Backlog-Based Random Access in Wireless Networks

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YEQT V
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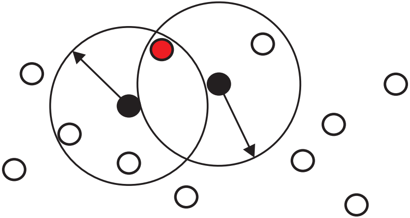
Wireless signals



Interference may result in data not being received correctly

In large-scale wireless networks

- Centralized control is unfeasible
- Nodes operate autonomously, and share medium in distributed fashion

Distributed medium-access control

Randomized algorithms provide popular mechanism for distributed medium-access control

CSMA (Carrier-Sense Multiple-Access) algorithms

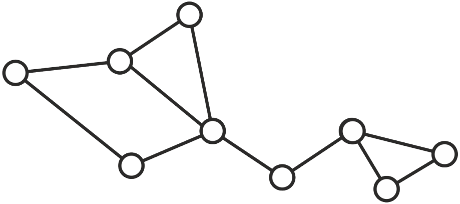
- Each node attempts to activate after random back-off time
- A node activates only if no interfering node is active
- Low implementation complexity, but highly complex behavior on macroscopic level

Interference graph

The network is described by an undirected graph $G = (V, E)$

- The vertices represent the various nodes of the network
- The edges indicate which pairs of nodes interfere

[Boorstyn *et al.* (1980), Wang & Kar (2005), Durvy & Thiran (2006)]



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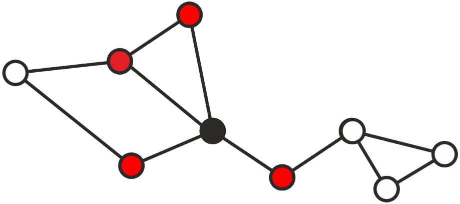


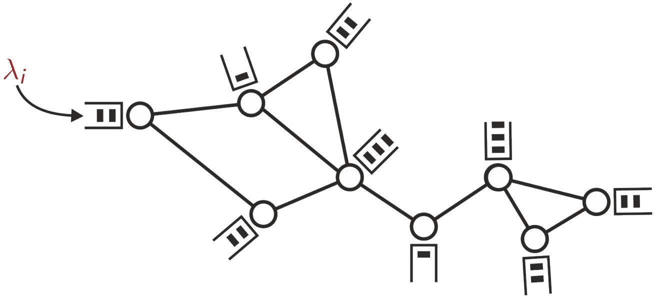
Model description

As opposed to most previous work that assumed saturated buffers, we consider queueing dynamics

Packets arrive at node i according to a Poisson process with rate λ_i

Packet transmission times of node i are exponentially distributed with mean $1/\mu_i$. Once a packet has been transmitted, it leaves the system

Denote by $\rho_i = \lambda_i/\mu_i$ the traffic intensity of node i



Backlog-based CSMA algorithms

Denote by $L_i(t)$ the number of packets at node i at time t

We consider random CSMA algorithms where

- node i attempts to activate at exponential rate $f_i(L_i(t))$, if inactive
- $f_i(0) = 0$
- $f_i(\cdot)$ is called the activation function of node i

- node i de-activates at exponential rate $g_i(L_i(t)) = p_i(L_i(t))\mu_i$, if active
- $p_i(L_i(t))$ is the probability that node i releases the medium if a packet transmission ends at time t
- $g_i(1) = \mu_i$
- $g_i(\cdot)$ is called the de-activation function of node i

Markov process

Define $\Omega \subseteq \{0,1\}^M$ as the set of all feasible joint activity states,

$$\Omega = \{x \in \{0,1\}^M : x_i + x_j \leq 1 \; \forall i,j \in E\},$$

with M the number of nodes in the network

Let $\sigma(t) = (\sigma_1(t), \ldots, \sigma_M(t)) \in \Omega$ represent the activity state of the network, with $\sigma_i(t)$ indicating whether node i is active at time t ($\sigma_i(t) = 1$) or not ($\sigma_i(t) = 0$)

