Eurandom workshop "Queues, Fluid Queues and Extremes"

Queueing theory meets optimization and control: subtleties of the minimum norm problem encountered by Gaussian queues

Ilkka Norros (VTT, Finland)
in collaboration with
Michel Mandjes, Petteri Mannersalo and Miranda van Uitert

The following minimum norm problem pops up in the context of large deviations of Gaussian queues. Consider a reproducing kernel Hilbert space R with kernel $\Gamma(s,t) = \mathrm{E} Z_s Z_t$, where Z is a continuous centered Gaussian process with stationary increments. Assume $\zeta \in R$ and $S \subset \Re$ compact. The problem is to find the minimum norm element in $\{f \in R : \ f(s) \geq \zeta(s) \ \forall s \in S\}$. It turns out that a crucial role is played by the infinitesimal space

$$\partial R_0 = \bigcap_{\epsilon>0} \overline{\mathrm{span}\{\Gamma(t,\cdot) : \ |t| < \epsilon\}}.$$

Roughly speaking, ∂R_0 is trivial if and only if the process Z has differentiable paths. Denote by $S^* = \{s \in S : \ \beta^*(s) = \zeta(s)\}$ the set of points where the condition is met tightly. We show that

$$\beta^* \in \bigcap_{\epsilon>0} \overline{\mathrm{span}\{\Gamma(t,\cdot) : \ d(t,S^*) < \epsilon\}}.$$

Thus, β^* is determined by its behavior on S^* *plus* on its infinitesimal surroundings. The result has a control theoretic interpretation: to achieve $f(s) \geq \zeta(s)$ on S, it is sufficient to push f to ζ on S^* and, in the smooth case $\partial R_0 \neq \{0\}$ only, possibly control the derivatives of f at the isolated points of S^*. We present an approximation method that can be used to find the minimum norm element in the case $\partial R_0 = \{0\}$. The basic examples considered are fractional Brownian motion for $\partial R_0 = \{0\}$ and the integrated Ornstein-Uhlenbeck process for $\partial R_0 \neq \{0\}$.