Compensation and Staffing and to Trade Off Speed and Quality in Large Service Systems

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Slowdown?

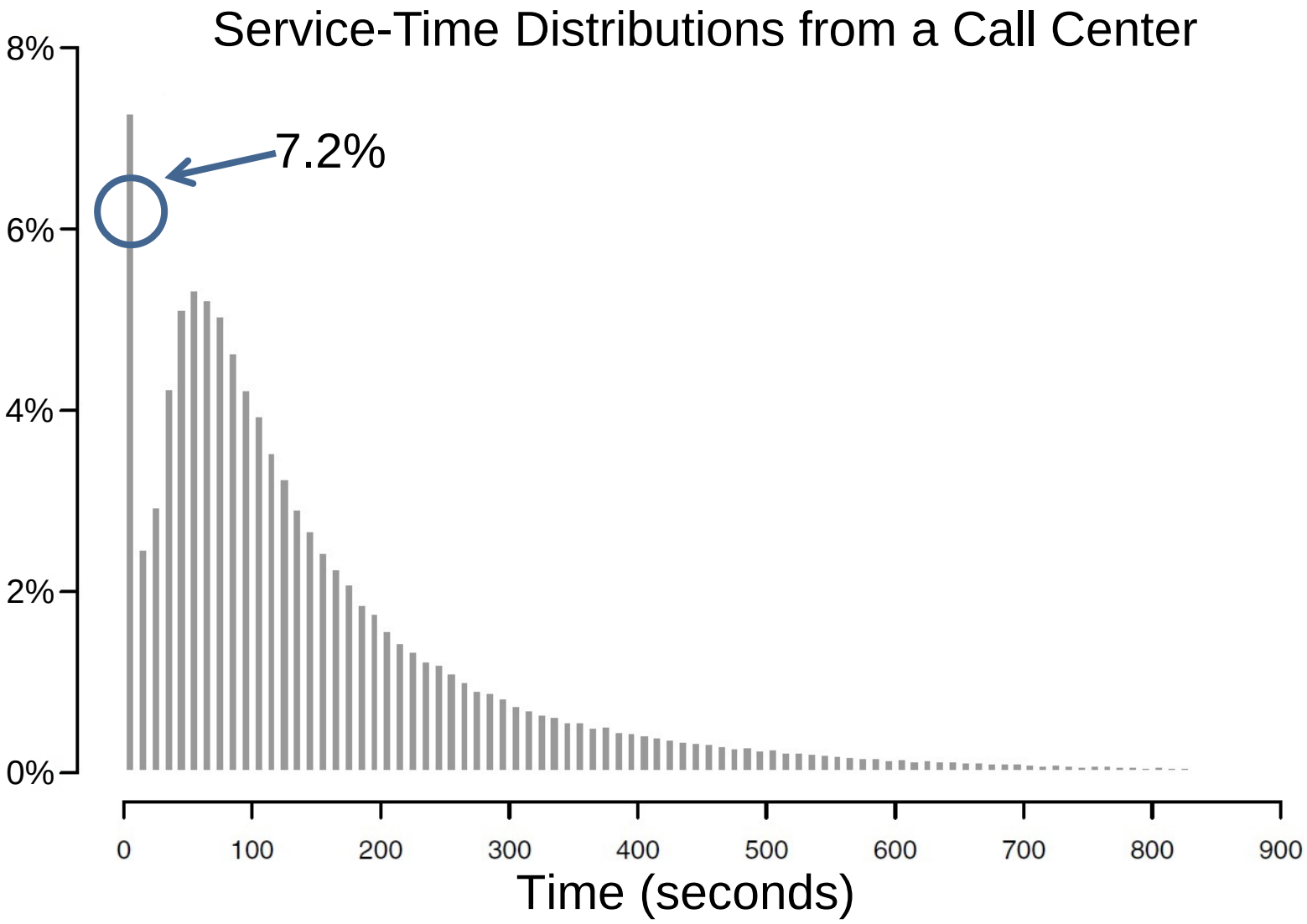
Imagine:

How do you respond?

You may respond strategically.