Under these CSMA algorithms, $(L(t), \sigma(t), t \geq 0)$ is a continuous-time Markov process with state space $\mathbb{N}^M \times \Omega$

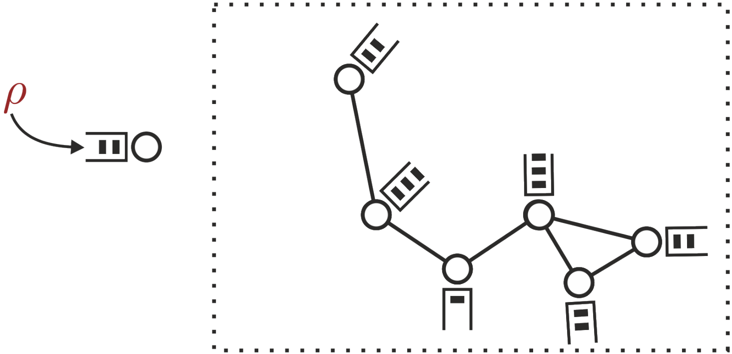
Stability

$\rho \in \mathrm{conv}(\Omega)$, with $\rho = (\rho_1, \ldots, \rho_M)$, is a necessary condition for stability

For suitable choices of $f_i(\cdot)$ and $g_i(\cdot)$ (e.g., $g_i(n) = 1/(1 + \log(n + 1))$ and $f_i(n) = \log(n + 1)/(1 + \log(n + 1))$), any network is stable as long as $\rho \in \mathrm{int}(\mathrm{conv}(\Omega))$
[Rajagopalan *et al.* (2009), Jiang *et al.* (2010), Ghaderi & Srikant (2010)]

However, simulation experiments demonstrate that these choices cause excessive backlogs and delays!

Isolated node

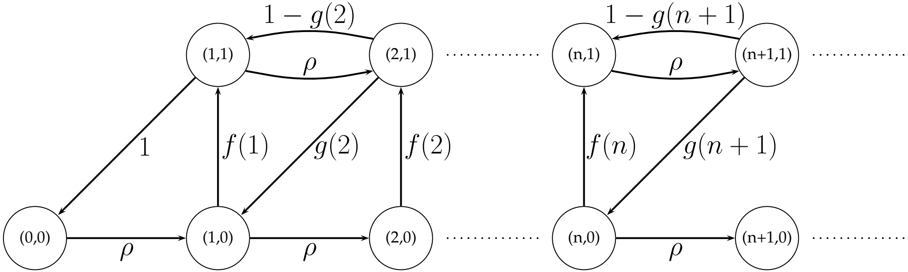


To better understand how the activity functions affect the queueing behavior, we first consider an isolated node

Assume $\mu = 1$, so that $\lambda = \rho$

Isolated node: Markov process

Markov process $(L(t), \sigma(t), t \geq 0)$ has state space $\mathbb{N} \times \{0, 1\}$



This system is stable if $\rho < 1$ and $f(n)/g(n) \to \infty$ as $n \to \infty$

Isolated node: Linear functions

Denote by L the total number of packets in the system in steady state, i.e.

$$\mathbb{P}\{L = n\} = \lim_{t\to\infty} \mathbb{P}\{L(t) = n\}$$

Theorem

(i) *If* $f(n) = n$ *and* $g(n) = 1$, $n \geq 1$,

$$\mathbf{E}\{z^L\} = \left(\frac{1-\rho}{1-\rho z}\right)^{\rho+1} \mathrm{e}^{\rho(z-1)}$$

(ii) *If* $f(n) = 1$ *and* $g(n) = \frac{1}{n}$, $n \geq 1$,

$$\mathbf{E}\{z^L\} = \left(\frac{1-\rho}{1-\rho z}\right)^{2}$$

Isolated node: Linear functions

When $f(\cdot) \equiv f$ and $g(\cdot) \equiv g$, the node would be active a fraction $\frac{f/g}{1+f/g}$ of the time

