Vehicle Interference Effects in Warehousing Systems with Autonomous Vehicles

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Stochastic Models for Warehousing Systems
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1. Autonomous Vehicle Technology
2. Focus of Current Research
3. Queuing Model
4. Numerical Results
5. Summary
Outline

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AVS/R System Overview

- AVS/RS: Uses autonomous vehicles instead of aisle-captive cranes
- System configuration
  - Rectilinear movement
  - Horizontal movement (x and y axes) by autonomous vehicles
  - Vertical movement (z axis) by lifts
  - Vehicles move between tiers using lifts
- Modular and adaptive design
Components of an AVS/R System

Depth (in number of aisles)

Width (in number of columns)

Load/Unload Point

Cross-aisle

Tier

Aisle

Rack

Krishnamurthy et al. (UW-Madison)
Design Parameters in AVS/RS

System Sizing Decisions
- Number of vehicles and lifts
- Depth/Width ratio
- Location of cross-aisle and load/unload points
- Number of zones

Operational Decisions
- Vehicle assignment rule
- Dwell point policy
- Command cycle
- Storage policy
- Transaction scheduling policy (FCFS, Random)

Key Performance Measures
- Transaction cycle time, Queue lengths, Throughput, Vehicle utilization
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Review of Analytical Models for AVS/RS

<table>
<thead>
<tr>
<th>Author</th>
<th>Method</th>
</tr>
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<tbody>
<tr>
<td>Malmborg (2002, 2003)</td>
<td>State equation based models</td>
</tr>
<tr>
<td>Kuo et al. (2004)</td>
<td>Probabilistic approach</td>
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<tr>
<td>Zhang et al. (2008)</td>
<td>Variance based approximations</td>
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<tr>
<td>Roy et al. (2009)</td>
<td>Semi-open queuing networks</td>
</tr>
</tbody>
</table>

Objective of these models

- Model vehicle-lift interface and its effect on cycle times
- Quantify performance benefits of AVS/R systems

Limitation: These models does not account for possible vehicle interference and its effect on system performance
Focus of Current Research

Types of Vehicle Interference

On the Cross-aisle

Within an Aisle

At an intersection of Aisle and Cross-aisle

Krishnamurthy et al. (UW-Madison)
Current Research: Analyze the Effect of Vehicle Interference

- **Is the effect of vehicle interference significant?**
- Efficient single tier systems form effective building blocks for multi-tier systems

Krishnamurthy et al. (UW-Madison)
Research Approach

- Develop protocols for vehicle interference
- Develop a semi-open queuing network model of a single tier
- Solve the model using a decomposition based approach
- Validate the analytical model against simulations
- Analyze the effect of vehicle interference on performance
Protocols for Vehicle Interference

Each half of the cross-aisle has atmost one vehicle at any time $t$:

Vehicles within an aisle yield to other incoming vehicles:
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Assumptions

- **System Design Assumptions**
  - One load/unload point
  - Single command cycle
  - Random vehicle assignment
  - LU dwell point policy
  - Random storage policy
  - FCFS transaction scheduling

- **Model Assumptions**
  - Poisson arrivals
## Description of Vehicle Classes

<table>
<thead>
<tr>
<th>Vehicle Class Prior to Start of Service</th>
<th>Transaction Type</th>
<th>Vehicle Class After Start of Service</th>
</tr>
</thead>
<tbody>
<tr>
<td>Store ((s))</td>
<td>Retrieval</td>
<td>Retrieve ((r))</td>
</tr>
<tr>
<td>Store ((s))</td>
<td>Storage</td>
<td>Store ((s))</td>
</tr>
<tr>
<td>Retrieve ((r))</td>
<td>Retrieval</td>
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</tr>
<tr>
<td>Retrieve ((r))</td>
<td>Storage</td>
<td>Store ((s))</td>
</tr>
</tbody>
</table>
Vehicle Class Switching

\[ pr = \frac{\lambda_r}{\lambda_r + \lambda_s} \]

\[ ps = \frac{\lambda_s}{\lambda_r + \lambda_s} \]
Nodes of the Queuing Model

- Aisle 1
- Cross-Aisle Left ($CA_L$)
- Cross-Aisle Right ($CA_R$)
- Aisle $N$
- LUPoint
Nodes of the Queuing Model

LUPoint

Krishnamurthy et al. (UW-Madison)
Nodes of the Queuing Model
Queuing Model for a Single Tier

\[ \mu_{A1} \]

\[ \mu_{A2} \]

\[ \mu_{A_{\lceil \frac{N}{2} \rceil}} \]

\[ \mu_{A_{\lceil \frac{N}{2} \rceil} + 1} \]

\[ \mu_{AN} \]

\[ LCFS - PR \]

Class: \( r \)

\[ \lambda_r \]

\[ FCFS \]

\[ IS \]

\[ LCFS \]

\[ PR \]

\[ \mu_{LU} \]

\[ \mu_{CAL} \]

\[ \mu_{CAR} \]

\[ \mu_{CA} \]

\[ N - 1 \]

\[ N \]

\[ \lambda \]

\[ \mu \]
Queuing Model for a Single Tier

\[ \lambda_s \]

\[ \mu_{LU} \]

\[ \mu_{CAL} \]

\[ \mu_{CAR} \]

\[ \mu_{A_1} \]

\[ \mu_{A_2} \]

\[ \mu_{A_{\lfloor N/2 \rfloor}} \]

\[ \mu_{A_{\lfloor N/2 \rfloor+1}} \]

\[ \mu_{A_{N-1}} \]

\[ \mu_{A_N} \]

\[ \text{Class: } s \]
Queuing Model for a Single Tier

\[ \lambda_s + \lambda_r \]
Queuing Model for a Single Tier: Decomposition

\[ x_1 = \text{Number of transactions waiting in Buffer } B_1 \]
\[ x_2 = \text{Number of idle vehicles in Buffer } B_2 \]
\[ y = x_1 - x_2 \]

