A new approach to LIBOR modeling

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Based on joint work with Martin Keller-Ressel and Josef Teichmann

Workshop on Statistical Inference for Lévy processes and Applications to Finance
Outline of the talk

1. Interest rate markets
2. LIBOR model: Axioms
3. LIBOR and Forward price model
4. Affine processes
5. Affine martingales
6. Affine LIBOR model
7. Example: CIR martingales
8. Summary and Outlook
Interest rates – Notation

- $B(t, T)$: time-$t$ price of a zero coupon bond for $T$; $B(T, T) = 1$;
- $L(t, T)$: time-$t$ forward LIBOR for $[T, T + \delta]$;

$$L(t, T) = \frac{1}{\delta} \left( \frac{B(t, T)}{B(t, T + \delta)} - 1 \right)$$

- $F(t, T, U)$: time-$t$ forward price for $T$ and $U$; $F(t, T, U) = \frac{B(t, T)}{B(t, U)}$

“Master” relationship

$$F(t, T, T + \delta) = \frac{B(t, T)}{B(t, T + \delta)} = 1 + \delta L(t, T) \quad (1)$$
Interest rates evolution

Evolution of interest rate term structure, 2003 – 2004 (picture: Th. Steiner)
1. Implied volatilities are constant neither across strike nor across maturity
2. Variance scales non-linearly over time (see e.g. D. Skovmand)
Economic thought dictates:

**Axiom 1**

The LIBOR rate should be non-negative, i.e. $L(t, T) \geq 0$ for all $t$.

**Axiom 2**

The LIBOR rate process should be a martingale under the corresponding forward measure, i.e. $L(\cdot, T) \in \mathcal{M}(P_{T+\delta})$.

Practical applications require:

**Axiom 3**

Models should be analytically tractable ($\leadsto$ fast calibration).

Models should have rich structural properties ($\leadsto$ good calibration).

• What axioms do the existing models satisfy?
LIBOR models I (Sandmann et al, Brace et al, . . . , Eberlein & Özkan)

Ansatz: model the LIBOR rate as the exponential of a semimartingale $H$:

$$L(t, T_k) = L(0, T_k) \exp \left( \int_0^t b(s, T_k) ds + \int_0^t \lambda(s, T_k) dH^{T_{k+1}}_s \right), \quad (2)$$

where $b(s, T_k)$ ensures that $L(\cdot, T_k) \in \mathcal{M}(P_{T_{k+1}})$.

$H^{T_{k+1}}$ has the $P_{T_{k+1}}$-canonical decomposition

$$H^{T_{k+1}}_t = \int_0^t \sqrt{c_s} dW^{T_{k+1}}_s + \int_0^t \int_{\mathbb{R}} x(\mu^H - \nu^{T_{k+1}})(ds, dx), \quad (3)$$

where the $P_{T_{k+1}}$-Brownian motion is

$$W^{T_{k+1}}_t = W^{T*}_t - \int_0^t \left( \sum_{l=k+1}^N \frac{\delta_l L(t-, T_l)}{1 + \delta_l L(t-, T_l)} \lambda(t, T_l) \right) \sqrt{c_s} ds, \quad (4)$$
and the $P_{T_{k+1}}$-compensator of $\mu^H$ is

$$\nu^{T_{k+1}}(ds, dx) = \left( \prod_{l=k+1}^{N} \frac{\delta_l L(t-, T_l)}{1 + \delta_l L(t-, T_l)} \left( e^{\lambda(t, T_l)x} - 1 \right) + 1 \right) \nu^{T_*}(ds, dx).$$
and the $P_{T_k+1}$-compensator of $\mu^H$ is

$$
\nu^{T_{k+1}}(ds, dx) = \left( \prod_{l=k+1}^{N} \frac{\delta_l L(t-, T_l)}{1 + \delta_l L(t-, T_l)} \left( e^{\lambda(t, T_l) x} - 1 \right) + 1 \right) \nu^{T_\ast}(ds, dx).
$$

**Consequences for continuous semimartingales:**

1. Caplets can be priced in closed form;
2. Swaptions and multi-LIBOR products cannot be priced in closed form;
3. Monte-Carlo pricing is very time consuming $\sim$ coupled high dimensional SDEs!
LIBOR and Forward price model

