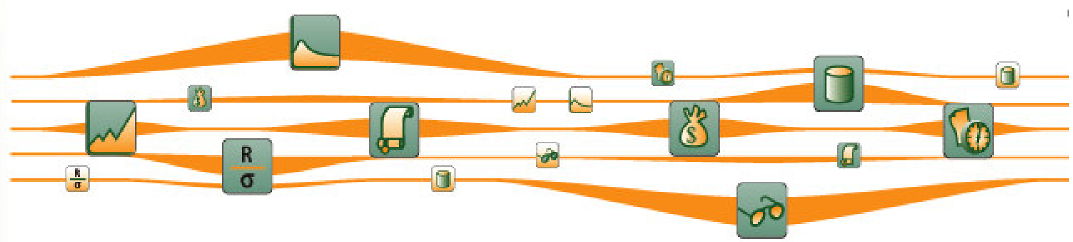
FINALYSE

Composing Solutions for Finance



Historical calibration of the Equivalent Martingale Measure

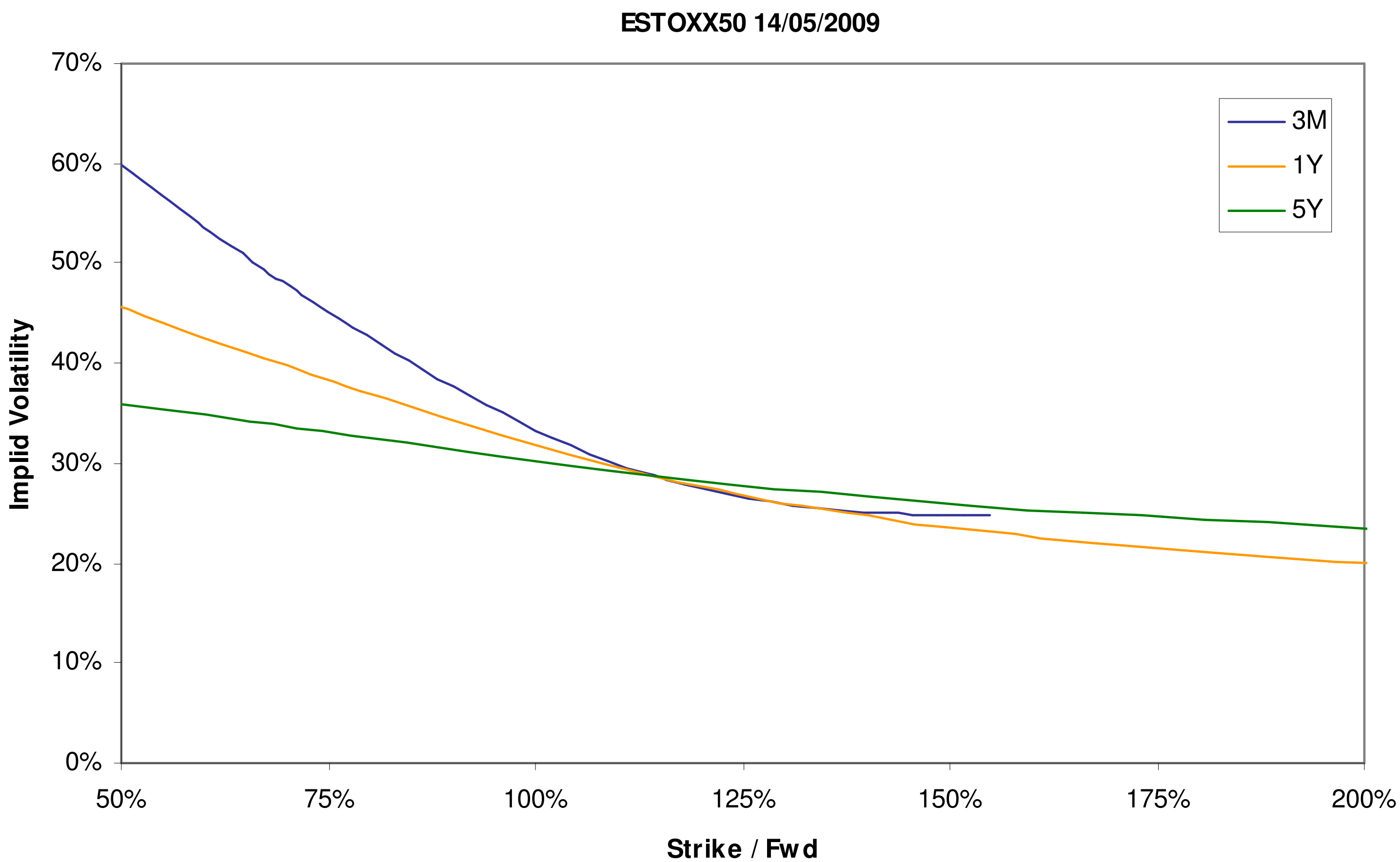
Péter Dobránszky

15th July, Eindhoven

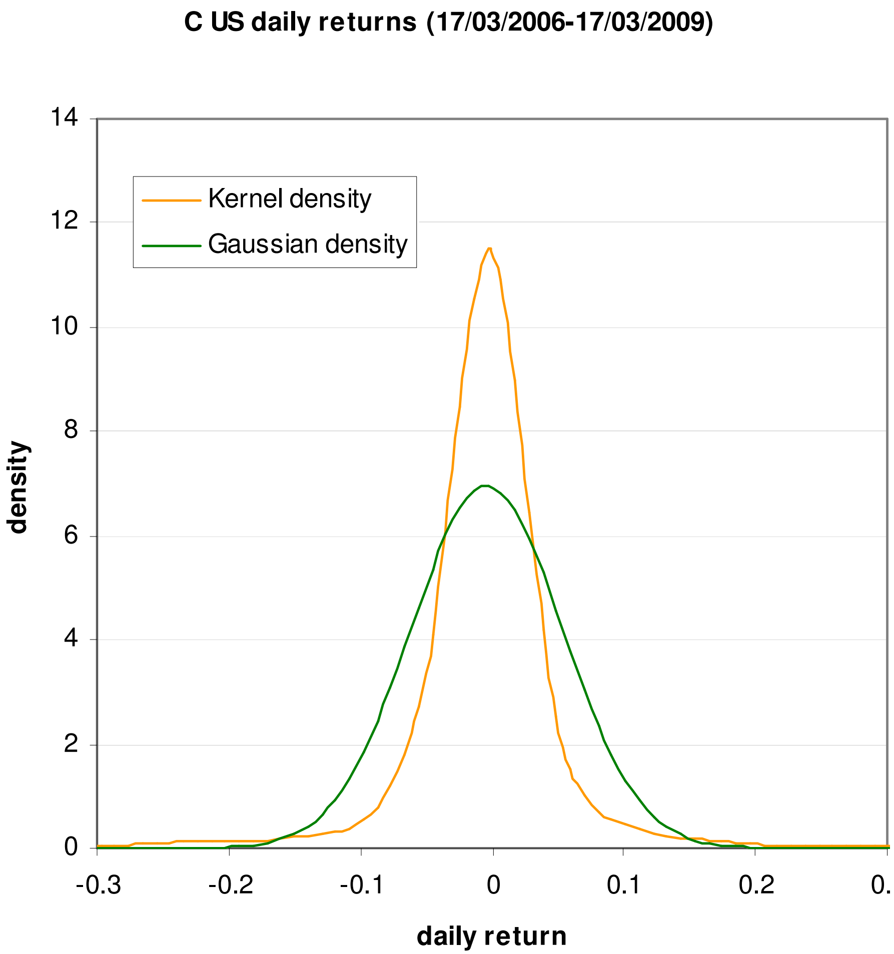
Agenda presentation

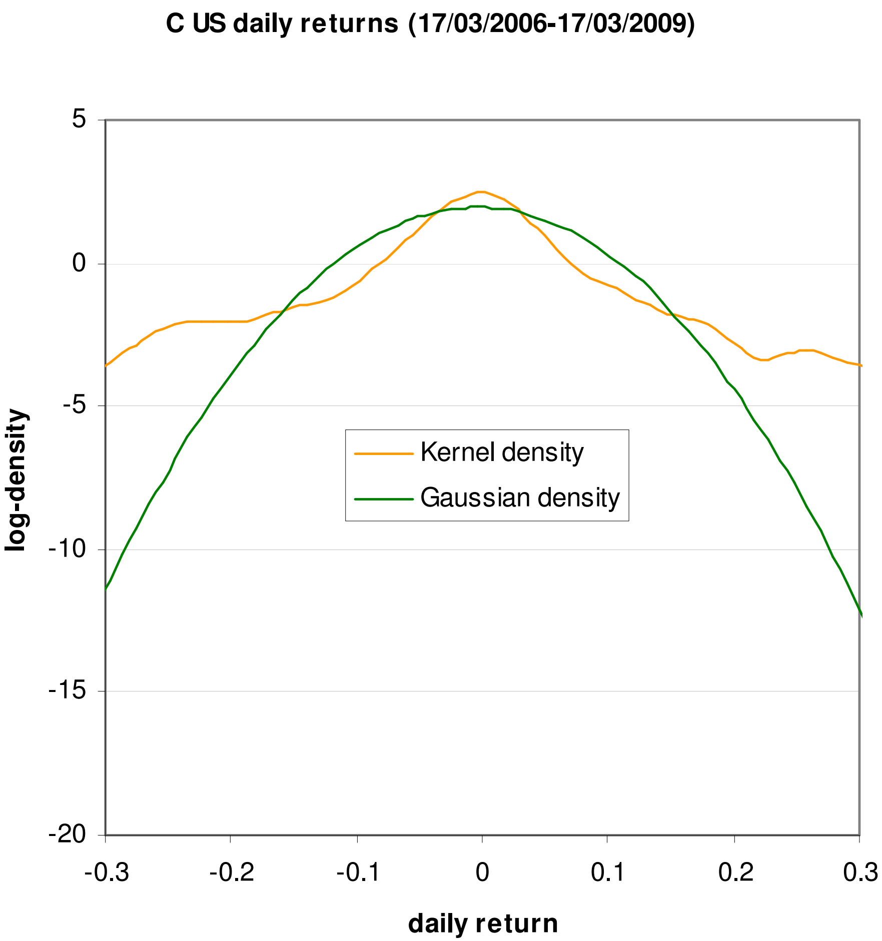
1. Usual Pitfalls of Financial Models
2. Why Is There a Problem?
3. How to Avoid Common Mistakes
4. Questions

