Similarity metrics based on *n*-gram statistics and applications to authorship attribution problems

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1. The problem
    - Quantitative A.A.
    - The Gramsci Project

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   - The Gramsci Project

2. Similarity metrics
   - Definitions
   - A model
   - *n*-gram distances
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3. Experiments
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Stylometry

Stylometry = Quantitative Authorship Attribution = detection of the author of an anonymous/apocryphal text by counting some "quantities" inside the text itself and comparing these measures to those performed on texts of known attribution (*reference set*).

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De falso credita et ementita Constantini donatione declamatio - L. Valla, 1440

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Examples:

De Morgan, 1882, math	Mean length of words
Mosteller & Wallace, 1888, stat	Frequency of “typical” words
Yule, 1944, stat	Lexical richness
Ledger, 1989, stat	Many indicators with no syntactical meaning
Kešelj, 2003, cs	*n*-grams

Our problem

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Joint work with M. Degli Esposti (Univ. of Bologna), D. Benedetto and E. Caglioti (*La Sapienza*, Rome) and M. Lana (Univ. of Western Piedmont, Vercelli).

Texts as symbol sequences

Some definitions:

$$\mathcal{A} = \text{a finite alphabet}$$
$$\mathcal{A}^n = \{x = (x_1, \ldots, x_n) \mid x_j \in \mathcal{A}\}$$
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A similarity measure for biological signals: new applications to HRV analysis, *JP J Biostat., vol 1, n 1, pp 53-78 (2007)*

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Sequence distance via parsing complexity: Heartbeat signals,
Chaos, Solitons and Fractals (2007), in press

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From our point of view a text is an element of $\mathcal{A}^*$... no grammatical structure is taken into consideration.

(Pseudo) similarity distances

A distance is any function

$$d : \mathcal{A}^* \times \mathcal{A}^* \longrightarrow \mathbb{R}$$

with three properties:

symmetric: $d(x, y) = d(y, x)$

positive: $d(x, y) \geq 0$ and $d(x, y) = 0 \Leftrightarrow x = y$

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Often we use pseudo-distances...

Authors as Markov sources

A very strong hypothesis:

suppose an author writes his texts choosing at each step a new character according to some transition probabilities depending only on the last n characters (the last *n-gram*) of the generated sequence.

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A simpler idea (Kešelj, 2003): comparing *n*-gram frequencies through a similarity metric...

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Example: `feel` has $D_1 = \{\texttt{f}, \texttt{e}, \texttt{l}\}$, $D_2 = \{\texttt{fe}, \texttt{ee}, \texttt{el}\}, \ldots$

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Kešelj won the AAAC (Juola, 2004)...

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Problem: for "short" texts and "large" n, most n-grams appear just once $\Rightarrow$ we do not cut the dictionaries but add a factor to Kešelj's formula:

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$\hookrightarrow$ Benedetto, Caglioti, Loreto (2002): K-L divergence approximation (?) through data compression.

▸ tell me more

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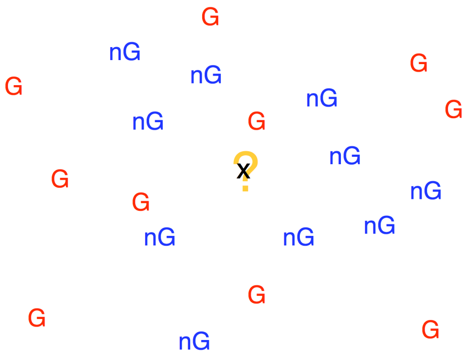
e.g. Gramscian corpus (A = Gramsci, B = "not Gramsci")



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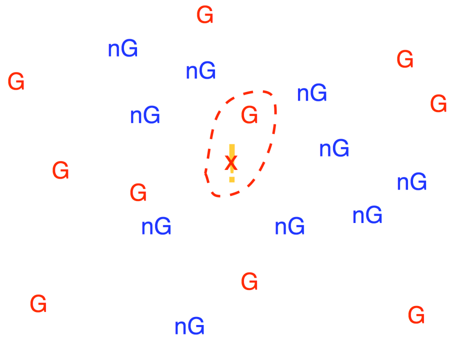
First idea: nearest neighbour



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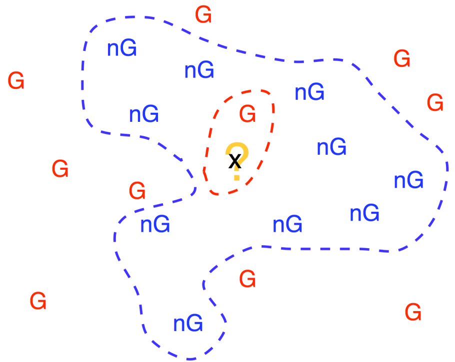

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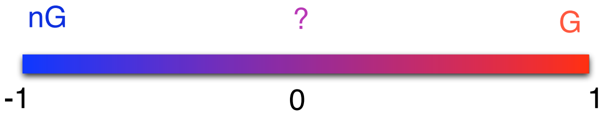