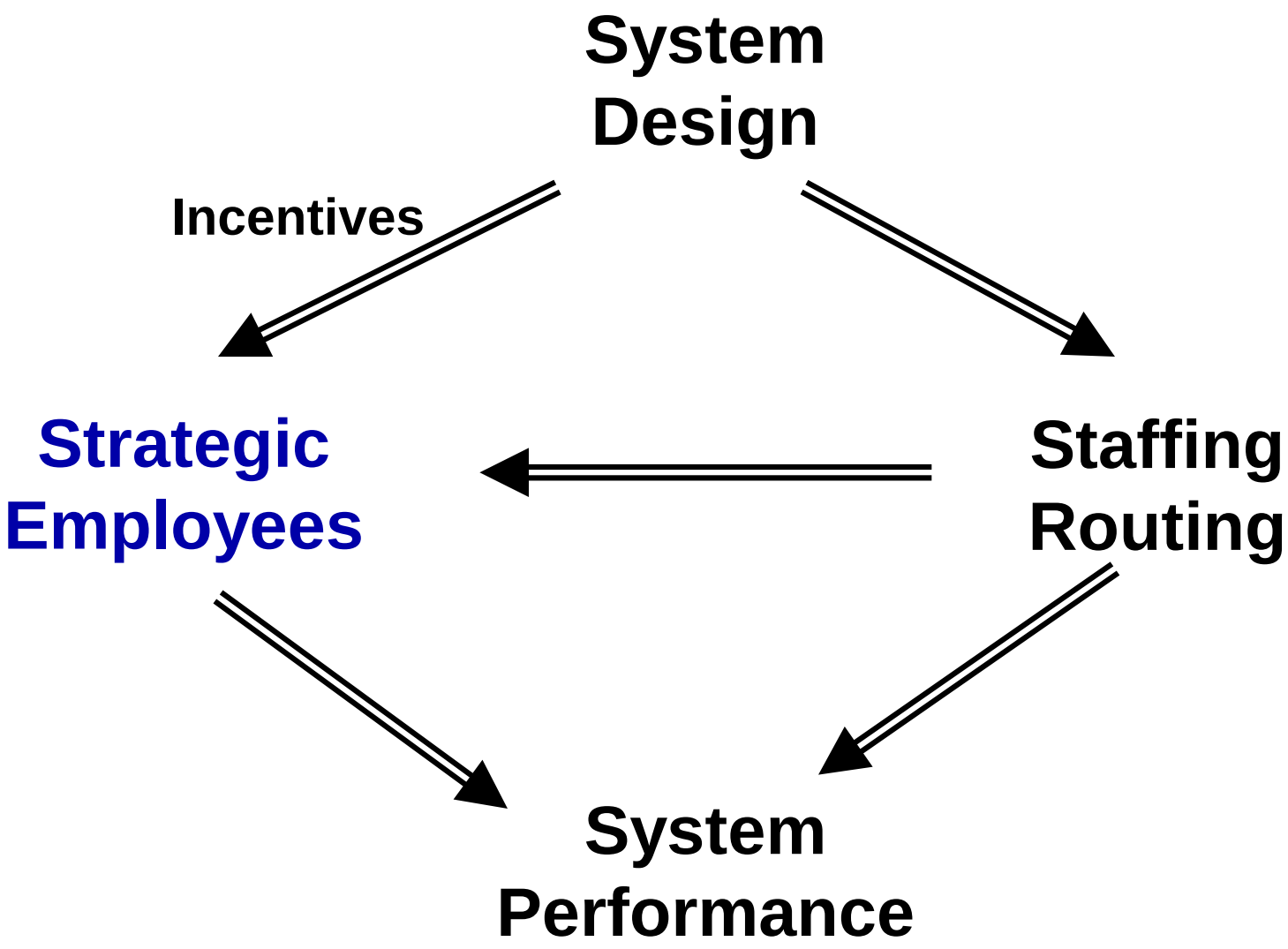
What if you were paid per review?

Employee Strategic Behavior



From Gans, Koole, Mandelbaum (2003)

The Big Picture



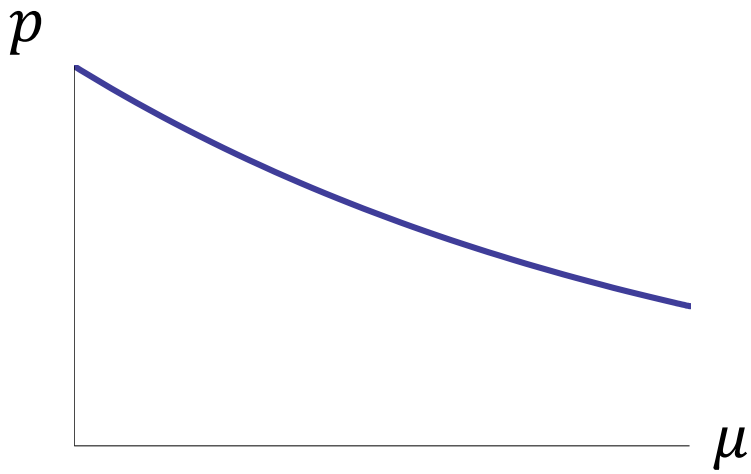
Outline

- The Employee Payment Model
- Employee Behavior: the Equilibrium Service Rate
- Optimal Design: Compensation and Staffing
- Generalizations

The Speed Quality Trade-off

Lower speed ⟹ Higher quality

$p(\mu)$: The probability of successful service.