Implied volatility surface by strike price



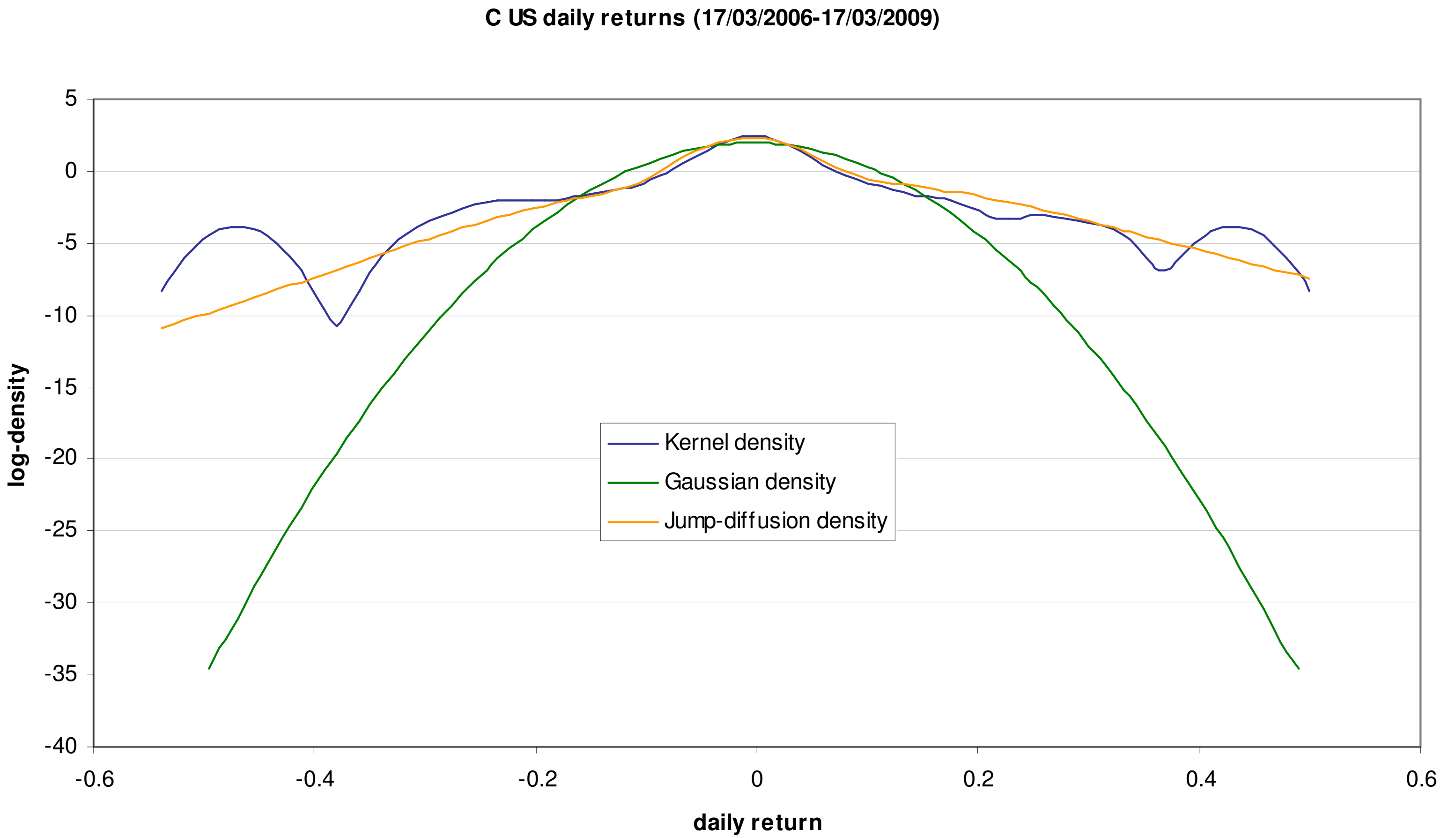
Fat tailed daily returns





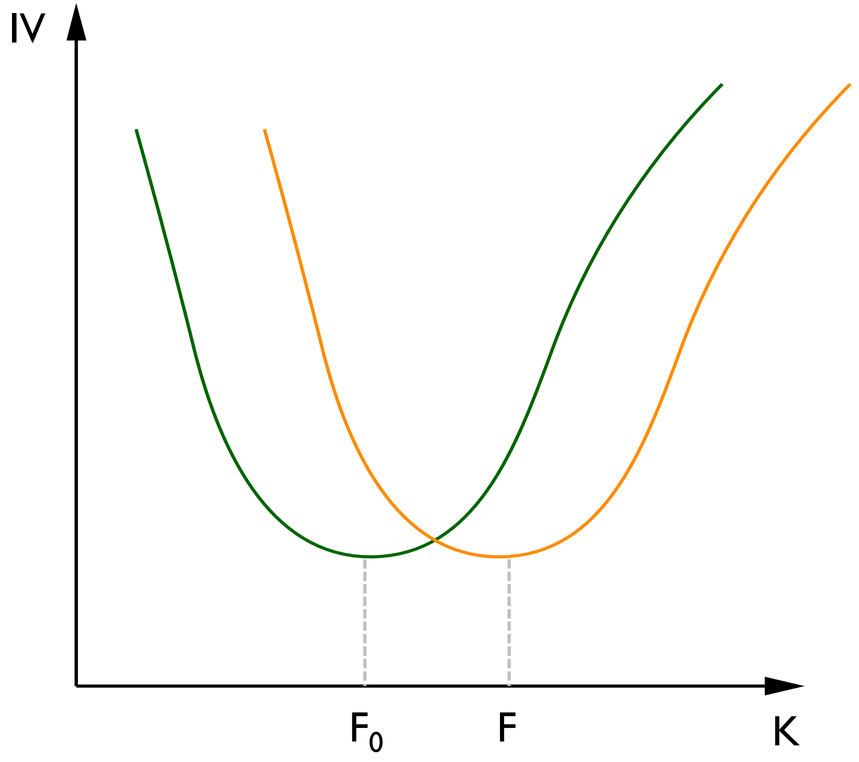
Lévy processes

- Generalization of jumps-diffusion processes with various jump types

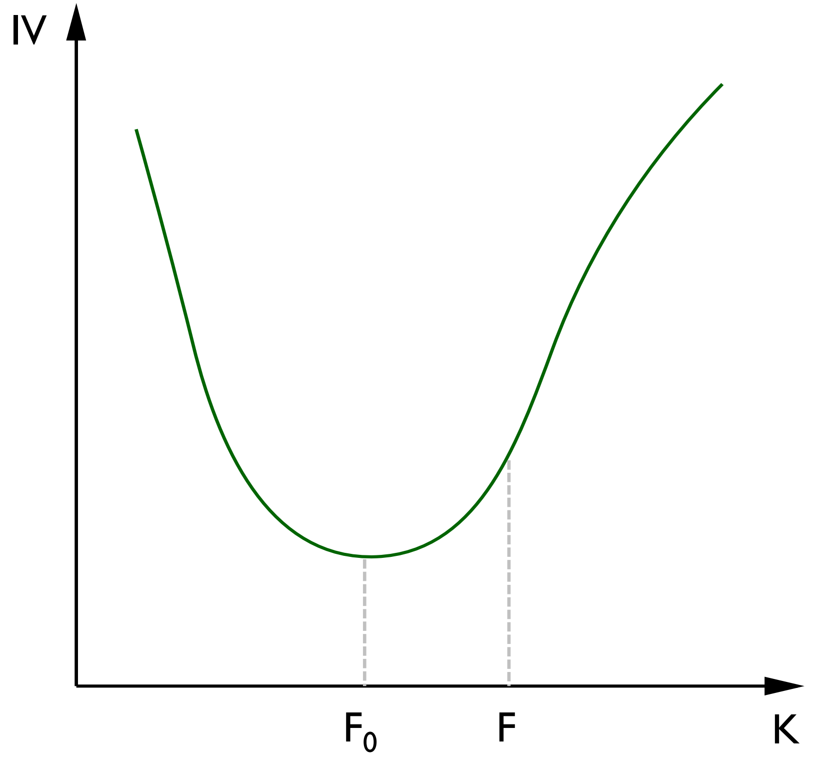


Sticky-strike versus sticky-moneyness

sticky-moneyness



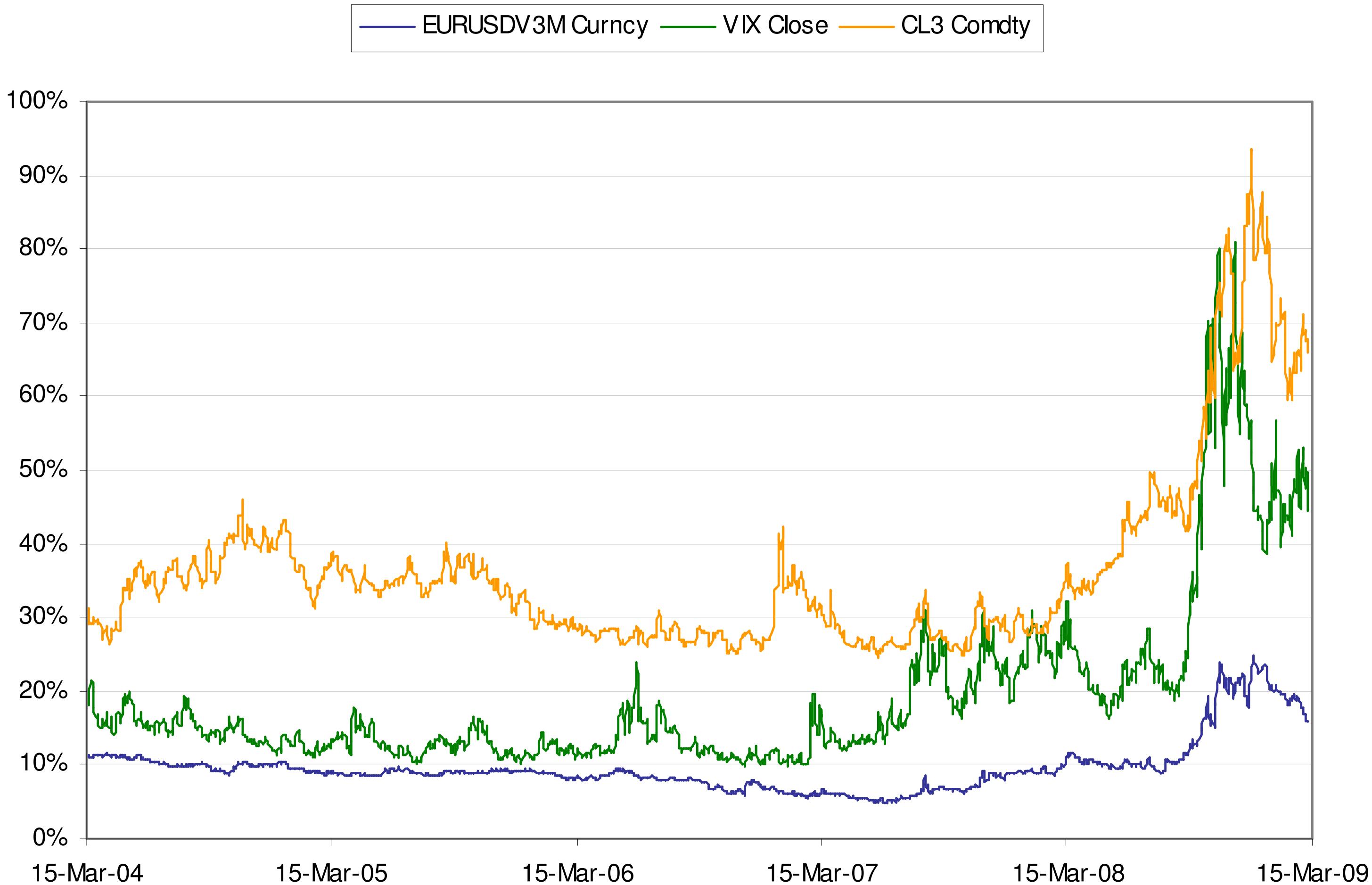
sticky-strike