**Case 1:** \( y \geq 0 \)

**Case 2:** \( y \leq 0 \)
Decomposition Based Approach for Solving the Model

1. For case \( y \leq 0 \): Solve the closed queuing network with two classes of vehicles: Store \((s)\), and Retrieve \((r)\) using an Approximate MVA (AMVA) algorithm.

2. For case \( y \geq 0 \): Solve the open queue as an M/G/1 queue.

3. Link results from the above two cases and obtain the steady state distribution of the vehicles and transactions in the original semi-open queuing network.

4. Obtain the performance measures (cycle time, vehicle distribution in the network and vehicle utilization).
CQN for case \((y \leq 0)\)
Solution for case \( (y \leq 0) \)

Node Characteristics:

- Aisle nodes \((Q_1, \ldots, Q_N)\): LCFS-PR with exponential service times 
  \( (\mu_{A_1} = \mu_{A_2} = \ldots = \mu_{A_N}) \), where \( N \) is the number of aisles

- Cross-aisle nodes \((Q_{N+1} \text{ and } Q_{N+2})\): FCFS with uniform service times 
  \( (\mu_{CA_L} = \mu_{CA_R} \text{ and CV of 0.58}) \)

- LU nodes \((Q_{N+3})\): IS with exponential service times

- Wait for transaction node \((Q_{N+4})\): FCFS node with exponential service time

Therefore, the network is non-product form (Baskett et al. (1975)) and an Approximate MVA algorithm (Lazowska et al. (1984)) is used to obtain conditional measures.
Solution for case \((y \geq 0)\)

- When \(y \geq 0\), arriving transactions wait for a vehicle

\[
\lambda_s + \lambda_r \quad \mu_T = ?
\]

- Approach
  - Solve as an open queue with a single server station

- Challenges
  - Determine \(\mu_T^{-1}\) (average service time) and coefficient of variation (CV) of the service time
Determining $\mu_T$ for case ($y \geq 0$)

Set $\mu_T$ to be the throughput of the closed queuing network with $V$ vehicles.
Solution for case \((y \geq 0)\)

- CV of service time:
  - From simulation studies, the CV of the service time is 0.6-0.8
  - We estimate the CV by analyzing the vehicle distribution in a reduced closed queuing network

- Solution of open queue:
  - Open queue is analyzed as an M/G/1 queue
  - Determine \(\pi(i|y \geq 0)\) by analyzing an M/G/1 queue with service rate \(\mu_T\)
Unconditional Probabilities

- For Case 1 \((y \leq 0)\):
  \[
  \pi(y = i) = \sum_{q:|Q_{N+4}| = -i} \pi_q(|Q_1|, |Q_2|, \ldots, |Q_{N+4}| |y \leq 0) \pi(y \leq 0) \\
  \forall i = 0, \ldots, -V \text{ where } |Q_m| \text{ denote the number of vehicles at node } m
  \]

- For Case 2 \((y \geq 0)\):
  \[
  \pi(y = i) = \pi(i | y \geq 0) \pi(y \geq 0) \forall i = 0, \ldots, \infty
  \]

Two unknowns \(\pi(y \geq 0)\) and \(\pi(y \leq 0)\)

1. \(\pi(y = 0)\) is common to both cases
2. \(\sum_{k=-V}^{\infty} \pi(y = k) = 1\)
Using $\pi(y = i)$, we can obtain the following performance measures:

- Vehicle utilization
- Average number of transactions waiting for service
- Expected storage cycle time and retrieval cycle time
Model Validation against Simulation

- **Design Parameters**
  - Vehicles = 3, 5
  - $\frac{D}{W} = 0.5, 1.5$
  - $\lambda_s + \lambda_r = 45 - 100$ pallet./hr in increments of 5 pallet./hr
  - Number of storage locations = 7300

- Analyzed 40 cases where vehicle utilizations range between 60% to 90%

- Simulation: Modeled using AUTOMOD path mover system
  (15 replications for each scenario, 96000 transactions per run)
Model Validation: Results

\[ \% Error = \frac{A - S}{S} \], where S = Simulation Value and A = Analytical Value
Effect of Vehicle Interference on Cycle Times: $\frac{D}{W} = 1.5$

Tier Configuration:
7300 Locations, 45 Aisles, 81 Columns, 5 Vehicles
Numerical Results

Effect of Vehicle Interference on Cycle Times: $\frac{D}{W} = 0.5$

Tier Configuration:
7300 Locations, 27 Aisles, 135 Columns, 5 Vehicles
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Summary and Next Steps

Conclusions:

- Developed analytical model of single tier with vehicle interference
- Vehicle interference increase cycle times

Next Steps:

- Refine analytical model and validate against detailed simulations
- Use analytical model to obtain design insights
- Model to account for lift interactions in multi-tier systems
Thank You!

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Questions or Comments?
CQN to determine CV for the Open Queue