LIBOR models II

and the $P_{T_{k+1}}$-compensator of $\mu^H$ is

$$
\nu^{T_{k+1}}(ds, dx) = \left( \prod_{l=k+1}^{N} \frac{\delta_l L(t-, T_l)}{1 + \delta_l L(t-, T_l)} \left( e^{\lambda(t, T_l) x} - 1 \right) + 1 \right) \nu^{T*}(ds, dx).
$$

Consequences for continuous semimartingales:

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Consequences for general semimartingales:

1. even caplets cannot be priced in closed form!
2. ditto for Monte-Carlo pricing.
“Frozen drift” approximation

Brace et al, Schlögl, Glassermann et al, ...
replace the random terms by their deterministic initial values:

\[
\frac{\delta_i L(t-, T_i)}{1 + \delta_i L(t-, T_i)} \approx \frac{\delta_i L(0, T_i)}{1 + \delta_i L(0, T_i)}
\]  \hspace{1cm} (5)

(+): deterministic characteristics \(\Rightarrow\) closed form pricing

(−): “ad hoc” approximation, no error estimates, compounded error ...

Log-normal and/or Monte Carlo methods

best log-normal approximation (e.g. Schoenmakers)
interpolations and predictor-corrector MC methods
Joshi and Stacey (2008): overview paper
**LIBOR models IV: Remedies**

3. **Strong Taylor approximation**
   - approximate the LIBOR rates in the drift by
     \[
     L(t, T_l) \approx L(0, T_l) + Y(t, T_l)
     \]
     where \( Y \) is the (scaled) exponential transform of \( H \) (\( Y = \log e^H \))
   - theoretical foundation, error estimates, simpler equations for MC
   - Siopacha & Teichmann (2007); Papapantoleon & Siopacha (2009)

Difference in implied vols between full SDE vs frozen drift and full SDE vs strong Taylor.
Forward price model I (Eberlein & Özkan, Kluge)

Ansatz: model the forward price as the exponential of a semimartingale $H$:

$$F(t, T_k) = F(0, T_k) \exp \left( \int_0^t b(s, T_k)ds + \int_0^t \lambda(s, T_k)dH_s^{T_{k+1}} \right),$$

where $b(s, T_k)$ ensures that $F(\cdot, T_k) = 1 + \delta L(\cdot, T_k) \in \mathcal{M}(P_{T_{k+1}})$. $H^{T_{k+1}}$ has the $P_{T_{k+1}}$-canonical decomposition

$$H_t^{T_{k+1}} = \int_0^t \sqrt{c_s}dW_s^{T_{k+1}} + \int_0^t \int_\mathbb{R} x(\mu^H - \nu^{T_{k+1}})(ds, dx),$$

where the $P_{T_{k+1}}$-Brownian motion is

$$W_t^{T_{k+1}} = W_t^{T_*} - \int_0^t \left( \sum_{l=k+1}^N \lambda(t, T_l) \right) \sqrt{c_s}ds.$$
and the $P_{T_{k+1}}$-compensator of $\mu^H$ is

$$\nu^{T_{k+1}}(ds, dx) = \exp \left( x \sum_{l=k+1}^{N} \lambda(t, T_l) \right) \nu^{T^*}(ds, dx).$$

**Consequences:**

1. the model structure is preserved;
2. caps, swaptions and multi-LIBOR products priced in closed form.

**So, what is wrong?**
Forward price model II

and the $P_{T_{k+1}}$-compensator of $\mu^H$ is

$$\nu^{T_{k+1}}(ds, dx) = \exp\left( x \sum_{l=k+1}^{N} \lambda(t, T_l) \right) \nu^{T*}(ds, dx).$$

Consequences:

1. the model structure is preserved;
2. caps, swaptions and multi-LIBOR products priced in closed form.

So, what is wrong?

Negative LIBOR rates can occur!
LIBOR and Forward price model: other questions

1. Modeling concerns: model $L(t, T)$ or $1 + \delta L(t, T)$ as $e^H$?

2. Distributional concerns: log-normal vs. normal . . .
1 Modeling concerns: model $L(t, T)$ or $1 + \delta L(t, T)$ as $e^H$?

