# Multiresolution based Smoothing

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- Nonparametric Regression
- Data Closeness

### 2 Multiresolution Conditions (MRC) for Smoothing

- MRC for one-dimensional Problems
- Multiresolution based Smoothing

#### MR-Smoothing by inhomogeneous Diffusion

- One-dimensional Diffusion Process
- Two-dimensional Diffusion Process
- MRC for two-dimensional Problems

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### Nonparametric Regression

#### Situation

- Data  $(x_i, y_i)$ ,  $i = 0, \ldots, n$  with  $x_i \in \mathbb{R}^k$ , k = 1, 2 and  $y_i \in \mathbb{R}$ .
- Model  $Y_i = f(x_i) + \sigma \varepsilon_i$ ,  $\varepsilon_i$  i.i.d.  $\mathcal{N}(0, 1)$
- Wanted: Decomposition  $y_i = f_n(x_i) + r_n(x_i)$

#### Smoothing Methods

- kernel estimator
- local polynomials
- smoothing splines
- wavelets

• ...

Different methods to balance **data closeness** and **simplicity**.

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Data Closeness

MRC for Smoothing 00000 MR-Smoothing by inhomogeneous Diffusion



- Data closeness can best be evaluated considering the residuals of an approximation.
- A multiresolution analysis of the residuals can be used to decide whether an approximation is close enough to the data or not.

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MRC for Smoothing

MR-Smoothing by inhomogeneous Diffusion

# 1-dim. Multiresolution Conditions

White noise MR criterion (Davies and Kovac, 2001)

• residuals  $r_n(x_i) = y_i - f_n(x_i)$ 

• MR coefficients

$$w_l(r) = \frac{1}{\sqrt{|I|}} \sum_{x_i \in I} r_n(x_i), \qquad |I| = \#\{x_i \mid x_i \in I\}$$

• MR constraints (modulus of continuity of the Brownian motion)

 $r_n$  look like Gaussian white noise  $\Rightarrow |w_l| \le \sigma_n \sqrt{2.3 \log(n)} \quad \forall l$ 

• 
$$\sigma_n = \frac{1.48}{\sqrt{2}} \text{MED}(|y_2 - y_1|, \dots, |y_n - y_{n-1}|) \text{ (e.g. } \sigma = 0.4, \sigma_n = 0.4056)$$

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MRC for Smoothing

MR-Smoothing by inhomogeneous Diffusion

# 1-dim. Multiresolution Conditions

- Different sets of Intervals I can be used for the MR conditions.
- A dyadic scheme is usually sufficient.
- MRC on different scales with threshold  $\sigma_n\sqrt{2.3\log(n)}pprox 1.62$

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MR-Smoothing by inhomogeneous Diffusion

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 Outline of the Problem
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 An Example: Locally Weighted Smoothing Splines

Iterative method to determine smoothing parameters  $\lambda_i$  automatically in

$$\min_{g} \sum_{i=1}^{n} \lambda_{i} (y_{i} - g(x_{i}))^{2} + \int_{a}^{b} (g''(x))^{2} dx.$$





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$$\min_{g} \sum_{i=1}^{n} \lambda_{i} (y_{i} - g(x_{i}))^{2} + \int_{a}^{b} (g''(x))^{2} dx.$$



### Multiresolution based Smoothing

In general: MR conditions can be used to improve or automate smoothing methods. They work as:

- stopping criterion in iterative procedures
  - to determine locally defined smoothing parameters
    - kernel estimators, local polynomials, smoothing splines
    - inhomogeneos diffusion processes (cf. Stichtenoth)
  - for inverse problems (cf. Mildenberger, Weinert)
- linear side conditions in (linear) optimization methods
  - min  $TV(f^{(k)})$  (cf. Kovac)
  - linear inverse problems

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MR-Smoothing by inhomogeneous Diffusion

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MRC for Smoothing

MR-Smoothing by inhomogeneous Diffusion

### An Example: min TV for Inverse Problems

• Model:  $y = Kf + \varepsilon$  with  $\varepsilon_i$  i.i.d.  $\mathcal{N}(0, \sigma^2)$ 

 Method: (find f with minimal total variation s.t. Kf satisfies the MRC) min<sub>f</sub> TV(f<sup>(k)</sup>) s.t. MKf ≤ b.