If $f(n) = n$ and $g(n) = 1$, $n \geq 1$,

$$\mathbf{E}\{L\} = \frac{2\rho}{1-\rho},$$

and,

$$\mathrm{Var}\{L\} = \frac{2\rho - \rho^2(1-\rho)}{(1-\rho)^2}$$

If $f(n) = 1$ and $g(n) = \frac{1}{n}$, $n \geq 1$,

$$\mathbf{E}\{L\} = \frac{2\rho}{1-\rho},$$

and,

$$\mathrm{Var}\{L\} = \frac{2\rho}{(1-\rho)^2}$$

Isolated node: Linear functions

Let E_k be a random variable having an Erlang distribution with k exponential phases, each with unit mean

Denote by S the stationary delay

Theorem

If $f(n) = n$ *and* $g(n) = 1$, $n \geq 1$, *or* $f(n) = 1$ *and* $g(n) = \frac{1}{n}$, $n \geq 1$,

$$(1-\rho)L \xrightarrow{d} E_2 \text{ as } \rho \uparrow 1, \text{ and}$$

$$(1-\rho)S \xrightarrow{d} E_2 \text{ as } \rho \uparrow 1$$

Isolated node: General de-activation functions

Assume $f(n) = 1$ for $n \geq 1$ and $f(0) = 0$

Theorem

If $g(\cdot)$ is a strictly decreasing convex function,

$$\mathbf{E}\{L\} \geq g^{-1}\left(\frac{1-\rho}{\rho}\right) + \rho$$

Isolated node: General activation functions

Assume $g(\cdot) \equiv 1$

Theorem

(i) *If $f(\cdot)$ is a strictly increasing, continuous and convex function,*

$$\mathbf{E}\{L\} \leq \frac{\rho}{1-\rho} + f^{-1}\left(\frac{\rho}{1-\rho}\right)$$

(ii) *If $f(\cdot)$ is a strictly increasing and concave function,*

$$\mathbf{E}\{L\} \geq \frac{\rho}{1-\rho} + f^{-1}\left(\frac{\rho}{1-\rho}\right)$$

Note: when $f(n) = n$ the inequalities are in fact equalities

Isolated node: General activation functions

Theorem

(i) *If* $f(\cdot)$ *is a strictly increasing, continuous and convex function with* $\lim_{n\to\infty} f^{-1}(n)/n = 0$,

$$\lim_{\rho\uparrow 1}(1-\rho)\mathbf{E}\{L\} = 1$$

(ii) *If* $f(n) = n^{\alpha}$, $0 < \alpha < 1$,

$$\lim_{\rho\uparrow 1}\frac{\mathbf{E}\{L\}}{f^{-1}(\rho/(1-\rho))} = \lim_{\rho\uparrow 1}(1-\rho)^{1/\alpha}\mathbf{E}\{L\} = 1$$

Remember: for $f(n) = n$ we found

$$\lim_{\rho\uparrow 1}(1-\rho)\mathbf{E}\{L\} = 2$$

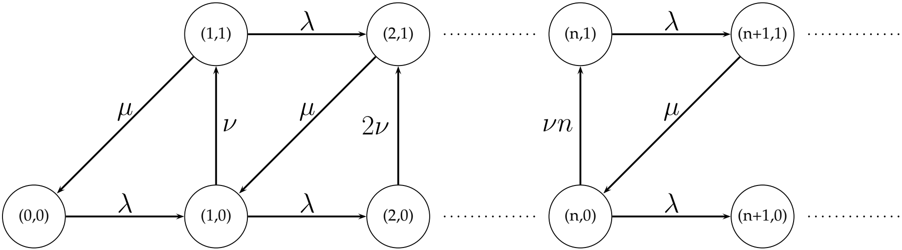
Full interference graph: Linear activation function

Assume complete interference graph and $g_i(n) = \mu$ and $f_i(n) = \nu n$

The system can be described by a continuous-time Markov process with state space $\{0, 1, 2, \ldots\} \times \{0, 1\}$

- First component represents the total number of packets in the system, $L(t) = \sum_{i=1}^{M} L_i(t)$
- Second component indicates whether one of the nodes is active

