The dynamics of a low-order model for the Atlantic Multidecadal Oscillation

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Joint work with Henk Broer (Groningen), Henk Dijkstra (Utrecht), Carles Simó (Barcelona) & Renato Vitolo (Exeter)
The Atlantic Multidecadal Oscillation (AMO)
Pattern of sea surface temperature variability in North Atlantic Ocean


Average SST anomaly over entire North Atlantic basin
The AMO as an internal mode of the ocean
Te Raa & Dijkstra (2002) : interaction between temperature & transport anomalies
Model

Essential ingredients

Diagnostic equations (no time derivatives)

- Coriolis force \((-v, u)\) balances \((p_x, p_y)\)
- \(p_z\) linearly related with \(T\)
- mass conservation: \(\text{div} (u, v, w) = 0\)

Temperature advection in rectangular basin (6000 \(\times\) 6000 \(\times\) 4 km)

\[ T_t + u T_x + v T_y + w T_z = \text{Diffusion} + \text{Forcing} \]

\[ \text{Forcing} = Bi \left[ 15 + \frac{\Delta}{2} \cos \left( \frac{\pi y}{L_y} \right) \right] - T \]

\[ \begin{aligned} &\text{atmosphere temp.} \\ \end{aligned} \]
expand $T$ in Fourier modes:

$$T(x, y, z, t) = \sum \hat{T}_{m,n,k}(t) \cos \left( \frac{m\pi x}{L_x} \right) \cos \left( \frac{n\pi y}{L_y} \right) \cos \left( \frac{k\pi z}{L_z} \right)$$

truncate: $0 \leq m, n, k \leq 2 \implies 27$ ODEs for $\hat{T}_{m,n,k}$

$(p_x, p_y, p_z)$ and $(u, v, w)$ are eliminated
Stable steady state for $\Delta = 20^\circ C$

Velocity field: upwelling in the south & downwelling in the north
Modify the forcing term
Changes stability of the steady state...

\[ \text{Forcing} = Bi \left( 15 + \frac{\Delta}{2} \cos \left( \frac{\pi y}{L} \right) - (1 - \gamma) T - \gamma T_{eq} \right) \]

What happens if \( \gamma \) increases from 0 to 1?

- \( \Delta = 20 \): Hopf bifurcation at \( \gamma \approx 0.95 \) \( \Rightarrow \) AMO!
- \( \Delta > 20 \): period doubling bifurcations & Hénon-like SA
Periodic orbit has physical characteristics of the AMO

1) Westward travelling temperature anomalies
2) phase difference between anomalous E–W & S–N transports
The Poincaré return map

Intersections of integral curves with a hyperplane

\[ P : \mathbb{R}^{27} \rightarrow \mathbb{R}^{27} \] maps each intersection to the next

- fixed points correspond with periodic orbits
Attractors of the Poincaré map as a function of $\gamma$

Two period doublings for $\Delta = 22^\circ C$, a cascade for $\Delta = 24^\circ C$
Hénon-like strange attractors after PD cascade

$SA = \text{closure of the unstable manifold of an unstable fixed point}$
Next step: add annual variation to the forcing term

Two periods (AMO & forcing) : dynamics on a 2-torus

![Graph showing basin averaged SST over time. The graph has x-axis labeled 'time (years)' and y-axis labeled 'basin averaged SST'. There are two periodic oscillations visible.](image-url)
The Poincaré stroboscopic map
Snapshots of integral curves at multiples of forcing period
Dynamics of the stroboscopic map on invariant circles

Periodic or quasi-periodic
Doublings of invariant circles ($\Delta = 24^\circ \text{C}$, increasing $\gamma$)

Inherited from the period doublings of periodic orbits

(A) $T_{0,0,1}$ vs $T_{0,0,0}$
(B) $T_{0,0,1}$ vs $T_{0,0,0}$
(C) $T_{0,0,1}$ vs $T_{0,0,0}$
(D) $T_{0,0,1}$ vs $T_{0,0,0}$
A quasi-periodic Hénon-like strange attractor

$SA = \text{closure of unstable manifold of invariant circle?}$


Appendix: model equations

Relation velocity & temperature gradients

\[-v + p_x = E_H(u_{xx} + u_{yy}) + E_V u_{zz}\]
\[u + p_y = E_H(v_{xx} + v_{yy}) + E_V v_{zz}\]
\[p_z = Ra T\]

Mass conservation

\[u_x + v_y + w_z = 0\]

Temperature advection

\[T_t + u T_x + v T_y + w T_z = P_H(T_{xx} + T_{yy}) + P_V T_{zz} + B(T_S - T)G(z)\]