2 Distributional concerns: log-normal vs. normal . . .

3 Does there exist a “quick fix”? **No!**
LIBOR and Forward price model: other questions

1. Modeling concerns: model $L(t, T)$ or $1 + \delta L(t, T)$ as $e^H$?
2. Distributional concerns: log-normal vs. normal . . .
3. Does there exist a “quick fix”? No!

Aim: design a model where the model structure is preserved and LIBOR rates are positive.

Tool: Affine processes on $\mathbb{R}_{\geq 0}^d$. 
Affine processes

Let \( X = (X_t)_{0 \leq t \leq T} \) be a time-homogeneous Markov process taking values in \( D = \mathbb{R}_d^d \geq 0 \); and \((P_x)_{x \in D}\) a family of probability measures on \((\Omega, \mathcal{F})\), such that \( X_0 = x \), \( P_x \)-a.s. for every \( x \in D \). Setting

\[
\mathcal{I}_T := \left\{ u \in \mathbb{R}^d : E_x \left[ e^{\langle u, X_T \rangle} \right] < \infty, \text{ for all } x \in D \right\},
\]

we assume that

(i) \( 0 \in \mathcal{I}_T^0 \);

(ii) the conditional moment generating function of \( X_t \) under \( P_x \) has exponentially-affine dependence on \( x \); i.e. there exist functions \( \phi_t(u) : [0, T] \times \mathcal{I}_T \to \mathbb{R} \) and \( \psi_t(u) : [0, T] \times \mathcal{I}_T \to \mathbb{R}^d \) such that

\[
E_x \left[ \exp \langle u, X_t \rangle \right] = \exp \left( \phi_t(u) + \langle \psi_t(u), x \rangle \right)
\]

for all \((t, u, x) \in [0, T] \times \mathcal{I}_T \times D\).
Affine processes II

The process $X$ is a regular affine process in the spirit of DFS. We can show that

$$F(u) := \frac{\partial}{\partial t} \bigg|_{t=0^+} \phi_t(u) \quad \text{and} \quad R(u) := \frac{\partial}{\partial t} \bigg|_{t=0^+} \psi_t(u) \quad (12)$$

exist for all $u \in \mathcal{I}_\mathcal{T}$ and are continuous in $u$. Moreover, $F$ and $R$ satisfy Lévy–Khintchine-type equations:

$$F(u) = \langle b, u \rangle + \int_{D} \left( e^{\langle \xi, u \rangle} - 1 \right) m(d\xi) \quad (13)$$

and

$$R_i(u) = \langle \beta_i, u \rangle + \left\langle \frac{\alpha_i}{2} u, u \right\rangle + \int_{D} \left( e^{\langle \xi, u \rangle} - 1 - \langle u, h^i(\xi) \rangle \right) \mu_i(d\xi), \quad (14)$$

where $(b, m, \alpha_i, \beta_i, \mu_i)_{1 \leq i \leq d}$ are admissible parameters.
Lemma (Flow property)

The functions \( \phi \) and \( \psi \) satisfy the semi-flow equations:

\[
\begin{align*}
\phi_{t+s}(u) &= \phi_t(u) + \phi_s(\psi_t(u)) \\
\psi_{t+s}(u) &= \psi_s(\psi_t(u))
\end{align*}
\] (15)

with initial condition

\[
\begin{align*}
\phi_0(u) &= 0 \quad \text{and} \quad \psi_0(u) = u,
\end{align*}
\] (16)

for all suitable \( 0 \leq t + s \leq T \) and \( u \in \mathcal{I}_T \).
Affine martingales

Affine LIBOR model: martingales $\geq 1$

Idea:
1. insert an affine process in its moment generating function with inverted time; the resulting process is a martingale;
2. if the affine process is positive, the martingale is greater than one.

