

Fast Valuation and Calibration of Credit Default Swaps Under Lévy Dynamics

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Abstract

Credit default swaps (CDSs), the basic building block of the credit risk market, offer investors the opportunity to either buy or sell default protection on a reference entity. The protection buyer pays a premium periodically for the possibility to get compensation if there is a credit event on the reference entity until maturity or the default time, whichever is first. If there is a credit event the protection seller covers the losses by returning the par value. The premium payments are based on the CDS spread.

The spread of CDSs depends on the default probability of the underlying reference entity and it is possible to back out the market view of default probabilities for individual names from quoted market prices. It is therefore essential to be able to use advanced models in credit default modeling.

Lately Lévy models have attracted attention in the field of credit risk, see e.g. Cariboni (2007), Cariboni and Schoutens (2007) and Madan and Schoutens (2007). Cariboni and Schoutens (2007) price CDSs using the structural approach with a Variance Gamma model driving the firm value. To calculate the default probabilities they derive the partial integro-differential equation (PIDE) satisfied by the barrier option price and solve the equation by adapting a numerical scheme developed

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by Hirta and Madan (2004) for pricing American options. If the driving Lévy process in the firm value model only has negative jumps, i.e., it is a single-sided or spectrally negative Lévy process, then the default probabilities can be found by numerically performing a double Fourier inversion as shown in Madan and Schoutens (2007).

We take a structural approach towards the modelling of credit risk, following the same methodology as Black and Cox (1976), which defines the credit event to be the first time the value of the reference entity is below a predefined lower barrier representing the total debt of the firm. In contrast to Black and Cox (1976), which used a Geometric Brownian motion to drive the firm value, we set up a firm value model driven by an exponential Lévy process. In particular, we study the model developed by Carr *et al* (2002) (CGMY), and the Normal Inverse Gaussian (NIG) processes.

We will show that default probabilities can be efficiently recovered from the Fourier-cosine series expansion of the underlying density, following the path of the COS method for European options in Fang and Oosterlee (2008a) and that for Bermudan options and discretely monitored barrier options in Fang and Oosterlee (2008b). We can price a single CDS within fractions of a second and several CDSs in less than two seconds with a high accuracy. Switching from one underlying model to another is furthermore as easy as switching from one characteristic function to another. This enables us to calibrate the Lévy models to market quotes of CDS prices easily and efficiently.

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