Probabilistic Analysis of Hierarchical Cluster Protocols for Wireless Sensor Networks

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Overview

- Wireless sensor networks, Energy aspects
- Cluster based protocol design; LEACH
- Probabilistic analysis
  - Cluster head selection
  - Cluster formation/Voronoi tessellation
  - Lost rounds
- Energy dissipation model
  - Renewal-reward model
  - Low energy optimization
  - Numerical illustration
- Multilevel LEACH

Wireless Sensor Networks

- Hundreds or thousands of small embedded units deployed in spatial region;
- Sensors extract data from the environment - sampled regularly over time, or in response to special events;
- Data aggregated and transmitted within network and forwarded to base station;
- Nodes contain sophisticated electronics: radio transciever, antenna, computer, memory, sensors, battery, ...
  Current vision: size $5 \times 4 \times 5$ mm, manufactoring cost 1 euro.

Wireless Sensor Networks, Energy aspect

- Usually not an option to replace/reload batteries;
- Energy efficiency is crucial for long life time sensor networks;
- Most of the energy-use in node is for wireless communication;
- Power consumption increases with distance:
  - reduce data transmission between nodes far apart;
  - minimize upload traffic from sensor nodes to distant base station;
- Network protocol *distributed*, i.e. based on information available to the nodes without need for central processing;

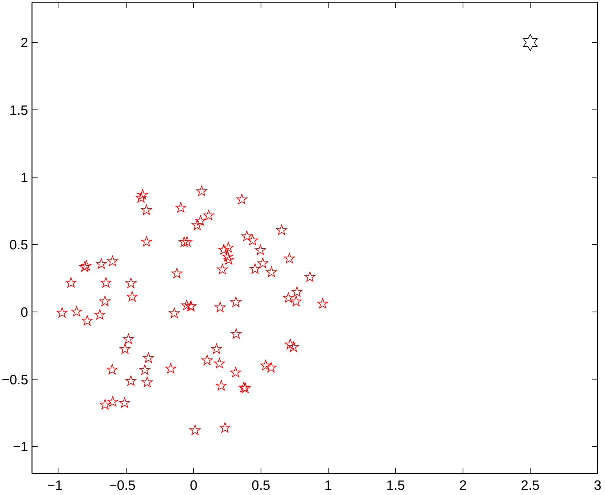
Cluster-based architectures

- In a hierarchical clustering algorithm all nodes are organized in clusters, each with designated cluster head node (CH);
- Nodes within cluster only communicate with CH, the aggregated data is forwarded by CH to base station;
- In this way any substantial energy dissipation is limited to CH nodes: to avoid battery drainage the task of being CH must rotate among nodes.

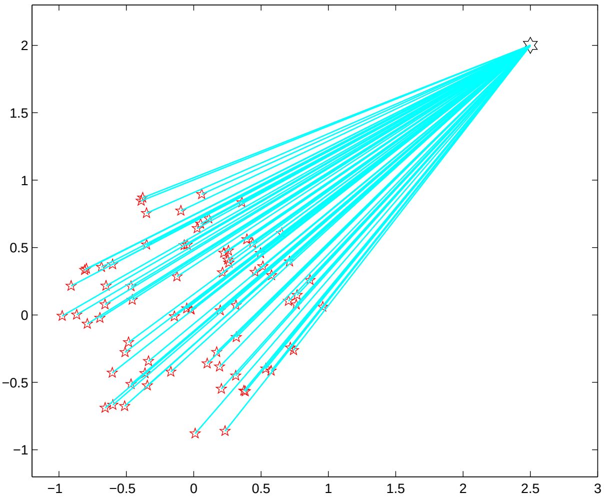
Proposed solution:
LEACH: Low-Energy Adapted Clustering Hierarchy
Heinzelmann, Chandrakasan, Balakrishnan, (2000,2002)

It is assumed that any node can communicate with any other node and with BS, single-hop mode.