Theorem

The process $M^u = (M^u_t)_{0 \leq t \leq T}$ defined by

$$M^u_t = \exp \left( \phi_{T-t}(u) + \langle \psi_{T-t}(u), X_t \rangle \right),$$

is a martingale. Moreover, if $u \in \mathcal{I}_T \cap \mathbb{R}^d_{\geq 0}$ then $M_t \geq 1$ a.s. for all $t \in [0, T]$, for any $X_0 \in \mathbb{R}^d_{\geq 0}$. 
Proof. Using (16) and the Markov property we have that:

\[
E_x[M_T^u | \mathcal{F}_t] = E_x[\exp\langle u, X_T \rangle | \mathcal{F}_t] \\
= \exp(\phi_{T-t}(u) + \langle \psi_{T-t}(u), X_t \rangle) = M_t^u.
\]

Regarding \( M_t^u \geq 1 \) for all \( t \in [0, T] \): note that if \( u \in \mathcal{I}_T \cap \mathbb{R}^{d}_{\geq 0} \), then

\[
M_t^u = E_x[\exp\langle u, X_T \rangle | \mathcal{F}_t] \geq 1. \tag{18}
\]
Affine LIBOR model: Ansatz

Consider a discrete tenor structure \(0 = T_0 < T_1 < T_2 < \cdots < T_N;\) discounted bond prices must satisfy:

\[
\frac{B(\cdot, T_k)}{B(\cdot, T_N)} \in \mathcal{M}(P_{T_N}), \quad \text{for all } k \in \{1, \ldots, N - 1\}. \tag{19}
\]

**Ansatz**

We model quotients of bond prices using the martingales \(M:\)

\[
\frac{B(t, T_1)}{B(t, T_N)} = M^u_1 \tag{20}
\]

\[\vdots\]

\[
\frac{B(t, T_{N-1})}{B(t, T_N)} = M^u_{N-1}, \tag{21}
\]

with initial conditions: \(\frac{B(0, T_k)}{B(0, T_N)} = M^u_0, \text{ for all } k \in \{1, \ldots, N - 1\}.\)
Proposition

Let $L(0, T_1), \ldots, L(0, T_N)$ be a tenor structure of non-negative initial LIBOR rates; let $X$ be an affine process starting at the canonical value 1.

1. If $\gamma_X := \sup_{u \in \mathcal{I}_T \cap \mathbb{R}_d^+} E_1 \left[ e^{\langle u, X_T \rangle} \right] > \frac{B(0, T_1)}{B(0, T_N)}$, then there exists a decreasing sequence $u_1 \geq u_2 \geq \cdots \geq u_N = 0$ in $\mathcal{I}_T \cap \mathbb{R}_d^+$, such that

$$M_0^{u_k} = \frac{B(0, T_k)}{B(0, T_N)}, \quad \text{for all } k \in \{1, \ldots, N\}. \quad (22)$$

In particular, if $\gamma_X = \infty$, then the affine LIBOR model can fit any term structure of non-negative initial LIBOR rates.

2. If $X$ is one-dimensional, the sequence $(u_k)_{k \in \{1, \ldots, N\}}$ is unique.

3. If all initial LIBOR rates are positive, the sequence $(u_k)_{k \in \{1, \ldots, N\}}$ is strictly decreasing.
Affine LIBOR model: forward prices

Forward prices have the following form

\[
\frac{B(t, T_k)}{B(t, T_{k+1})} = \frac{B(t, T_k)}{B(t, T_N)} \frac{B(t, T_N)}{B(t, T_{k+1})} = \frac{M_t^{u_k}}{M_t^{u_{k+1}}}
\]

\[
= \exp \left( \phi_{T_N-t}(u_k) - \phi_{T_N-t}(u_{k+1}) \right. \\
\left. + \langle \psi_{T_N-t}(u_k) - \psi_{T_N-t}(u_{k+1}), X_t \rangle \right)
\]

(23)

Now, \( \phi_t(\cdot) \) and \( \psi_t(\cdot) \) are order-preserving, i.e.

\[ u \geq v \Rightarrow \phi_t(u) \geq \phi_t(v) \text{ and } \psi_t(u) \geq \psi_t(v). \]

**Consequently:** positive initial LIBOR rate yields positive LIBOR rates for all times.
Forward measures are related via:

\[
\frac{dP_{T_k}}{dP_{T_{k+1}}} \bigg|_{\mathcal{F}_t} = \frac{F(t, T_k, T_{k+1})}{F(0, T_k, T_{k+1})} = \frac{B(0, T_{k+1})}{B(0, T_k)} \times \frac{M_{u_k}^t}{M_{u_{k+1}}^t} \quad (24)
\]

or equivalently:

\[
\frac{dP_{T_{k+1}}}{dP_{T_N}} \bigg|_{\mathcal{F}_t} = \frac{B(0, T_N)}{B(0, T_{k+1})} \times \frac{B(t, T_{k+1})}{B(t, T_N)} = \frac{B(0, T_N)}{B(0, T_{k+1})} \times M_{u_{k+1}}^t. \quad (25)
\]