Denote $\lambda = \sum_{i=1}^{M} \lambda_i$ and $\rho = \sum_{i=1}^{M} \rho_i = \lambda/\mu$



Full interference graph: Linear activation function

Assume $\rho < 1$ and denote by L the total number of packets in the system in steady state

Theorem

The generating function of L is given by

$$\mathbf{E}\{z^L\} = \left(\frac{1-\rho}{1-\rho z}\right)^{\lambda/\nu+1} \mathrm{e}^{(z-1)\lambda/\nu}$$

In particular,

$$\mathbf{E}\{L\} = \frac{\rho + \frac{\lambda}{\nu}}{1-\rho} = \frac{\lambda(\mu+\nu)}{\nu(\mu-\lambda)}$$

Full interference graph: General de-activation functions

Assume $f_i(n) = \nu_i$ for $n \geq 1$ and $f_i(0) = 0$

Denote $\nu = \sum_{i=1}^{M} \nu_i$

Theorem

For $g_i(\cdot) \equiv g(\cdot)$ a strictly decreasing convex function,

$$\sum_{i=1}^{M} \rho_i \mathbf{E}\{L_i\} \geq \rho g^{-1}\left(\frac{\nu(1-\rho)}{\rho}\right) + \sum_{i=1}^{M} \rho_i^2$$

Full interference graph: General activation functions

Assume $g_i(n) = \mu$

Theorem

For $f_i(\cdot) \equiv f(\cdot)$ a strictly increasing and concave function,

$$\mathbf{E}\{L\} \geq \frac{\rho}{1-\rho} + M f^{-1}\left(\frac{1}{M}\frac{\lambda}{1-\rho}\right)$$

For $f_i(\cdot) \equiv f(\cdot)$ a strictly increasing, continuous and convex function,

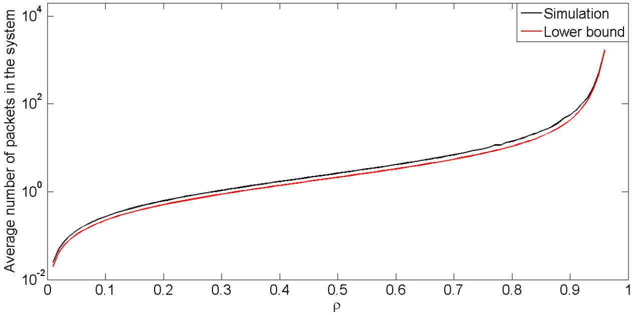
$$\mathbf{E}\{L\} \leq \frac{\rho}{1-\rho} + M f^{-1}\left(\frac{1}{M}\frac{\lambda}{1-\rho}\right)$$

Note: when $f(n) = \nu n$ the inequalities are in fact equalities

Full interference graph: General activation functions

The bounds turn out to be very tight for all values of ρ

Simulation for the average number of packets in the system for $f(n) = \log(n+1)$ with $M = 4$

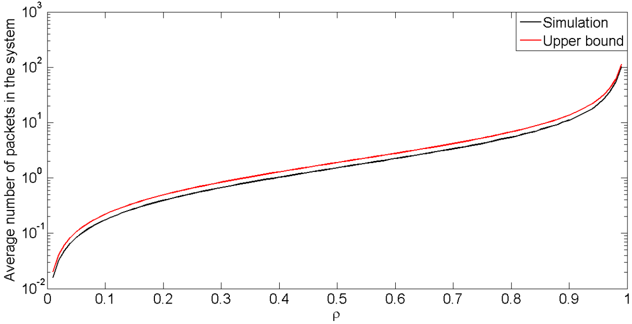


In heavy traffic, as $\rho \uparrow 1$, L grows like $M \exp(\frac{\rho}{M(1-\rho)})$

Full interference graph: General activation functions

The bounds turn out to be very tight for all values of ρ

Simulation for the average number of packets in the system for $f(n) = \mathrm{e}^n - 1$ with $M = 4$