Stationary Lévy processes

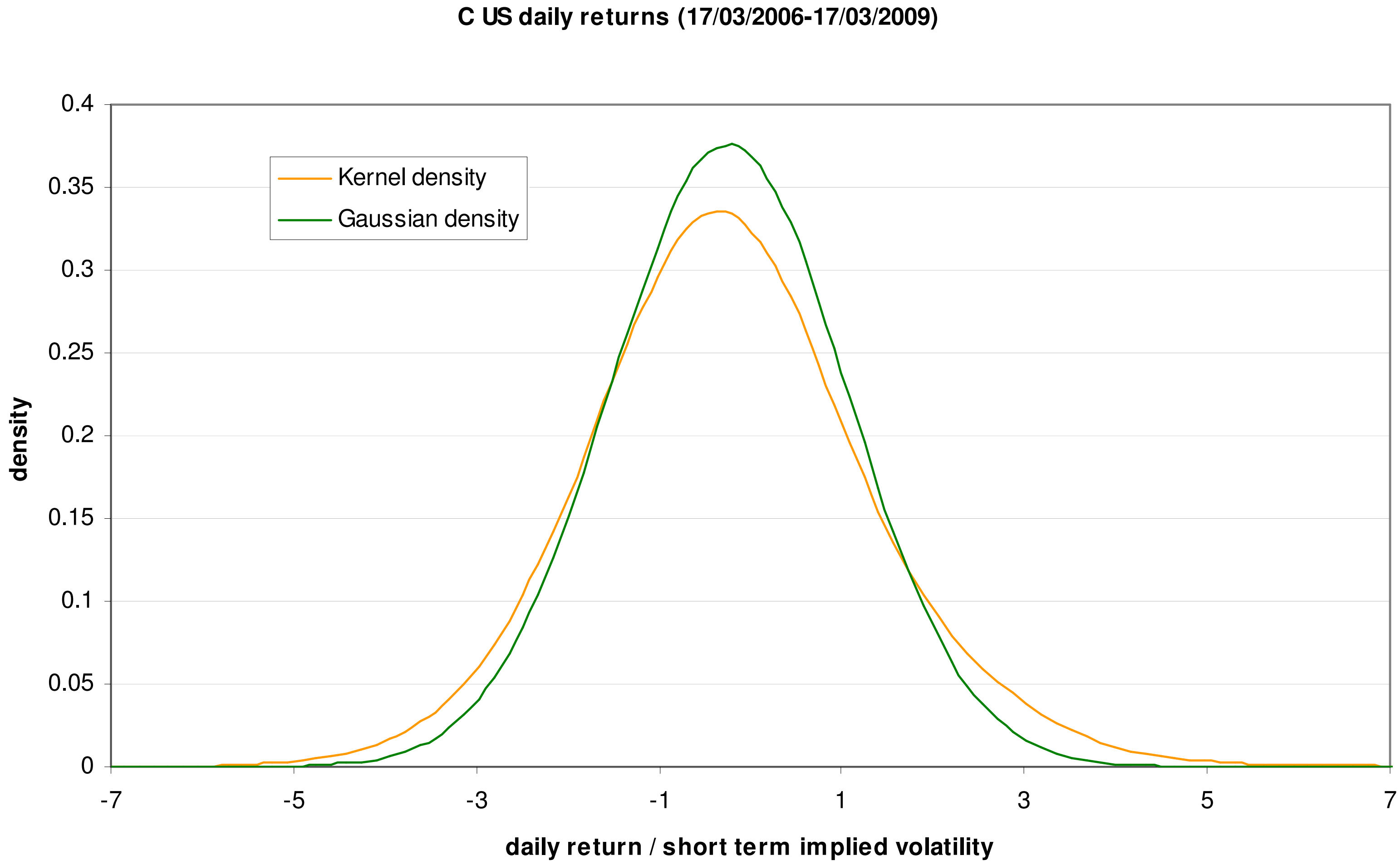
- FMLS, VG, CGMY, NIG
- Stationarity, independent daily returns
  - What happens today will not matter tomorrow
  - Law is the same every day
  - Implied volatility surface should not move
  - Everything is sticky-moneyness
- Non-stationary Lévy processes with random clock
  - Random clock is interesting only with persistence!