Hence, we can easily see that

\[
\frac{B(\cdot, T_k)}{B(\cdot, T_{k+1})} = \frac{M_{u_k}^t}{M_{u_{k+1}}^t} \in \mathcal{M}(P_{T_{k+1}}) \quad \text{since} \quad M_{u_k}^t \in \mathcal{M}(P_{T_N}). \quad (26)
\]
Affine LIBOR model: dynamics under forward measures

The moment generating function of $X_t$ under any forward measure is

$$E_{P_{T_{k+1}}} \left[ e^{\nu X_t} \right] = M_{0}^{u_{k+1}} E_{P_{T_N}} \left[ M_{t}^{u_{k+1}} e^{\nu X_t} \right]$$

$$= \exp \left( \phi_t (\psi_{T_N-t}(u_{k+1}) + \nu) - \phi_t (\psi_{T_N-t}(u_{k+1})) \right)$$

$$+ \langle \psi_t (\psi_{T_N-t}(u_{k+1}) + \nu) - \psi_t (\psi_{T_N-t}(u_{k+1})), x \rangle),$$

hence $X$ is time-inhomogeneous affine under any $P_{T_{k+1}}$. Note also the “Esscher structure”.
Affine LIBOR model: dynamics under forward measures

The moment generating function of $X_t$ under any forward measure is

$$E_{P_{T_{k+1}}} \left[ e^{vX_t} \right] = M_0^{u_{k+1}} E_{P_{T_N}} \left[ M_t^{u_{k+1}} e^{vX_t} \right]$$

$$= \exp \left( \phi_t (\psi_{T_N-t}(u_{k+1}) + v) - \phi_t (\psi_{T_N-t}(u_{k+1})) \right)$$

$$+ \left\langle \psi_t (\psi_{T_N-t}(u_{k+1}) + v) - \psi_t (\psi_{T_N-t}(u_{k+1))), x \right\rangle,$$  

hence $X$ is time-inhomogeneous affine under any $P_{T_{k+1}}$. Note also the “Esscher structure”.

Moreover, denote by $\frac{M_t^{u_k}}{M_t^{u_{k+1}}} = e^{A_k + B_k \cdot X_t}$; then

$$E_{P_{T_{k+1}}} \left[ e^{v(A_k + B_k \cdot X_t)} \right] = \frac{B(0, T_N)}{B(0, T_{k+1})} \exp \left( A'_k + \left\langle B'_k, x \right\rangle \right),$$

where $A'_k$ and $B'_k$ are explicitly known in terms of $\phi$ and $\psi$. 
Affine LIBOR model: caplet pricing

We can re-write the payoff of a caplet as follows (here $\mathcal{K} := 1 + \delta K$):

$$
\delta(L(T_k, T_k) - K)^+ = (1 + \delta L(T_k, T_k) - 1 + \delta K)^+
= \left( \frac{M_{u_k}^T}{M_{u_{k+1}}^T} - \mathcal{K} \right)^+ = \left( e^{A_k + B_k \cdot X_{T_k}} - \mathcal{K} \right)^+. \quad (29)
$$

Then we can price caplets by Fourier-transform methods:

$$
\mathbb{C}(T_k, K) = B(0, T_{k+1}) E_{P_{T_{k+1}}} \left[ \delta(L(T_k, T_k) - K)^+ \right]
= \mathcal{K} B(0, T_{k+1}) \int_{\mathbb{R}} \mathcal{K}^{-R} \frac{\Lambda_{A_k + B_k \cdot X_{T_k}} (R - i v)}{(R - iv)(R - 1 - iv)} dv \quad (30)
$$

where $\Lambda_{A_k + B_k \cdot X_{T_k}}$ is given by (28).