Discretionary task completion in Hopp, Iravani, Yuen, 2007

Customer-intensive service in Anand, Pac, Veeraraghavan, 2011.

Compensation and Research Question

Each risk neutral employee

- is paid P_S per service completion.
- is penalized P_F per failed service.
- maximizes her expected payment.

The promised expected payment is c_S per unit time.

What is employees' behavior?

What is the optimal system design?

OM Literature Review

Queueing games

- Hassin and Haviv (2003)

Service rate decisions when service providers compete

- Kalai, Kamien, Rubinovitch (1992), Gilbert and Weng (1998), Cachon and Harker (2002) , Cachon and Zhang (2007)

Speed and quality trade-off

- Hopp, Iravani, Yuen(2007), Ren and Zhou (2008), Lu, Van Mieghem, Savaskan (2009), Anand, Pac, Veeraraghavan (2011), Mehrotra, Ross, Ryder, Zhou (2012), Chan, Yom-Tov, Escobar(2014), Zhan and Ward (2014)

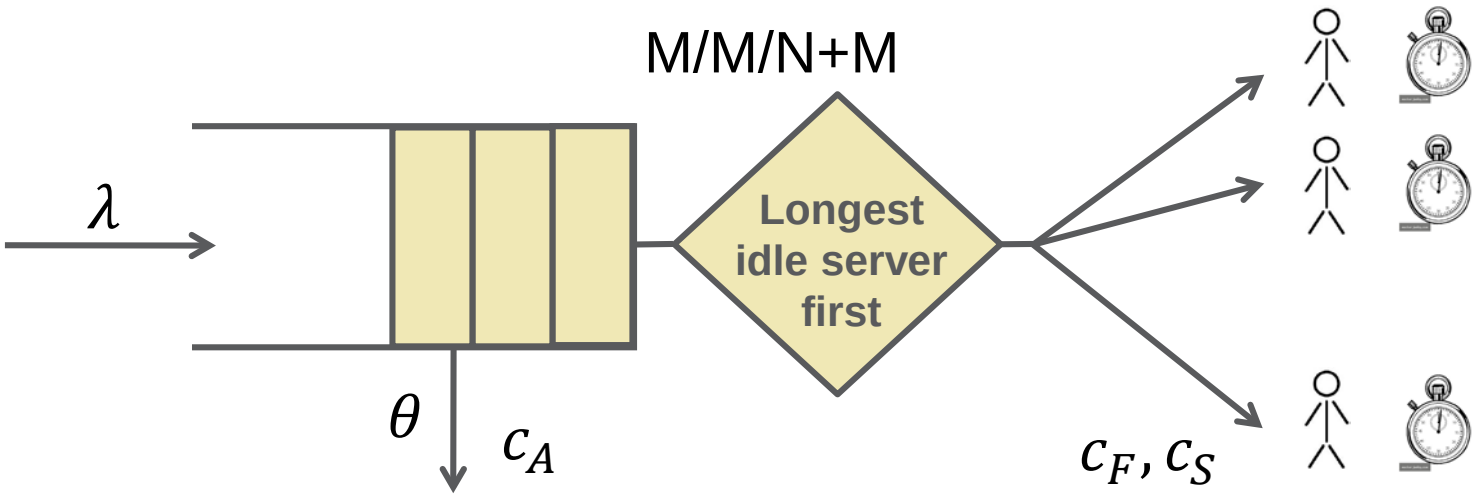
Large-scale service systems

- Haffin and Whitt (1981), Garnett, Mandelbaum, Reiman (2002), Gurvich and Whitt (2009), Tezcan and Dai (2010), Allon and Gurvich (2010), Mandelbaum and Armony (2011)

Strategic employees within an organization

- Buell, Kim, Tsay (2014), Song, Tucker, Murrell (2014), Shunko, Niederhoff, Rosokha (2014), Gopalakrishnan, Douroudi, Ward, Wierman (2014)

The Model



Given reward P_S, penalty P_F and staffing N, what service rate will the employees choose?