Stochastic volatility



Foreign exchange, equity and energy volatility references

Normalized daily returns

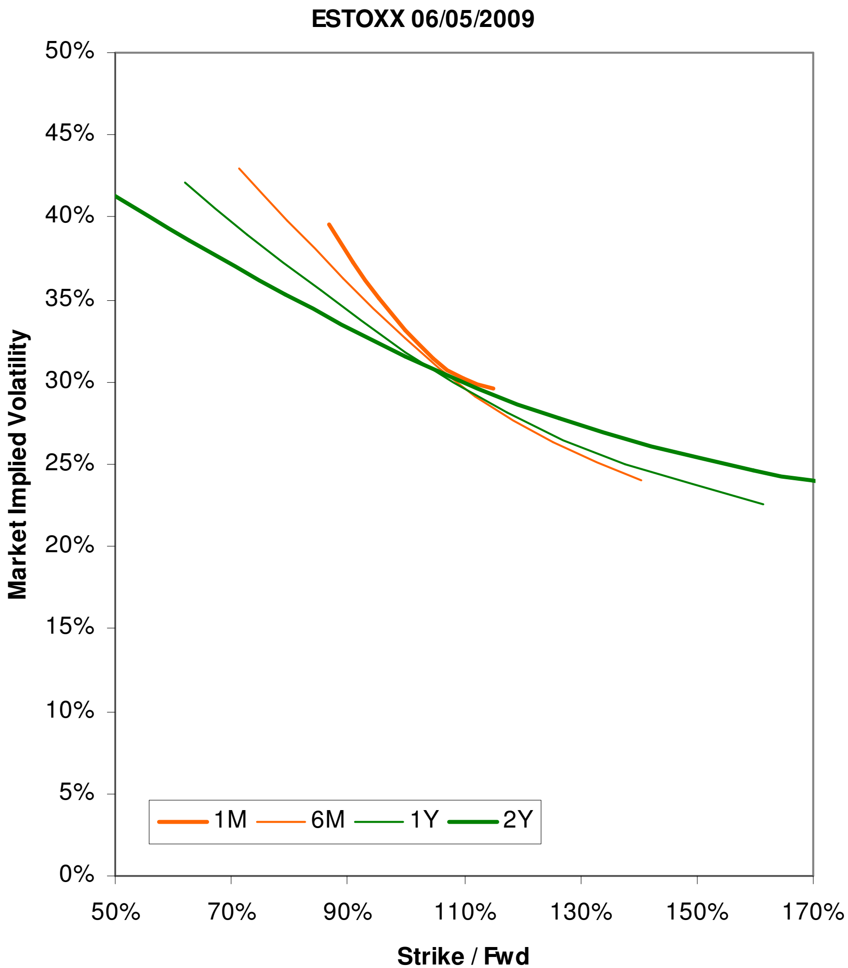


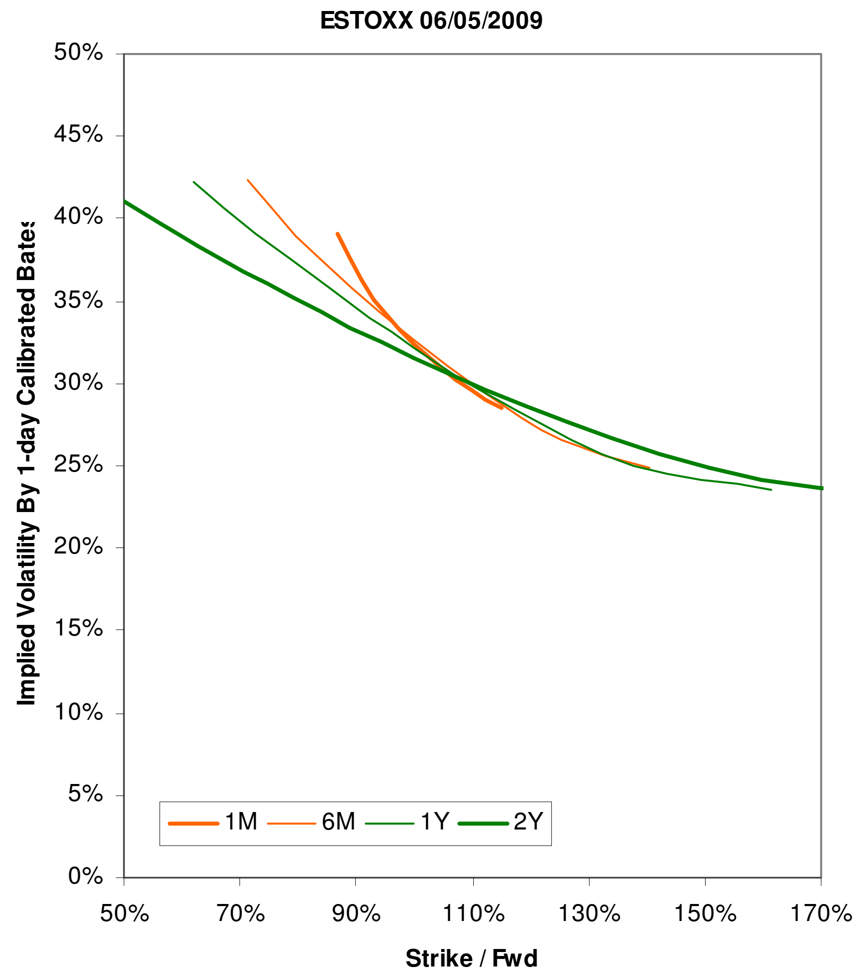
Heston (1993) and Bates (1996)

$$
\begin{aligned}
dS_t &= \mu_S S_t dt + \sqrt{\nu_t} S_t dW_t^S \\
d\nu_t &= \kappa\left(\theta - \nu_t\right) + \sigma\sqrt{\nu_t} dW_t^\nu
\end{aligned}
\qquad \left\langle dW_t^S, dW_t^\nu \right\rangle = \rho
$$

$$
\begin{aligned}
dS_t &= \mu_S S_t dt + \sqrt{\nu_t} S_t dW_t^S + \left(e^J - 1\right) S_t dN_t \\
d\nu_t &= \kappa\left(\theta - \nu_t\right) + \sigma\sqrt{\nu_t} dW_t^\nu
\end{aligned}
$$

One-day calibration





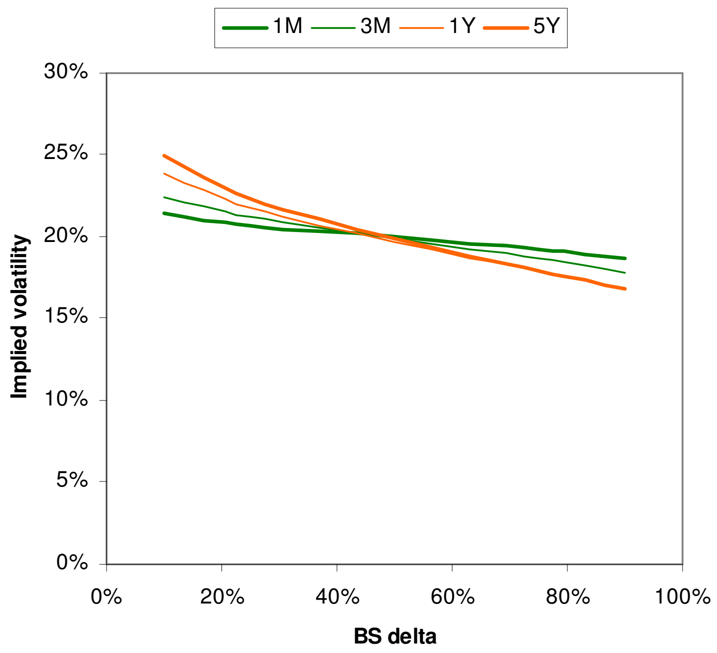
Local stochastic volatility and local Lévy

$$dS_t = \mu_S S_t dt + a\left(S_t, t\right)\sqrt{\nu_t} S_t dL_t^S$$
$$d\nu_t = \kappa\left(\theta - \nu_t\right) + \sigma\sqrt{\nu_t} dL_t^{\nu}$$

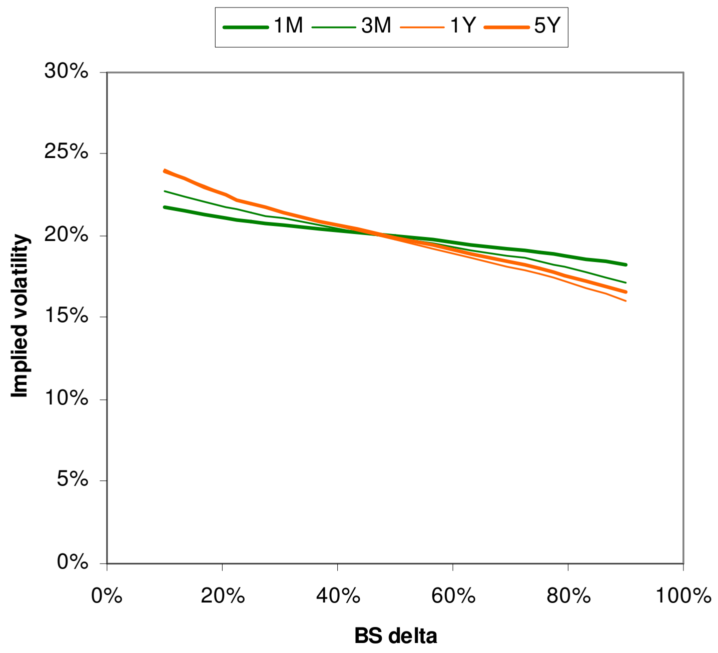
- Very popular nowadays – bunch of exotics can be easily priced
- Major challenge is to find the best calibration technique to get the forward skew as high as possible

Persistent, but how persistent?