Similar formula for swaptions (1D affine process).
CIR martingales

The Cox-Ingersoll-Ross (CIR) process is given by

\[ dX_t = -\lambda (X_t - \theta) \, dt + 2\eta \sqrt{X_t} \, dW_t, \quad X_0 = x \in \mathbb{R}_{\geq 0}, \quad (31) \]

where \( \lambda, \theta, \eta \in \mathbb{R}_{\geq 0} \). This is an affine process on \( \mathbb{R}_{\geq 0} \), with

\[ E_x[e^{uX_t}] = \exp \left( \phi_t(u) + x \cdot \psi_t(u) \right), \quad (32) \]

where

\[ \phi_t(u) = -\frac{\lambda \theta}{2\eta} \log \left( 1 - 2\eta b(t)u \right) \quad \text{and} \quad \psi_t(u) = \frac{a(t)u}{1 - 2\eta b(t)u}, \quad (33) \]

with

\[ b(t) = \begin{cases} 
  t, & \text{if } \lambda = 0 \\
  \frac{1-e^{-\lambda t}}{\lambda}, & \text{if } \lambda \neq 0 
\end{cases} \quad \text{and} \quad a(t) = e^{-\lambda t}. \]
CIR martingales: closed-form formula 1

**Definition**

A random variable $Y$ has **location-scale extended non-central chi-square distribution**, $Y \sim \text{LSNC-}\chi^2(\mu, \sigma, \nu, \alpha)$, if $\frac{Y-\mu}{\sigma} \sim \text{NC-}\chi^2(\nu, \alpha)$

Then we have that

$$X_t \overset{P_{T_N}}{\sim} \text{LSNC-}\chi^2 \left(0, \eta b(t), \frac{\lambda \theta}{\eta}, \frac{xa(t)}{\eta b(t)} \right),$$

and

$$X_t \overset{P_{T_{k+1}}}{\sim} \text{LSNC-}\chi^2 \left(0, \frac{\eta b(t)}{\zeta(t, T_N)}, \frac{\lambda \theta}{\eta}, \frac{xa(t)}{\eta b(t)\zeta(t, T_N)} \right),$$

hence

$$\log \left( \frac{B(t, T_k)}{B(t, T_{k+1})} \right) \overset{P_{T_{k+1}}}{\sim} \text{LSNC-}\chi^2 \left(A_k, \frac{B_k \eta b(t)}{\zeta(t, T_N)}, \frac{\lambda \theta}{\eta}, \frac{xa(t)}{\eta b(t)\zeta(t, T_N)} \right).$$
CIR martingales: closed-form formula II

Then, denoting by \( M = \log \left( \frac{B(T_k, T_k)}{B(T_k, T_{k+1})} \right) \) the log-forward rate, we arrive at:

\[
C(T_k, K) = B(0, T_{k+1}) \mathbb{E}_{P_{T_{k+1}}} \left[ \left( e^M - K \right)^+ \right]
= B(0, T_{k+1}) \left\{ \mathbb{E}_{P_{T_{k+1}}} \left[ e^M 1\{M \geq \log K \} \right] - K P_{T_{k+1}} [M \geq \log K] \right\}
= B(0, T_k) \cdot \frac{\chi^2_{\nu, \alpha_1}(\frac{\log K - A_k}{\sigma_1})}{\sigma_1} - K^\ast \cdot \frac{\chi^2_{\nu, \alpha_2}(\frac{\log K - A_k}{\sigma_2})}{\sigma_2},
\]

where \( K^\ast = K \cdot B(0, T_{k+1}) \) and \( \chi^2_{\nu, \alpha}(x) = 1 - \chi^2_{\nu, \alpha}(x) \), with \( \chi^2_{\nu, \alpha}(x) \) the non-central chi-square distribution function, and all the parameters are known explicitly.

Similar closed-form solution for swaptions!
Example of an implied volatility surface for the CIR martingales.
\[ dX_t = -\lambda (X_t - \theta) dt + dH_t, \quad X_0 = x \in \mathbb{R} \geq 0 \]

Example of an implied volatility surface for the Γ-OU martingales.
Summary and Outlook

1. We have presented a LIBOR model that
   - is very simple, and yet . . .
   - captures all the important features . . .
   - especially positivity and analytical tractability

2. Future work:
   - thorough empirical analysis
   - extensions: multiple currencies, default risk
   - connections to HJM framework and short rate models

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Thank you for your attention!