Recurrence of once edge-reinforced random walks with exponential weights on trees

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We introduce once edge-reinforced random walks with exponential weights on the general graphs. Let $G = (V, E)$ be infinite connected graphs with vertex set V and edge set E. Let $|v|$ be the graph distance from $o \in V$ to $v \in V$. We put $|e| = \min\{|v|, |u|\}$ for $e = \{v, u\} \in E$. We consider a stochastic process $\overrightarrow{X} = \{X_n\}_{n\geq 0}$ on G starting at $x_0 = o \in V$ and moving to one of the nearest neighboring vertices at each step. For parameters $\delta > 0$ and $\beta > 0$, a positive weight $w(n, e)$ is given at each edge $e \in E$ and at time n by

$$w(n,e) \quad = \quad \begin{cases} \delta^{|e|} & \text{for } \{X_l, X_{l+1}\} \neq e \text{ for all } l < n, \\ (\beta\delta)^{|e|} & \text{for } \{X_l, X_{l+1}\} = e \text{ for some } l < n. \end{cases}$$

For a given path $\{X_0, X_1, \ldots, X_n\}$, the transition probability of the process to a neighboring vertex is proportional to the weight $w(n, e)$ of the crossed edge e at time n. The process is called a once edge-reinforced random walk with exponential weights on G. We denote it by ORRW with parameters (δ, β) on G in short. A simpler once edge-reinforced random walk was originally introduced by R. Durrett, H. Kesten, V. Limic(2002). In their model the weight $w(n, e)$ takes 1 if the process has not crossed the edge e by time n, and β otherwise. An ORRW with parameters $(\delta, 1)$ becomes a Markov chain. We call this Markov chain as an MC with a parameter δ on G. It can be shown without difficulty that there exists a critical value δ_G such that if $\delta < \delta_G$ then an MC with a parameter δ on G is a.s. recurrent, and if $\delta > \delta_G$ then the MC is a.s. transient.

In this talk, we would like to mention the following result for a general tree G.

Theorem Let $\overrightarrow{X}$ be an ORRW with parameters (δ, β) on G. Then we have
(1) If $\beta \geq 1$ and an MC with a parameter δ on G is a.s. recurrent, then $P[\overrightarrow{X}$ is recurrent $] = 1$.
(2) If $\beta < 1$ or $\delta > 1$, then $P[\overrightarrow{X}$ is transient $] = 1$.

References

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