• Example: K represents the Abel-Integration, n = 400, k = 0,  $\sigma = 0.4$ 

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# 1-dim. Diffusion Process

$$\tilde{f}(x,t) = \frac{1}{\sqrt{4\pi t}} \int_{-\infty}^{\infty} y(s) \cdot \exp\left(-\frac{(x-s)^2}{4t}\right) ds$$

 $\widetilde{f}(x,t)$  satisfies the heat equation

$$\frac{\partial}{\partial t}\tilde{f}(x,t) - \frac{\partial^2}{\partial x^2}\tilde{f}(x,t) = 0$$

with the given data as starting points  $\tilde{f}(x_i, 0) = y_i$ .

 $\hat{f}_t$  = diffusion process with homogeneous Neumann conditions stopped at t t = smoothing parameter

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MRC for Smoothing

### Local Smoothing Parameter

### How can we realize a local smoothing parameter?

 $\implies$  inhomogeneous diffusion process

$$\frac{\partial}{\partial t}\tilde{f}(x,t) - \mathbf{a}(\mathbf{x}) \left(\frac{\partial^2}{\partial x^2}\tilde{f}(x,t)\right) = 0$$

a(x) = thermal diffusivity at x, new smoothing parameter

 $\widetilde{f}_a(x)$  := inhomogeneous diffusion process stopped at t=1

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MR-Smoothing by inhomogeneous Diffusion

# MRC to select local Smoothing Parameter automatically

#### goal:

find the smoothest among all the estimators  $\left\{ \tilde{f}_a; a(x) \ge 0 \right\}$  which satisfies the white noise MR criterion

i.e. find the largest  $a(x) \geq 0$ , so that  $\tilde{f}_a$  satisfies the MR criterion

 $\implies$  iterative algorithm

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MRC for Smoothing

MR-Smoothing by inhomogeneous Diffusion

### Diffusion Process with locally adapted a



computing time: 0.5 sec (kernel estimator: 163 sec)

# 2-dim. Diffusion Process

- inhomogeneous heat equation  $\frac{\partial}{\partial t}\tilde{f}(\mathbf{x},t) - a(\mathbf{x}) \Delta \tilde{f}(\mathbf{x},t) = 0$
- estimator  $\tilde{f}_a$ :

inhomogeneous diffusion process stopped at t = 1 with

- diffusivity a(x)
- starting values  $\tilde{f}(x_{ij}, 0) = y_{ij}$
- homogeneous Neumann conditions  $\nabla \tilde{f}(\mathbf{x}, t) = 0$  for  $\mathbf{x} \in \partial([0, 1] \times [0, 1])$

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Noisy Dataset

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# 2-dim. Multiresolution Conditions

• MR criterion for bivariate data:

$$w_{P} = \frac{1}{\sqrt{|P|}} \sum_{x_{ij} \in P} r_{n}(x_{ij})$$
$$|w_{P}| \leq \sigma_{n} \sqrt{2.3 \log n} \quad \forall P \in \mathcal{P}$$

• 
$$\sigma_n = \frac{1.48}{2} \operatorname{MED}(|y_{i+1,j+1} - y_{i+1,j} + y_{i,j+1} - y_{i,j}|)$$

• the partition  $\mathcal{P}$  should

- include different scales
- be fine enough to detect also small features
- allow a fast computation of the MR coefficients

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MR-Smoothing by inhomogeneous Diffusion

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  - be fine enough to detect also small features
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MRC for Smoothing

# MRC: different Partitions $\mathcal{P}$

different partitions  $\mathcal{P}$ :

- dyadic squares
- wedges
- dyadic squares on 2 levels

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# Dyadic Squares and Wedges

#### dyadic squares



#### wedges







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0 0 0

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# Dyadic Squares on 2 Levels

#### first level



0	0	0	0	0	0	٥	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	٥	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
٥	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
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#### second level

0 0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0 0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0 0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0 0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0 0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0 0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0 0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0 0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0 0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0 0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0 0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0 0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0 0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0 0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0 0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0 0	0	0	0	0	0	0	0	0	0	0	0	0	0	0





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MRC for Smoothing

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# $\tilde{f}_a$ for dyadic Squares on 2 Levels



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MRC for Smoothing

MR-Smoothing by inhomogeneous Diffusion

# $\tilde{f}_a$ for dyadic Squares on 2 Levels





#### Thank you for your attention!

contact: monika.meise@uni-due.de rahel.stichtenoth@uni-due.de www.stat-math.uni-essen.de

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# Iteration Steps





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