- What is the volatility mean-reversion level? $\left(\kappa = 0.988\right)$
- A 5/1 forward start option has really no risk exposure?
- What is the sensitivity to kappa? (flat TS → low, Hessian)

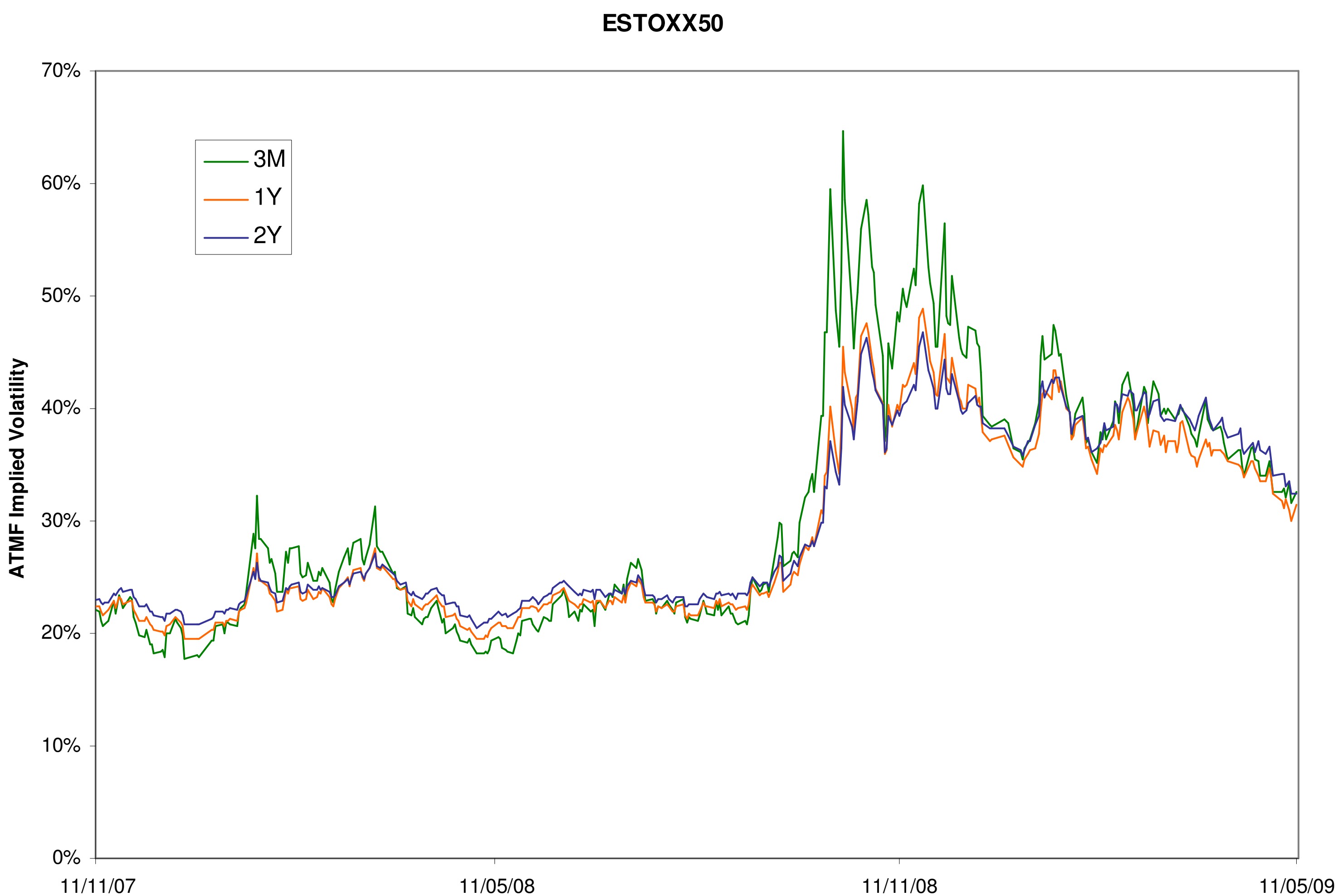


$$Heston : \nu_0 = 0.04, \theta = 0.06,$$
$$\boxed{\kappa = 0.4}, \sigma = 0.2, \rho = -0.7$$



$$Heston : \nu_0 = 0.04, \theta = 0.05,$$
$$\boxed{\kappa = 0.9}, \sigma = 0.2, \rho = -0.9$$

Look at the history!



Historical calibration of the EMM

- Incomplete market, but arbitrage freeness → EMM exists
- EMM selection: by model selection and by its calibration
- Assumption: risk-neutral measure is fixed not only through strikes and maturities, but also through trading days

- Model parameters (risk prices) are unique for the history
- Only state variables (risk factors) change from day to day

- Bates: 7 model parameters + 1 state variable

Calibration procedure I

Step 1

- Choose 7 maturities and 7 reasonable strikes (by BS delta) for each valuation date → 49 vanilla options
- Choose every 2 month a total of 20 valuation dates → 3Y
- Total of ~1000 option prices to calibrate to
- Calibrate model parameters and state variables

Step 2

- Involve all valuation dates and recalibrate
- As initial guess use the calibrated model parameters from step 1

Step 3

- Localize: time-dependent drift, risk premia for the current day

Calibration procedure II

<u>Stage 1</u>

- Set expectation range for each model parameter (hint for DE)
- Inside the engine normalize and transform the model parameters
  - ✓ log, logit, exp, … + apply Feller condition

- Calibration of the model parameters with Differential Evolution
- Inside function evaluations for each valuation date separately calibrate the state variables with Levenberg-Marquardt
- Reset the state variables after each generation/candidate

<u>Stage 2</u>

- Fine tune model parameters with Levenberg-Marquardt
- Calibrate the state variables only when the error function is evaluated, then cache them and use them to evaluate the Jacobian

Experience

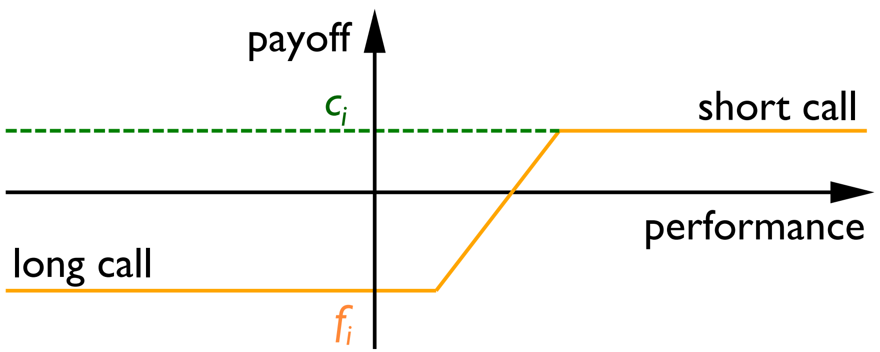
- To find hypothetical results (simulated option prices) LM (maybe with a first L-BFGS step) works pretty well
- When there are noises in target prices or identical factors (Christoffersen, Heston and Jacobs model) LM fails
- Model parameters should be calibrated only once for a while, thus slow DE is not a problem

- Bates calibration on ESTOXX50 $\kappa = 0.537$
- Forward starting straddle seems to show risk exposure