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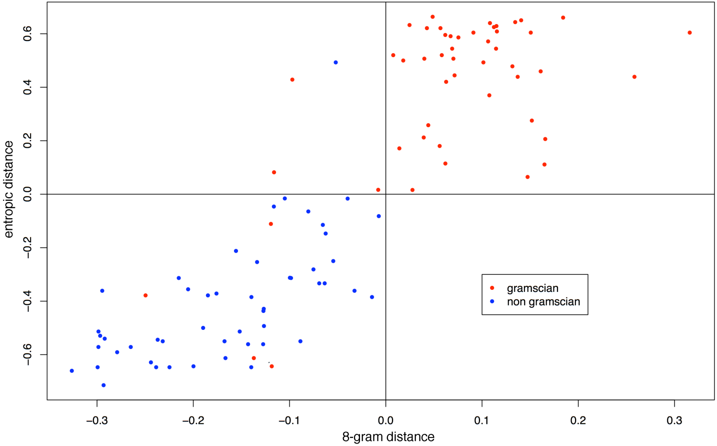

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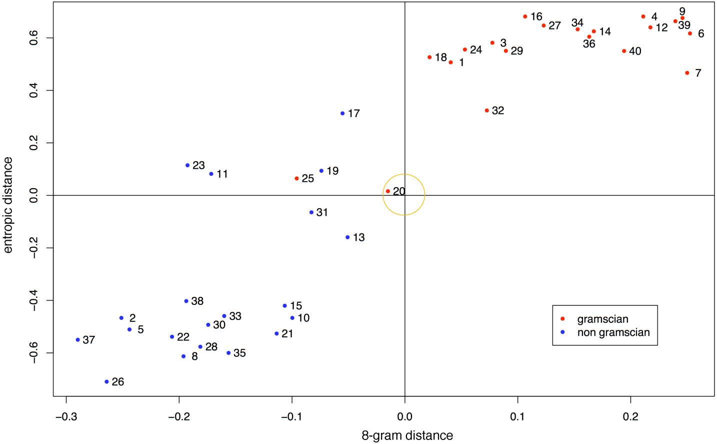

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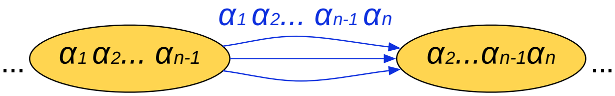
The answer comes from (multi-)graph theory.

n-gram graphs

For given $n \geq 2$ and $x \in \mathcal{A}^*$, the *n-gram graph* of x is the directed multigraph $G_n(x)$ with:

vertex set: $D_{n-1}(x)$

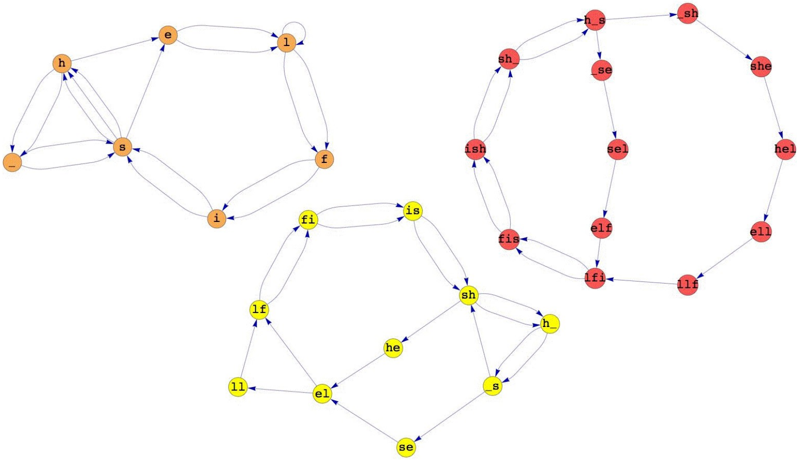
edge set: if the n-gram $(\alpha_1, \ldots, \alpha_n)$ appears m times in x, m edges link the vertex $(\alpha_1, \ldots, \alpha_{n-1})$ to the vertex $(\alpha_2, \ldots, \alpha_n)$.



$\#\{ i \mid x_i=\alpha_1 \ldots x_{i+n}=\alpha_n \} = 3$

n-gram graphs

Example: G_n(`selfish_shellfish_`) for $n = 2, 3, 4$.



Texts as Eulerian circuits

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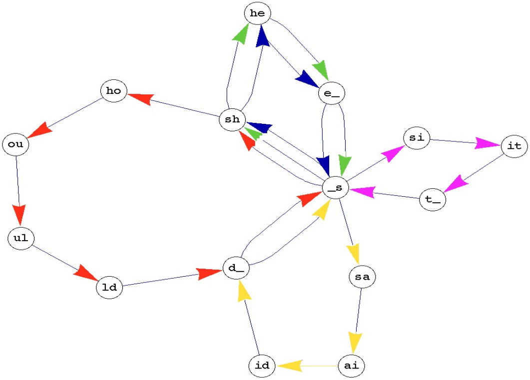

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Spanning tree directed towards v_i = spanning tree such that $\forall\, j$ the only path linking v_j and v_i is directed towards v_i.