In heavy traffic, as $\rho \uparrow 1$, L grows like $\frac{\rho}{1-\rho}$

General topologies: General functions

Let $\mathcal{N} \subseteq V$ be any clique and assume the system is stable

Denote $\lambda_\mathcal{N} = \sum_{j\in\mathcal{N}} \lambda_j$, $\nu_\mathcal{N} = \sum_{j\in\mathcal{N}} \nu_j$, and $\rho_\mathcal{N} = \sum_{j\in\mathcal{N}} \rho_j$

Theorem

(i) *If* $f_i(\cdot) \equiv f(\cdot)$ *is a strictly increasing concave function and* $g_i(\cdot) = \mu_i$,

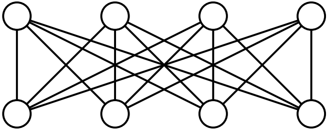
$$\sum_{i\in\mathcal{N}} \mathbf{E}\{L_i\} \geq \frac{\lambda_\mathcal{N} \sum_{i\in\mathcal{N}} \lambda_i/\mu_i^2}{1-\rho_\mathcal{N}} + \rho_\mathcal{N} + |\mathcal{N}| f^{-1}\left(\frac{1}{|\mathcal{N}|}\frac{\lambda_\mathcal{N}}{1-\rho_\mathcal{N}}\right)$$

(ii) *If* $f_i(0) = 0$, $f_i(n) = \nu_i$ *for all* $n \geq 1$ *and* $g_i(\cdot) \equiv g(\cdot)$ *a strictly decreasing convex function,*

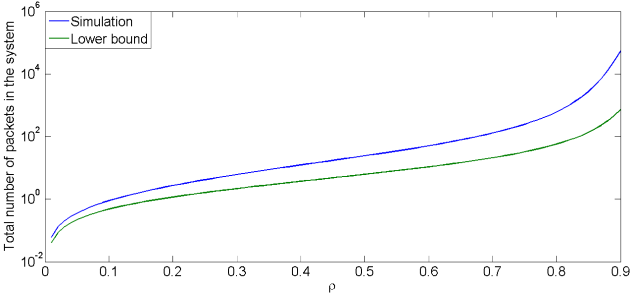
$$\sum_{i\in\mathcal{N}} \rho_i \mathbf{E}\{L_i\} \geq \rho_\mathcal{N} g^{-1}\left(\frac{(1-\rho_\mathcal{N})\nu_\mathcal{N}}{\rho_\mathcal{N}}\right) + \sum_{i\in\mathcal{N}} \rho_i^2$$

General topologies: General activation functions

Simulation for a system represented by a symmetric bipartite complete interference graph (subsets have cardinality 4)



$f(n) = \log(n+1)$ and $g(n) = 1$



Partite topologies: Fixed rates

Assume interference graph is complete P-partite, $f_i(\cdot) \equiv \nu_i$ and $g_i(\cdot) \equiv 1$

Let $V_p \subset V$ denote the subset of nodes that belong to the p-th component, $K_p = |V_p|$ and $K = \max_{p=1,\dots,P} K_p$

Assume $\rho_i = \hat{\rho}_p$ for all $i \in V_p$, and denote $\rho = \sum_{p=1}^{P} \hat{\rho}_p$, and $\rho_{\min} = \min_{p=1,\dots,P} \hat{\rho}_p$

Theorem

For any $w \in \mathbb{R}_+^M$, $V_0 \subseteq V_p$,

$$\sum_{i \in V_0} w_i \mathbf{E}\{L_i\} \geq \frac{1}{2K} (\rho_{\min})^{K+1} \sum_{i \in V_0} w_i \lambda_i \left(\frac{1}{1-\rho} \right)^{K-1},$$

