV, estimated using the full sample (May 2003–May 2005). 503 in-sample forecasts.

Benchmark $\sqrt{\text{IV}}$: TSRV estimates
Out-of-sample forecasts of $\sqrt{IV}$: IBM stock

Mincer-Zarnowitz regression:

$$\sqrt{IV}^{\text{true}} = b_0 + b_1 \cdot \text{TSRV}^{\text{forecast}} + b_2 \cdot \text{RV}^{\text{forecast}} + \text{error}$$

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<tr>
<td>RV 5mn</td>
<td>$-0.224$ (0.085)</td>
<td>$1.156$ (0.091)</td>
<td>$0.350$</td>
<td></td>
</tr>
<tr>
<td>TSRV 5mn</td>
<td>$-0.107$ (0.069)</td>
<td>$1.082$ (0.077)</td>
<td>$0.395$</td>
<td></td>
</tr>
<tr>
<td>vs. RV 5mn</td>
<td>$-0.131$ (0.084)</td>
<td>$0.984$ (0.206)</td>
<td>$0.121$ (0.234)</td>
<td>$0.396$</td>
</tr>
</tbody>
</table>

One day ahead forecasts of $\sqrt{IV}$: AR(1) model for IV re-estimated every day using the past 200 estimates of IV. 303 out-of-sample forecasts. Benchmark $\sqrt{IV}$: TSRV estimates
Out-of-sample forecasts of $\sqrt{IV}$: IBM stock

One day ahead forecasts of $\sqrt{IV}$. AR(1) model for IV re-estimated every day
Conclusions

- Monte Carlo evidence: TSRV outperforms RV in forecasting integrated variance *even* when volatility has jumps, long memory, . . .

- Empirical application confirms Monte Carlo results
Current research

- Dependent microstructure noise:
  \( \epsilon \) neither time-independent nor independent of latent price \( X \)

- Extend the empirical part