- Does there exist a symmetric Nash equilibrium?
- The utility is the expected payment (and later generalize).

How to choose P_S, P_F and N optimally?

- This depends on the cost structure the system manager faces.
- The costs are linear (and later generalize).

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Find the Service Rate When N=1

The server chooses her service rate to maximize her expected payment (utility):

$$U(\mu) = (P_S - P_F\big(1 - p(\mu)\big)\mu\, B(\mu)$$

Reward | Penalty | Busy time portion

$$B(\mu) = 1 - \frac{1}{1 + \sum_{i=1}^{\infty} \prod_{k=0}^{i-1} \frac{\lambda}{\mu + k\theta}}$$

Proposition:

Suppose $p(\mu)\mu$ is concave in μ, given P_S and P_F, the server chooses a **unique** service rate $\boldsymbol{\mu_E}$ to maximize the expected payment.

$\boldsymbol{\mu_E}$ is continuous and decreasing in P_F/P_S.

Find the Equilibrium Service Rate

When $N > 1$, the complication is that each server i has utility that depends on the service rates chosen by the other servers.

$$U_i(\vec{\mu}) = (P_S - P_F\big(1 - p(\mu_i)\big)\mu_i B_i(\vec{\mu})$$

What is the symmetric equilibrium service rate $\boldsymbol{\mu_E}$?

Server 1 busy, no queue — Server 1 busy, queue

$$B(\mu_1, \mu) = \frac{\sum_{i=0}^{N-1}\left(\frac{\lambda}{\mu}\right)^i \frac{1}{i!} + \left(\frac{\lambda}{\mu}\right)^i \frac{1}{(N-1)!}\sum_{i=1}^{\infty}\prod_{k=1}^{i}\frac{\lambda}{(N-1)\mu + \mu_1 + k\theta}}{\sum_{i=0}^{N-1}\left(\frac{\lambda}{\mu}\right)^i \frac{1}{i!} + \left(\frac{\lambda}{\mu}\right)^i \frac{1}{(N-1)!}\sum_{i=1}^{\infty}\prod_{k=1}^{i}\frac{\lambda}{(N-1)\mu + \mu_1 + k\theta} + \frac{\mu_1}{\lambda}\sum_{i=0}^{N-1}\left(\frac{\lambda}{\mu}\right)^i \frac{N-i}{i!}}$$

Server 1 idle, no queue

Wishful thinking: If $B_i(\vec{\mu}) = 1$, then

$$\mu_E = \mu^* := \text{argmax}_\mu(P_S - P_F\big(1 - p(\mu)\big)\mu$$

"No competition equilibrium."

Determined by $\frac{P_F}{P_S}$.

Busy Time Approximate

Staffing: $N^{\lambda} = b\lambda + o(\lambda), b > 0$

Note if $b = \frac{1}{\mu_E}$, then the first order term is identical to the square root staffing.

Define $\hat{B}(\mu_1, \mu) = \frac{\mu}{\mu + \mu_1[b\mu - 1]^+}$

Lemma:

As $\lambda \to \infty$,

$$\sup_{\mu_1, \mu \in [\underline{\mu}, \overline{\mu}]} \left| B(\mu_1, \mu) - \hat{B}(\mu_1, \mu) \right| \to 0$$

Therefore, define $\hat{U}(\mu_1, \mu) = (P_S - P_F(1 - p(\mu_1))\mu_1 \hat{B}(\mu_1, \mu)$, we have

$$U(\mu_1, \mu) \to \hat{U}(\mu_1, \mu) \text{ as } \lambda \to \infty.$$

The Approximate Equilibrium Service Rate

For the revised problem

$$\widehat{U}(\hat{\mu}_E, \hat{\mu}_E) = \max_{\mu \in [\underline{\mu}, \overline{\mu}]} \widehat{U}(\mu, \hat{\mu}_E)$$

There exists a unique $\hat{\mu}_E \in (\frac{\lambda}{N}, \mu^*]$.

Proposition: Suppose $p(\mu)\mu$ is concave in μ, for systems with large enough λ, there exists a symmetric equilibrium service rate $\mu_E^{\lambda}(N^{\lambda}, P_S, P_F)$. As $\lambda \to \infty$,

if $b \leq \frac{1}{\mu^*}$, $\mu_E^{\lambda}(N^{\lambda}, P_S, P_F) \to \hat{\mu}_E = \mu^*$;

if $b > \frac{1}{\mu^*}$, $\mu_E^{\lambda}(N^{\lambda}, P_S, P_F) \to \hat{\mu}_E \in (\frac{1}{b}, \mu^*)$.