Cliquet spreads

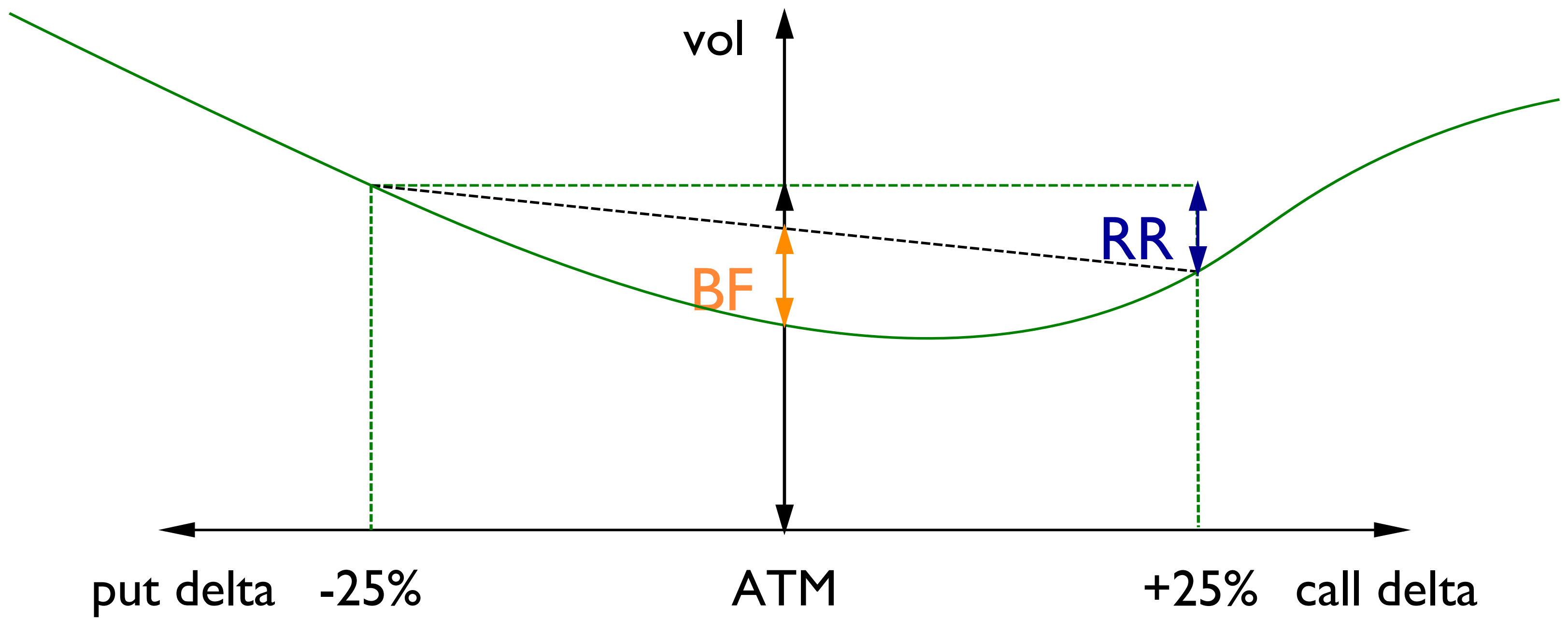
- Set of forward starting performance spread options

$$payoff = \sum_i \max\left(f_i, \min\left(c_i, \frac{S_i - S_{i-1}}{S_{i-1}}\right)\right)$$

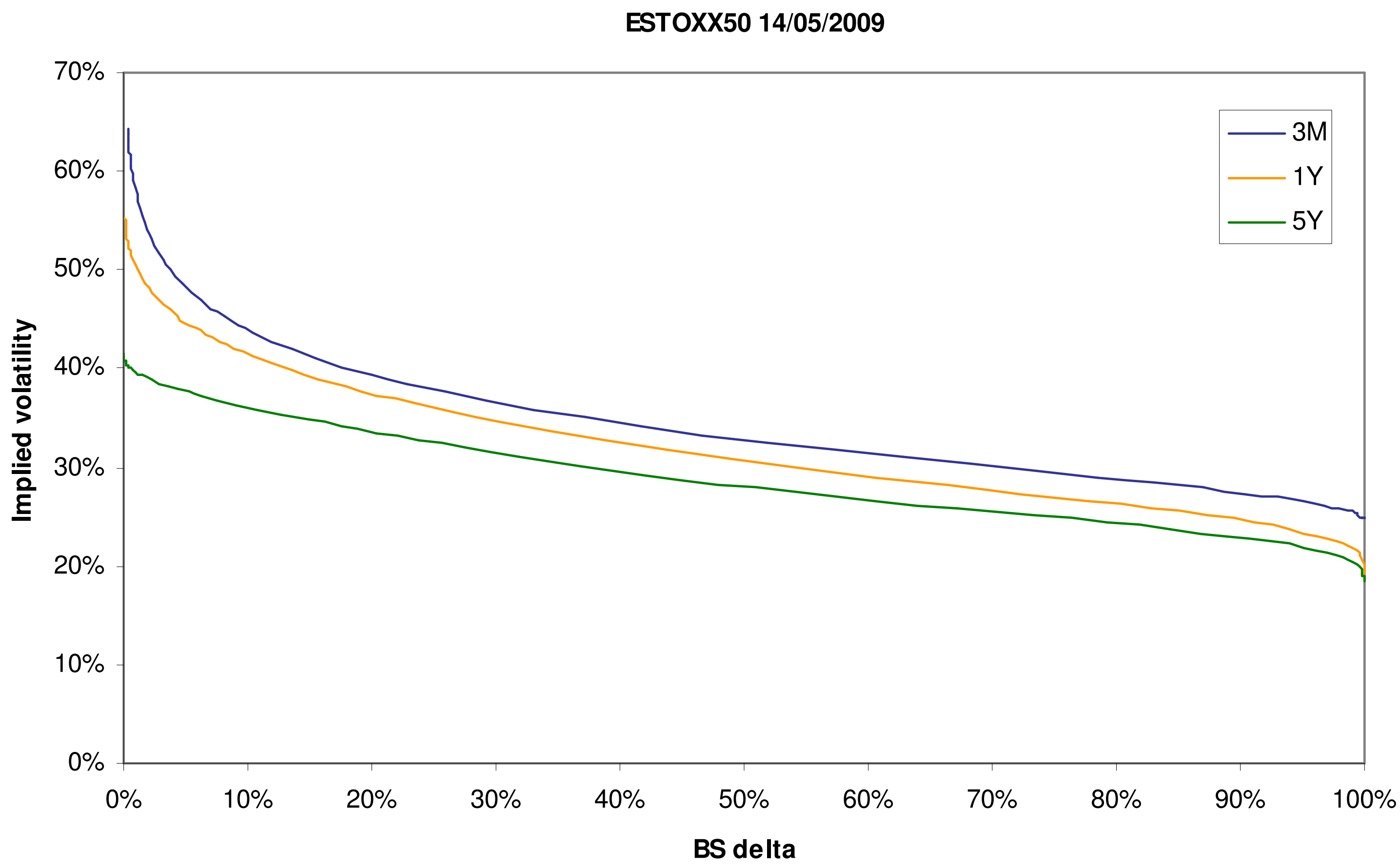


- Price not deductable from plain vanillas → we need a model
- Model should deliver forward skew
- Forward start → mainly delta neutral
- Spread option → use strikes to set them vega neutral
- More or less delta and vega neutral, where is the risk then?