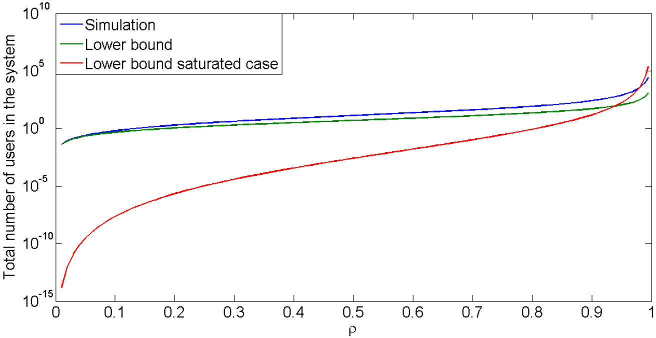
for all vectors $(\nu_1, \dots, \nu_M)$

Partite topologies: Fixed rates

The lower bound for $f(n) = n^{1/a}$, $a \geq 1$, and $g(n) = 1$ grows like $1/(1-\rho)^a$ as ρ approaches 1

The lower bound for fixed rates grows like $1/(1-\rho)^{K-1}$ as ρ approaches 1

Simulation for $f(n) = n$, $g(n) = 1$, $P = 2$ and $K_1 = K_2 = 4$



Fluid limits

More aggressive activation functions can improve the delay performance

Existing stability results involve slowly varying activation functions, e.g. $f(n) = \log(n + 1)$

Depending on the topology, how aggressive are the activation functions allowed to grow?

Examine the dynamics of the Markov process $Z(t) = (L(t), \sigma(t))$ using fluid limits

A fluid limit may be interpreted as a first-order approximation of the Markov process

Fluid limits

Consider a sequence of processes $Z^N(t)$, where the initial states satisfy $\sum_{i=1}^{M} L_i(0) = N$ and $L_i^N(0)/N \to q_i \geq 0$ as $N \to \infty$

The process $\bar{Z}^N(t) = (\frac{1}{N} L^N(Nt), \sigma^N(Nt))$ is the fluid-scaled version of the process $Z^N(t)$

$\xi(t) = \lim_{N\to\infty} \frac{1}{N} L^N(Nt)$ is called a fluid limit

We encounter different types of fluid limits depending on the mixing properties of the activity process $\sigma^N(t)$

These properties depend on the topology and activation functions

Fluid Limits

Assume constant de-activation function $g(\cdot) \equiv \mu$

Transition times between dominant activity states are $O(f(N)^H)$ when queue sizes are $O(N)$, where H depends on network structure, e.g.,

- Complete interference graph: $H = 0$
- Ring network: $H = 1$
- Grid network ($K \times K$): $H = K$ or $H = 2K$
- Complete bipartite graph (subsets have cardinality K): $H = K - 1$

Trichotomy

Fast mixing - Deterministic fluid limits

- Transitions between the various activity states are not observed at fluid level
- Slowly varying activation functions, $f(N)^H \ll N$

Slow mixing - Inhomogeneous Poisson fluid limits

- Transition times between the various activity states are driven by time-inhomogeneous Poisson processes
- Intermediate activation functions, $f(N)^H \sim N$

Torpid mixing - Pseudo-deterministic fluid limits

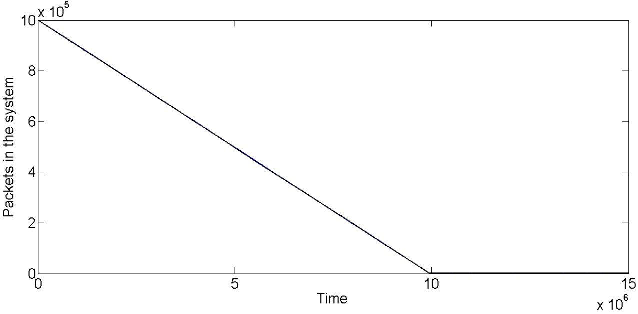
- Activity state seems to be frozen at fluid level
- Transitions only occur when a queue empties
- Aggressive activation functions, $f(N)^H \gg N$