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The System Cost

$$C(N, P_S, P_F) = \sum_{i=1}^{N} U_i(\mu_E) + c_F\big(1 - p(\mu_E)\big)\mu_E NB(\mu_E) + c_A(\lambda - \mu NB(\mu_E))$$

Controls | Staffing cost | Failed services cost | Abandonment cost

μ_E is determined by $N, \frac{P_F}{P_S}$, and is well approximated by $\hat{\mu}_E$.

Individual rationality constraint is: $U_i(\mu_E) \geqq c_S$.

$$C(N, P_S, P_F) = c_S N + c_F\big(1 - p(\mu_E)\big)\mu_E NB(\mu_E) + c_A(\lambda - \mu NB(\mu_E))$$

$$\mathrm{Min}_{N,P_S,P_F}\ C(N, P_S, P_F)$$

The First Best Control

In a **decentralized** system:

$$C(N, P_S, P_F) = c_S N + c_F\big(1 - p(\mu_E)\big)\mu_E NB(\mu_E) + c_A(\lambda - \mu NB(\mu_E))$$

In a **centralized** system:

$$C(N, \mu) = c_S N + c_F\big(1 - p(\mu)\big)\mu NB(\mu) + c_A(\lambda - \mu NB(\mu))$$

A **first best control** achieves:

$$\mathrm{Min}_{N,P_S,P_F}\, C(N, P_S, P_F) = \mathrm{Min}_{N,\mu}\, C(N, \mu)$$

First Best Compensation for M/M/1+M

No decision on staffing.

First best compensation:

$$P_S = c_A\Delta,\ P_F = c_F\Delta,$$

where Δ is set so that IR constraint binds.

Each service completion avoids the abandonment cost c_A.

Each service failure incurs the cost c_F.

Can this First Best Compensation Be Extended?

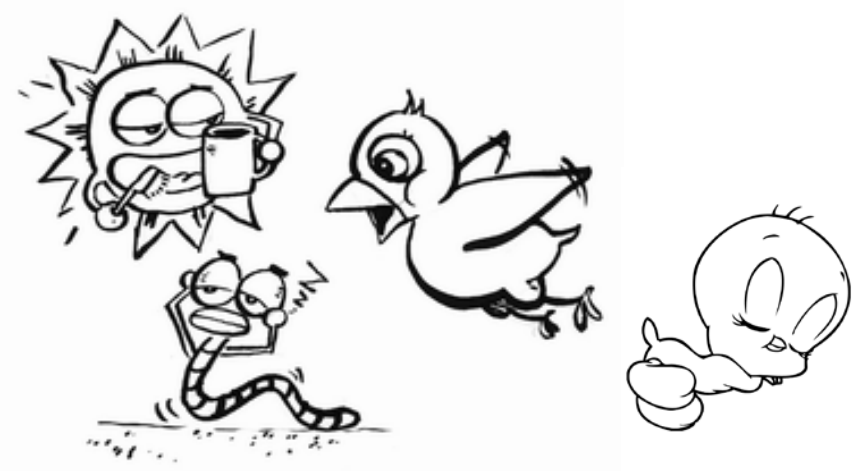
In an M/M/N+M system, is it the first best if the manager set

$$P_S = c_A\Delta,\ P_F = c_F\Delta\ ?$$

NO!

The employees have to compete for jobs.

The Early Bird Gets the Worm!



Competition ⇒ Speed up!

A **larger** $\frac{P_F}{P_S}$ is needed…

Cost of Serving One Customer

Each customer requires $\frac{1}{\mu}$ time in average, cost of salary is $\frac{c_S}{\mu}$.

The expected service failure cost is $c_F\big(1 - p(\mu)\big)$.

Define $\hat{\mu} \coloneqq \operatorname{argmin}_{\mu} \ \frac{c_S}{\mu} + c_F\big(1 - p(\mu)\big)$.

The manager wants the employees to work at $\hat{\mu}$.
The minimal cost of serving a customer is

$$\hat{c} \coloneqq \frac{c_S}{\hat{\mu}} + c_F\big(1 - p(\hat{\mu})\big).$$

Proposition (asymptotic optimal):

If $\hat{c} < c_A$, let the employees work at $\hat{\mu}$ and staff $N^{\lambda} = \left\lceil \frac{\lambda}{\hat{\mu}} \right\rceil$ in centralized systems. The policy is asymptotic optimal

$$\lim_{\lambda \to \infty} \frac{C\big(N^{\lambda}, \hat{\mu}\big)}{\lambda} = \hat{c}.$$

The Optimal Staffing and Compensation

Proposed policy: $N^{\lambda} = \left\lceil \frac{\lambda}{\hat{\mu}} \right\rceil$, $P_S = \hat{c}$, $P_F = c_F$.