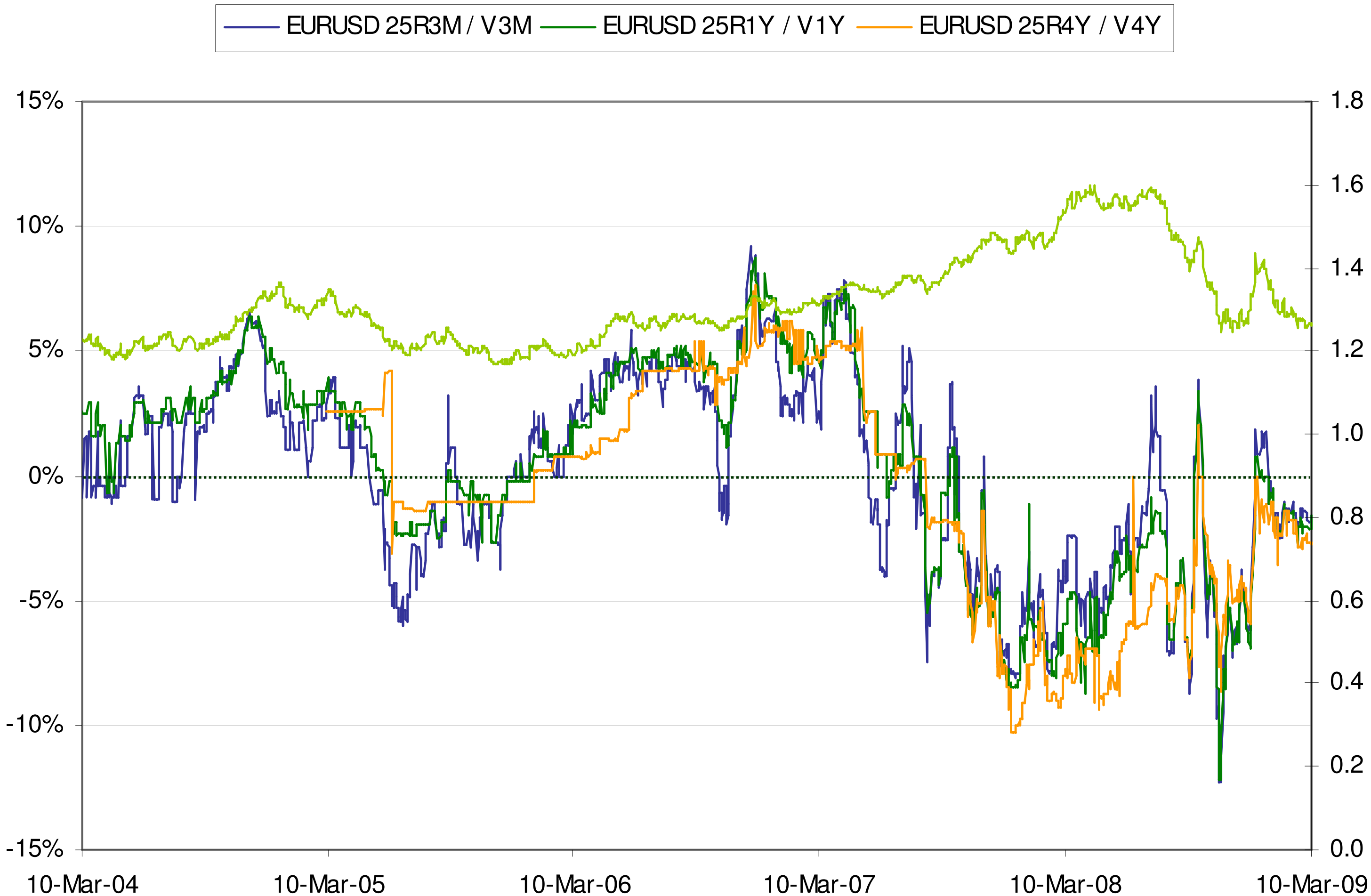
Quoting the smile by delta



Implied volatility surface by delta

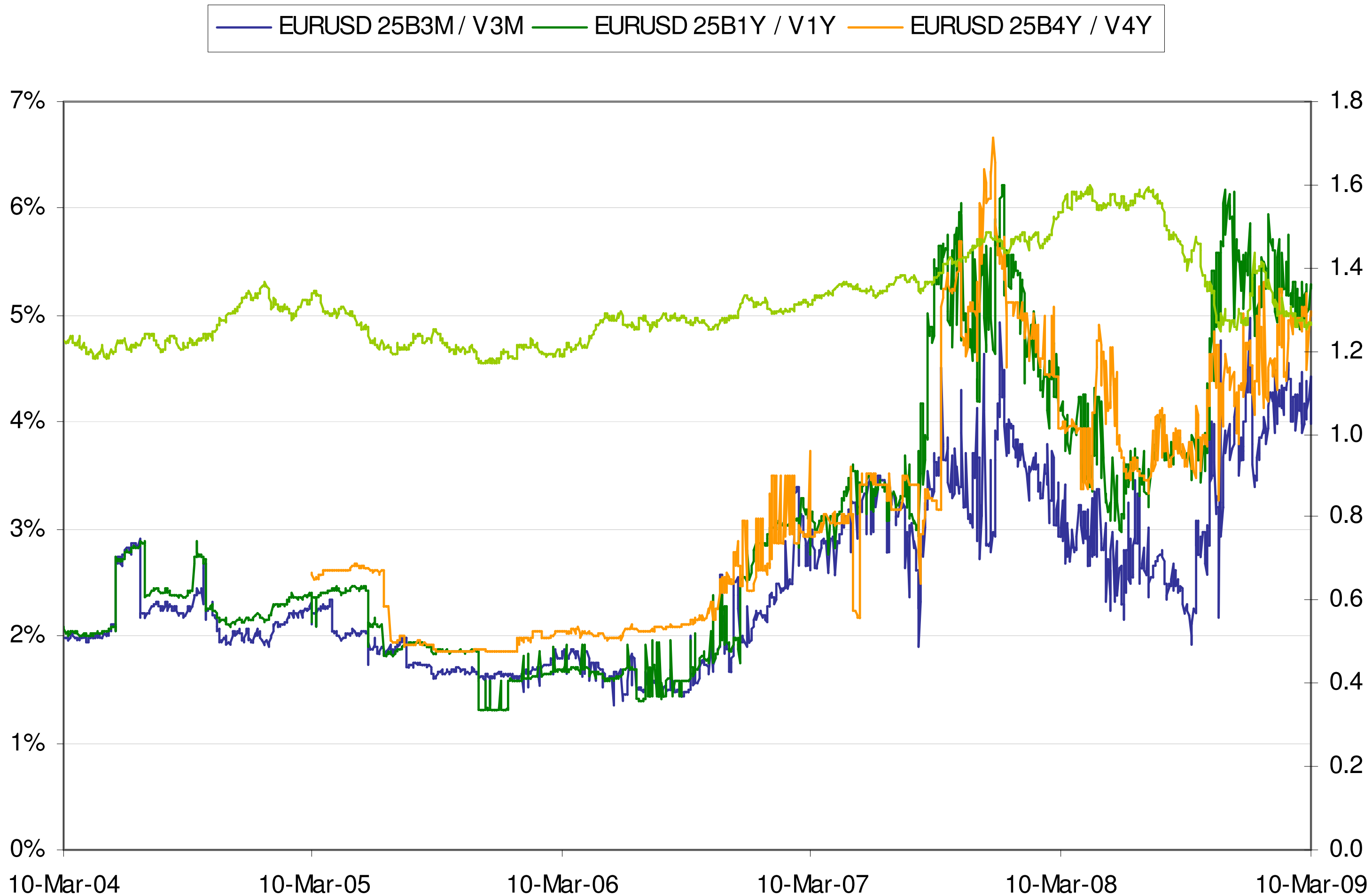


Stochastic skewness



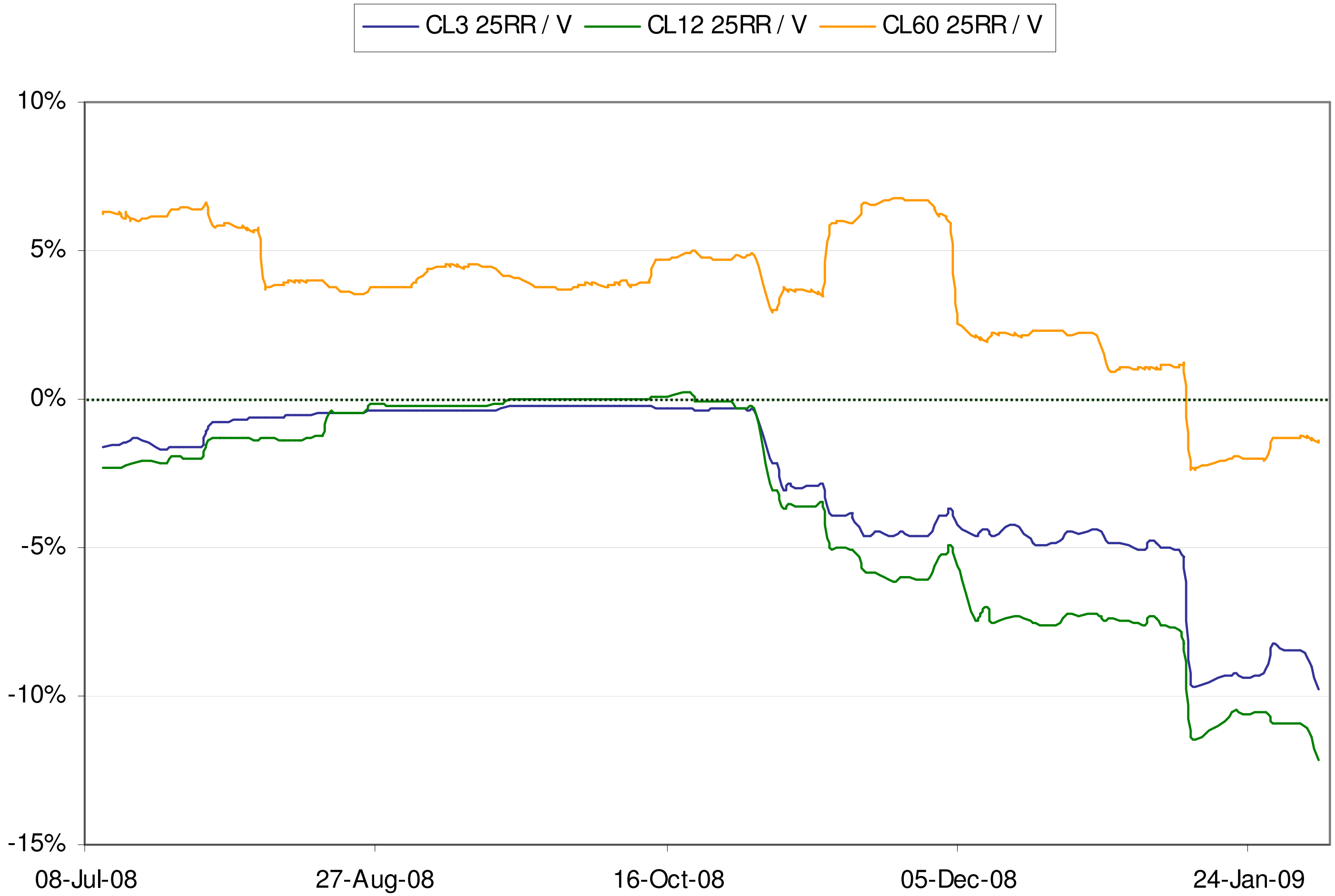
EUR/USD risk reversals over ATM volatility levels