Example of fast mixing

bipartite complete interference graph

Subsets have cardinality 1

$f(n) = n$ and $g(n) = 1$

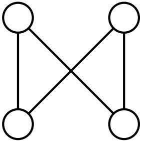


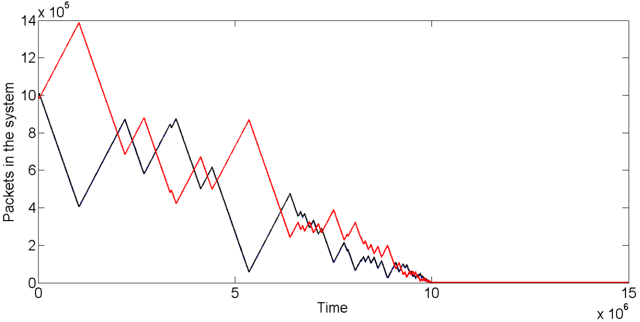
Example of slow mixing

bipartite complete interference graph

Subsets have cardinality 2

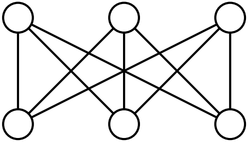
$f(n) = n$ and $g(n) = 1$





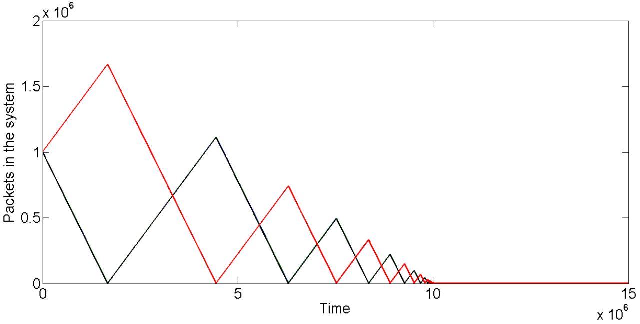
Example of torpid mixing

bipartite complete interference graph



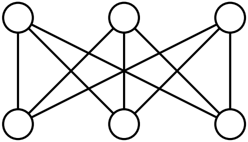
Subsets have cardinality 3

$f(n) = n$ and $g(n) = 1$



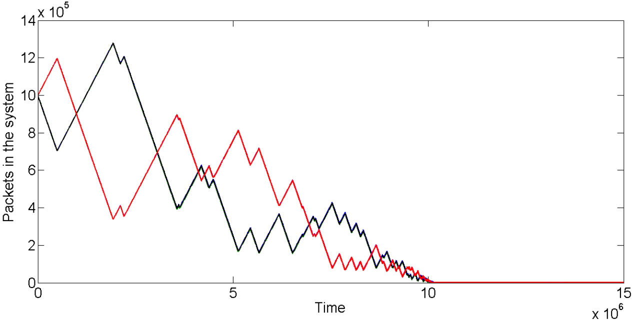
Example of slow mixing

bipartite complete interference graph



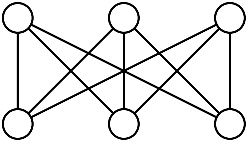
Subsets have cardinality 3

$f(n) = \sqrt{n}$ and $g(n) = 1$



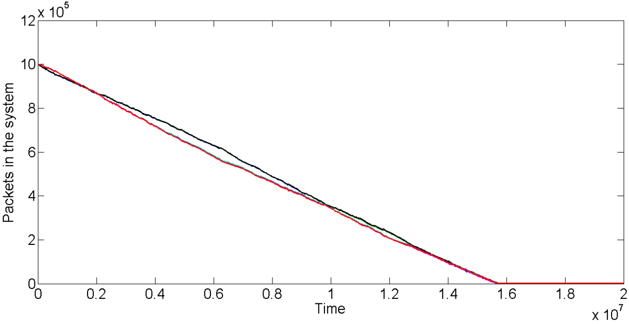
Example of fast mixing

bipartite complete interference graph



Subsets have cardinality 3

$f(n) = \log(n+1)$ and $g(n) = 1$



Conclusions

Backlog-based CSMA algorithms have been shown to guarantee maximum stability, if activity factors behave as logarithmic functions of the backlogs

We showed that more aggressive access schemes can improve the delay performance

How fast are the activity factors allowed to grow, depending on the topology, while retaining maximum stability?

As a first step we investigated fluid limits and identified three qualitative regimes that can arise