Theorem (asymptotic first best)

If $\hat{c} < c_A$, the policy is asymptotic first best:

$$\lim_{\lambda \to \infty} \frac{C\left(N^{\lambda}, P_S, P_F\right)}{\lambda} = \hat{c}.$$

The policy incentivizes the servers to work at $\mu^*(P_S, P_F) = \hat{\mu}$.

The system is critically loaded: $\lambda \approx N^{\lambda}\hat{\mu}$, $B(\mu^*) \approx 1$.

Since $\hat{c} < c_A$, we have $\frac{P_F}{P_S} = \frac{c_F}{\hat{c}} > \frac{c_F}{c_A}$.

M/M/1+M: $\frac{P_F}{P_S} = \frac{c_F}{c_A}$.

A larger $\frac{P_F}{P_S}$ to counteract speed-up effect!

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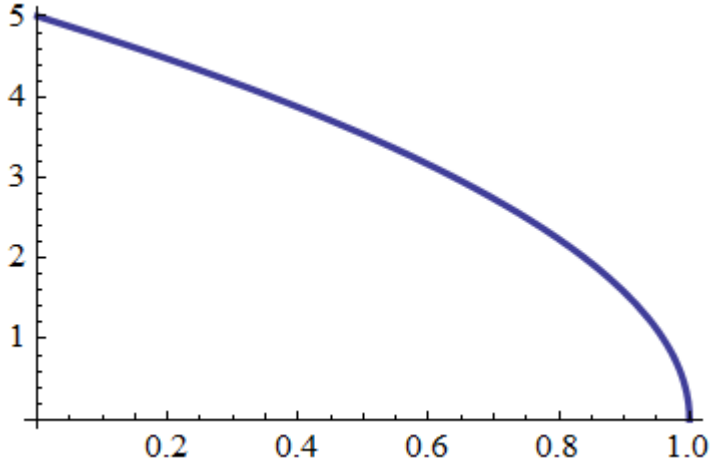
Extension on Utility and Cost Functions

$$U_i(\vec{\mu}) = (P_S - P_F\big(1 - p(\mu_i)\big)\mu_i B_i(\vec{\mu}) + U_I(B_i(\vec{\mu}))$$

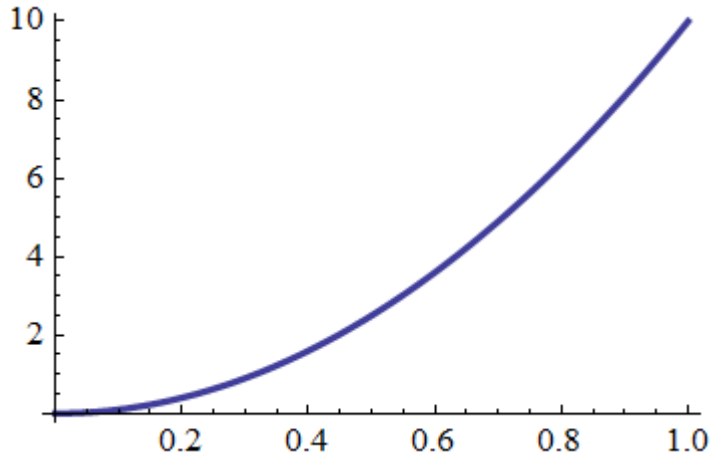
Monetary Non-monetary

Idleness value U_I



Busy time portion

Unit service failure / Abandonment cost



Service failure / Abandonment portion

An Approximate Symmetric Equilibrium

Replace B by $\hat{B}$ in the utility, so that

$$\hat{U}(\mu_1, \mu) = (P_S - P_F\big(1 - p(\mu_1)\big)\hat{B}(\mu_1, \mu) + U_I\left(\hat{B}(\mu_1, \mu)\right).$$

Find $\hat{\mu}_E$ such that

$$\hat{U}(\hat{\mu}_E, \hat{\mu}_E) = \max_{\mu \in [\underline{\mu}, \bar{\mu}]} \hat{U}(\mu, \hat{\mu}_E)$$

$\hat{\mu}_E$ is approximate symmetric equilibrium.