Stochastic volatility of volatility



EUR/USD butterflies over ATM volatility levels

Term structured stochastic skewness



WTI light sweet crude oil (CL) risk reversals over ATM volatility levels

Volatility - smile - maturity relationship

WTI light sweet crude oil (CL) butterflies over ATM volatility levels

Multi-factor asset price model

2 factors:

forward curve dynamics

$$S_T = F(t,T)e^{\chi_T+\xi_T}, \chi_t = 0, \xi_t = 0$$

$$F(t,T) = E_t^Q\left[S_T\right] = E_t^Q\left[F(t,T)e^{\chi_T+\xi_T}\right] = F(t,T)E_t^Q\left[e^{\chi_T+\xi_T}\right] = F(t,T)$$

6 factors:

volatility smile dynamics

$$d\chi_t = \left(\mu_{\chi,t} - \kappa\chi_t\right)dt + \sqrt{\nu_{\chi,t}}\,dW_t^\chi + \left(e^{J+} - 1\right)dN_t^{\chi+} + \left(e^{J-} - 1\right)dN_t^{\chi-}$$

$$d\xi_t = \mu_{\xi,t}dt + \sqrt{\nu_{\xi,t}}\,dW_t^\xi + \left(e^{J+} - 1\right)dN_t^{\xi+} + \left(e^{J-} - 1\right)dN_t^{\xi-}$$

- short-term volatility
- short-term skewness
- short-term smile
- long-term volatility
- long-term skewness
- long-term smile

$$\mu_{\chi,t} = -\frac{1}{2}\nu_{\chi,t} - \lambda_{\chi+,t}\left(e^{\eta_+ + \frac{1}{2}\gamma_+^2} - 1\right) - \lambda_{\chi-,t}\left(e^{\eta_- + \frac{1}{2}\gamma_-^2} - 1\right)$$

$$\mu_{\xi,t} = -\frac{1}{2}\nu_{\xi,t} - \lambda_{\xi+,t}\left(e^{\eta_+ + \frac{1}{2}\gamma_+^2} - 1\right) - \lambda_{\xi-,t}\left(e^{\eta_- + \frac{1}{2}\gamma_-^2} - 1\right)$$

$$d\nu_{\chi,t} = \kappa_{\chi,\nu}\left(\theta_{\chi,\nu} - \nu_{\chi,t}\right)dt + \sigma_{\chi,\nu}\sqrt{\nu_{\chi,t}}\,dW_t^{\chi,\nu}$$

$$d\nu_{\xi,t} = \kappa_{\xi,\nu}\left(\theta_{\xi,\nu} - \nu_{\xi,t}\right)dt + \sigma_{\xi,\nu}\sqrt{\nu_{\xi,t}}\,dW_t^{\xi,\nu}$$

$$d\lambda_{\chi+,t} = \kappa_{\chi+,\lambda}\left(\theta_{\chi+,\lambda} - \lambda_{\chi+,t}\right)dt + \sigma_{\chi+,\lambda}\sqrt{\lambda_{\chi+,t}}\,dW_t^{\chi+,\lambda}$$

$$d\lambda_{\chi-,t} = \kappa_{\chi-,\lambda}\left(\theta_{\chi-,\lambda} - \lambda_{\chi-,t}\right)dt + \sigma_{\chi-,\lambda}\sqrt{\lambda_{\chi-,t}}\,dW_t^{\chi-,\lambda}$$

$$d\lambda_{\xi+,t} = \kappa_{\xi+,\lambda}\left(\theta_{\xi+,\lambda} - \lambda_{\xi+,t}\right)dt + \sigma_{\xi+,\lambda}\sqrt{\lambda_{\xi+,t}}\,dW_t^{\xi+,\lambda}$$

$$d\lambda_{\xi-,t} = \kappa_{\xi-,\lambda}\left(\theta_{\xi-,\lambda} - \lambda_{\xi-,t}\right)dt + \sigma_{\xi-,\lambda}\sqrt{\lambda_{\xi-,t}}\,dW_t^{\xi-,\lambda}$$

- 8 state variables
- historical calibration

Pitfalls in historical calibration

- I have as many risk factors as characteristics of the volatility surface that I want to reproduce → nice fit every day
- How to calibrate the correlation between risk factors?
- By empirics unspanned stochastic volatility, but strong correlation between volatility, skew and smile factors
- The correlation problem: What are the illiquid risk factors that can be hedged by liquid assets?

- HC solved the problem of MR, but we need something extra to solve the problem of correlations
- Filtering the white noise in the historical calibration

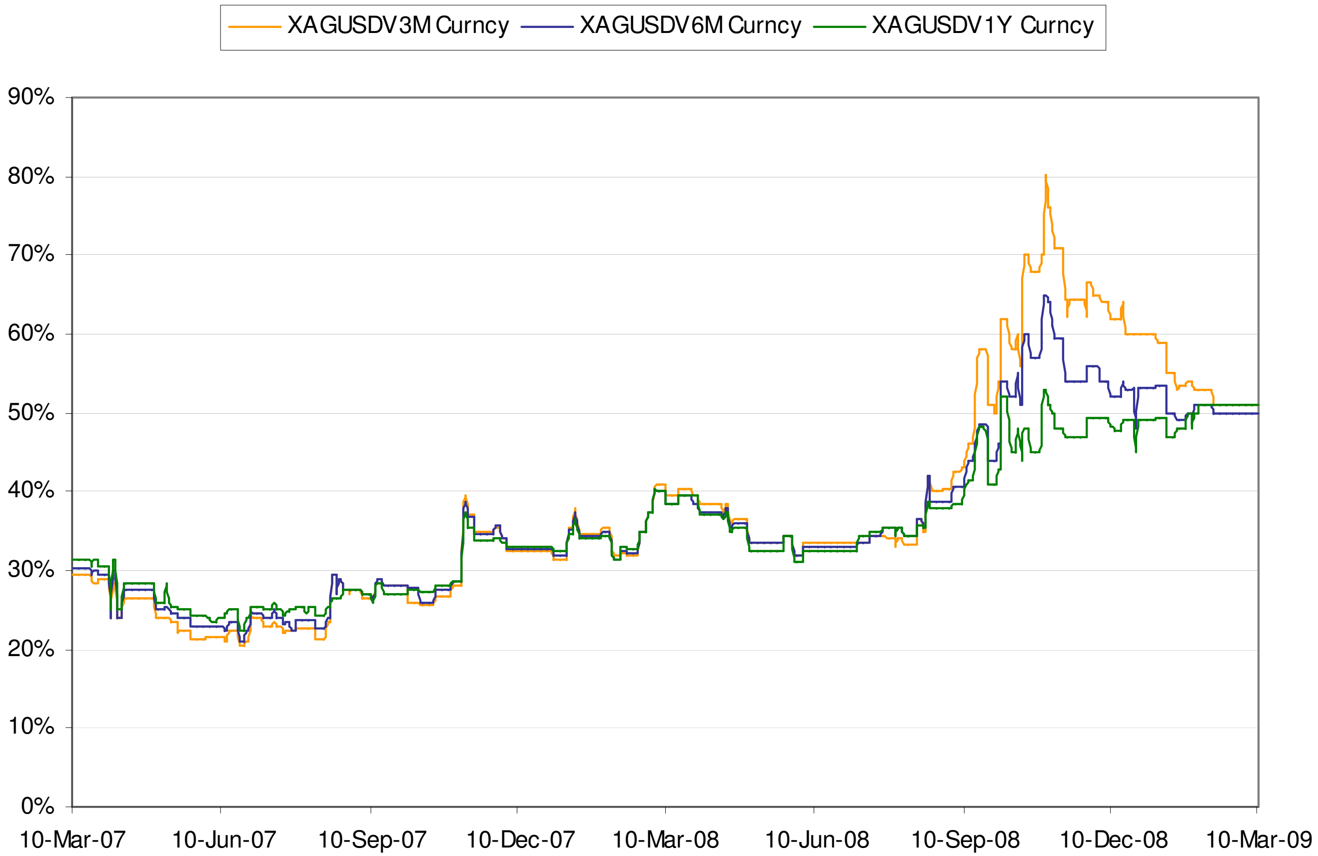
Conclusion

- Value of a derivative is the value of its replication
  - If I fail to describe the future asset price dynamics, I fail to price
- Exotic pricing is extrapolation of available information
  - Not enough to match market price, also match intuition
- Am I hedged? How my daily PnL fluctuate?
  - Did I forecast well the exotic price dynamics?

- Dynamics & Intuition

Arbitrage the volatility surface

- Arbitrage flat volatility curve (base metals)
- Arbitrage flat term structure (precious metals)



Agenda presentation

1. Usual Pitfalls of Financial Models
2. Why Is There a Problem?
3. How to Avoid Common Mistakes
4. Questions

Questions

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