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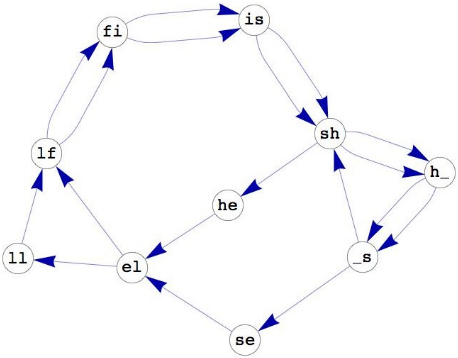

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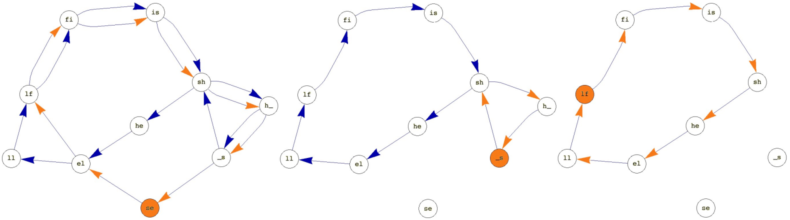

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A 6-equivalent of *Wish you were here*

So, so you think you can tell
heaven from hell, blue skies from pain,
can you tell a green field
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A smile from a veil?
Do you think you can tell?

Did they get you to trade,
your heroes for ghosts?
Hot ashes for trees?
Hot air for a cool breeze?
Cold comfort for change?
And did you exchange
a walk on part in the war,
for a lead role in a cage?

How I wish, how I wish you were here,
we're just two lost soles swimming in a fish bowl,
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7.	*gram_32*
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$\Rightarrow$

$$v_G(gram\_27) = \tfrac{1}{1} + \tfrac{2}{2} + \tfrac{3}{3} + \tfrac{6}{4} + \tfrac{7}{5} + \dots$$

$$v_{nG}(gram\_27) = \tfrac{4}{1} + \tfrac{5}{2} + \tfrac{9}{3} + \tfrac{10}{4} + \dots$$

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$$v_A(x) = \sum_{j=1}^{N} \frac{k_A(j)}{j}, \quad v_B(x) = \sum_{j=1}^{N} \frac{k_B(j)}{j},$$

   where $k_A(j)$ = position in the rank of the j-th text by A.

3. normalized vote:

$$v(x) = \frac{v_B(x) - v_A(x)}{v_B(x) + v_A(x)};$$

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back

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Even if authors were Markov sources with finite memory, we would not know their probability distributions and their memory length!

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where:

$$\begin{aligned} x &= \text{string of length } N \text{ generated by source } p; \\ y &= \text{string of length } N \text{ generated by source } q; \\ c(y) &= \text{cardinality of the LZ parsing of } y \\ &\quad \text{(Lempel \& Ziv, 1976, 1977, 1978);} \\ c(y \| x) &= \text{cardinality of the LZ parsing of } y \text{ performed} \\ &\quad \text{only with substrings from } x. \end{aligned}$$

LZ78

BCL method

LZ parsing algorithm is the base of today's data compressors (zippers): *gzip, winzip, . . .*

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given two (unknown) sources A and B, X a "long" sequence from A and y a "short" sequence from B, $X.y$ the string obtained appending y after X, L_X the length of the zipped file X and $\Delta_{Xy} := L_{X.y} - L_X$,

$$S_{AB} := \frac{\Delta_{Xy} - \Delta_{Yy}}{|y|}$$

could be an estimate of the divergence between A and B:

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could be an estimate of the divergence between A and B: while compressing $X.y$ the zipper, being sequential, first parses X and then parses y using mostly substrings from X.

BCL distance

BCL method and applications to classification problems:

D. Benedetto, E. Caglioti, V. Loreto, Language Trees and Zipping, *Physical Review Letters 88 (2002)*

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Distance function for two sources A and B:

$$\boxed{d_{BCL}(A,B) = \frac{\Delta_{Xy} - \Delta_{Yy}}{\Delta_{Yy}} + \frac{\Delta_{Yx} - \Delta_{Xx}}{\Delta_{Xx}}}$$

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A similar distance is used for the Gramscian corpus.

back

LZ78 parsing

Incremental LZ parsing: parse $y \in \mathcal{A}^*$ into $c(y)$ distinct substrings (phrases) such that each phrase is the shortest string which is not a previously parsed one.

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Similarity Metrics for A.A.

Chiara Basile

Appendix

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$$y = \mathtt{01111000110}$$
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$$c(y \parallel x) = |\{\ \mathtt{011},\ \mathtt{110},\ \mathtt{00110}\}| = 3$$

back