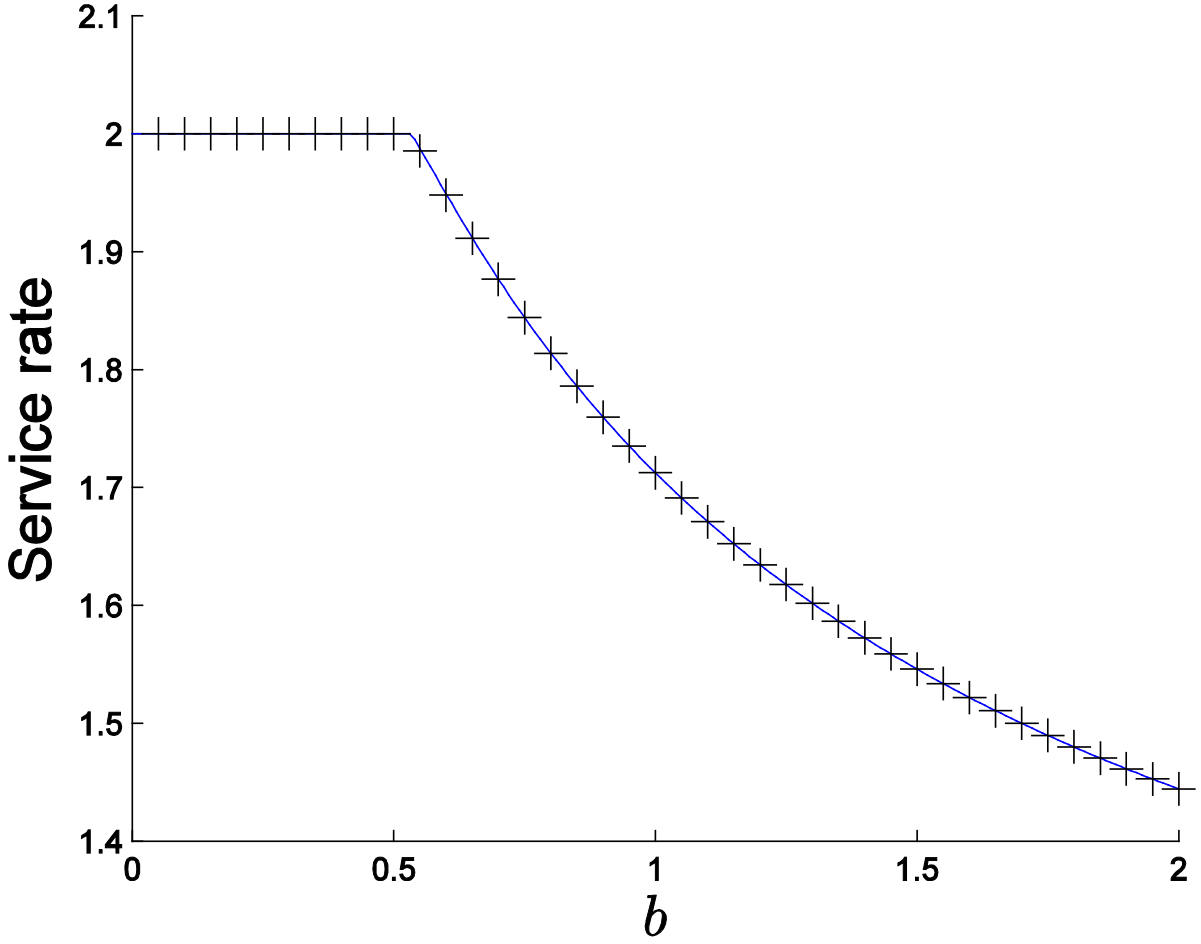
Proposition:

For any fixed $P_S > 0, P_F \geq 0$, the revised problem has a solution.

- If $b\mu^* \leq 1$, then $\hat{\mu}_E = \mu^*$ is one solution;
- Otherwise, any solution has $b\hat{\mu}_E > 1$.

Uniqueness?

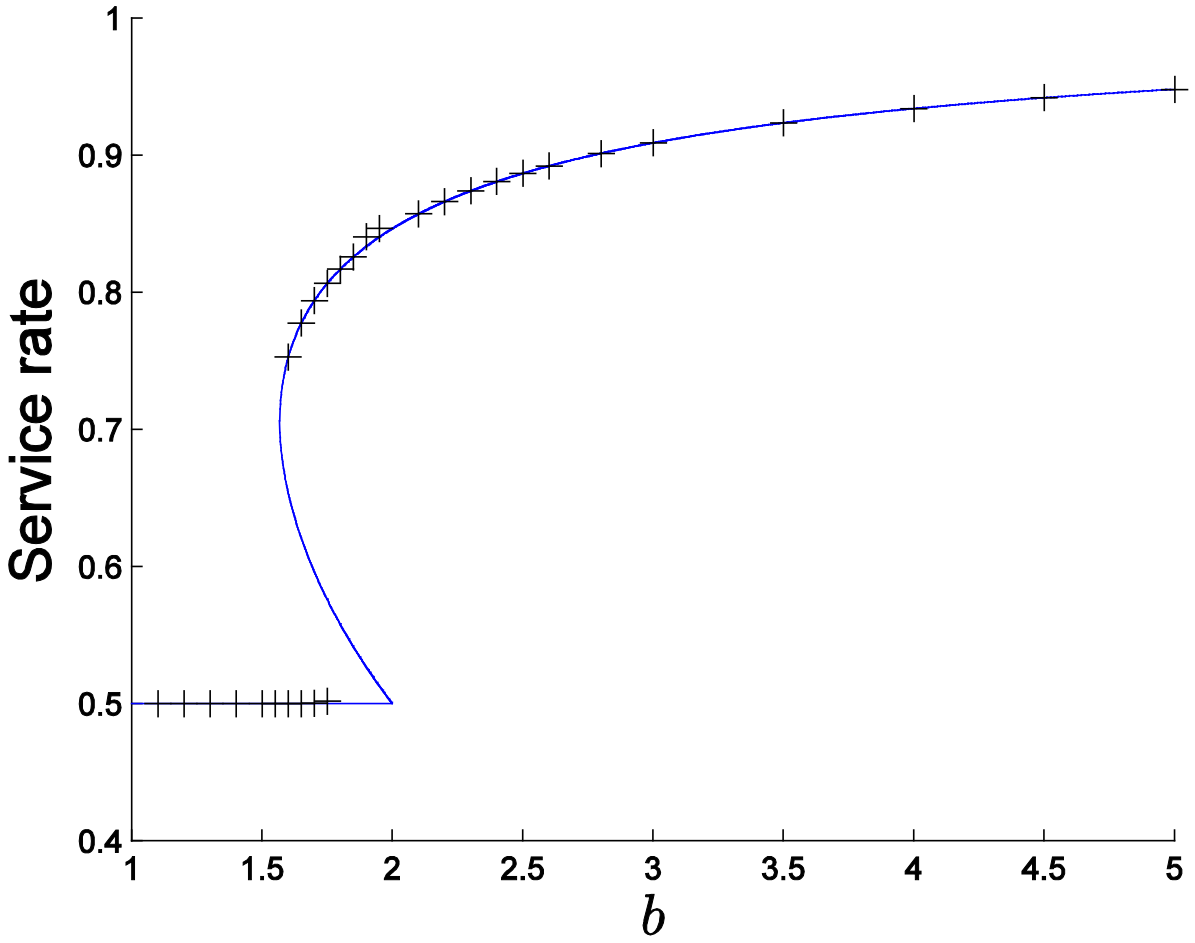
Comparison of μ_E^{λ}(cross) and $\widehat{\mu}_E$(line)



$\lambda = 1000, N^{\lambda} = b\lambda, \theta = 0.1, P_S = 20, P_F = 10;$

$p(\mu) = 1 - 0.5\mu, \mu \in [0,2]$, and $U_I(x) = 10\sqrt{1-x}$.

Comparison of μ_E^{λ}(cross) and $\widehat{\mu}_E$(line)



$\lambda = 1000, N^{\lambda} = b\lambda, \theta = 0.1, P_S = 5, P_F = 10;$

$p(\mu) = 1 - 0.5\mu, \mu \in [0,2]$, and $U_I(x) = 10\sqrt{1-x}$.

A Panorama of Optimal Limit Regimes

Utility and cost structure	Optimal Regime	Property
Linear Costs	Critically loaded	No idleness, no abandonment.
Utility = pay + concave idleness value	Underloaded	Servers enjoy idle time
Increasing abandonment cost	Overloaded	Some customers abandon
Concave idleness value + increasing abandonment cost	Underloaded + Overloaded	Customer waiting and server idling are simultaneous

Takeaways

We propose a first best compensation and staffing scheme for a large-scale service system with strategic employees.

Facing multiple strategic employees, the optimal compensation design needs to counteract the speed-up effect due to competition.

Different optimal regimes arise depending on the cost structure and employee utility function.

Thank you!

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