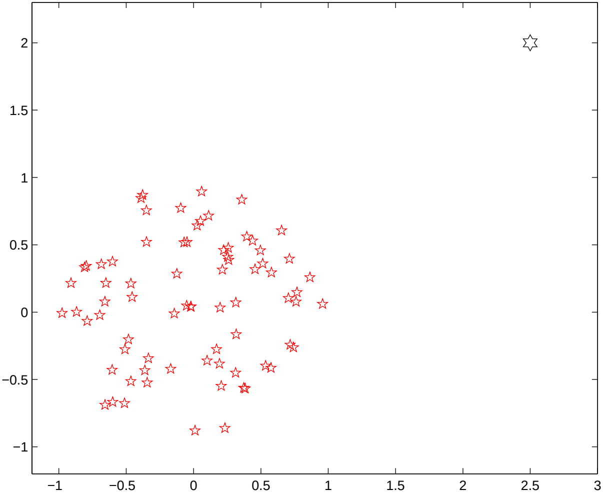
LEACH Principle



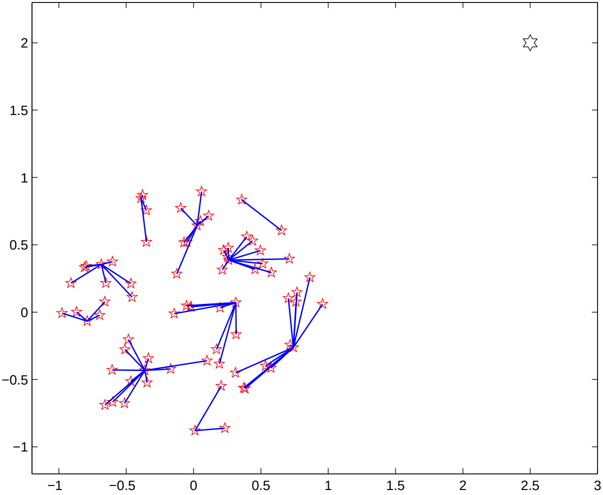
LEACH Principle



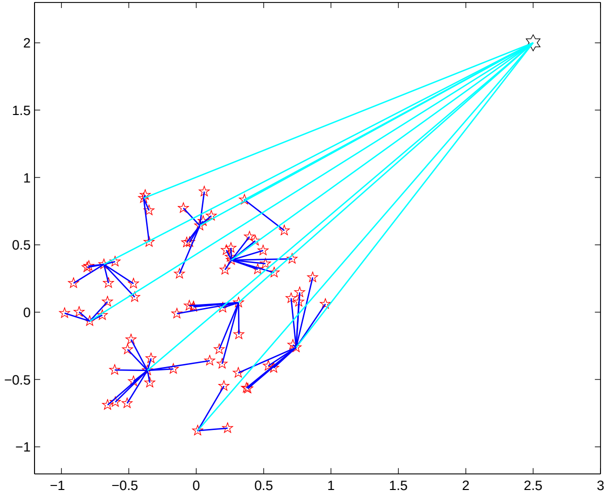
LEACH Principle



LEACH Principle



LEACH Principle



Cluster selection in LEACH

Operation of n node network is managed in consecutive cycles each consisting of r rounds. Each round has set-up and steady-state phase. During set-up:

- CH selection by random announcement;
- Cluster formation, by associating with nearest CH.

The selection algorithm of LEACH may be recast into

$$X_i = \text{the number of CH nodes in round } i, \quad i = 1, \ldots, r,$$

where

$$\begin{aligned}
X_1 &\sim \mathrm{Bin}(n, \frac{1}{r}) \\
X_2|X_1 &\sim \mathrm{Bin}(n - X_1, \frac{1}{r-1}) \\
X_3|X_1, X_2 &\sim \mathrm{Bin}(n - X_1 - X_2, \frac{1}{r-2}) \\
&\vdots \\
X_r|X_1, \ldots, X_{r-1} &\sim \mathrm{Bin}(n - X_1 - \cdots - X_{r-1}, 1) = n - X_1 - \cdots - X_{r-1}
\end{aligned}$$

Reformulation of CH Selection Algorithm

Design principle: For any $r = 1, 2, \ldots,$ it is possible to pick CH nodes so that each node acts as CH exactly once per cycle.

Proposition The joint distribution of the number of cluster heads in consecutive rounds of a cycle is given by the multinomial distribution

$$(X_1, \ldots, X_r) \sim \mathrm{Multnom}\Big(n, \Big\{\frac{1}{r} \ldots, \frac{1}{r}\Big\}\Big)$$

Moreover, letting $Z_1, \ldots, Z_r$ be i.i.d. Poisson distributed random variables with parameter n/r,

$$(X_1, \ldots, X_r) \overset{d}{=} \Big(Z_1, \ldots, Z_r \Big| \sum_{i=1}^{r} Z_i = n\Big)$$

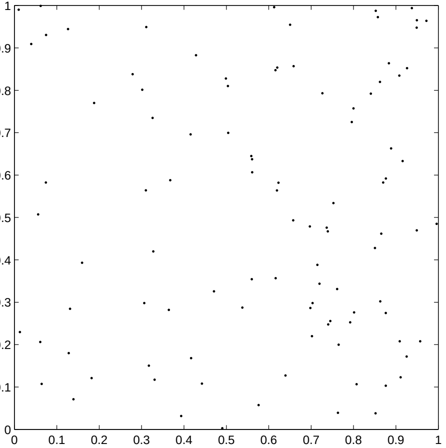
Cluster formation

To complete set-up of a round, each nonCH node ranks all CH annoucenments in order of increasing signal strength and sends a message to the CH on top: 'I belong to your cluster'.
Assuming that signal strength is proportional to distance this creates a Voronoi tessellation of the spatial region generated by the CH nodes.

Assuming that the spatial locations of nodes are uniform over the deployment region we are led to the Poisson-Voronoi Model (conditioned on the number of nodes).

2D Poisson Voronoi Model

$n = 100$, $r = 5$, first round



2D Poisson Voronoi Model

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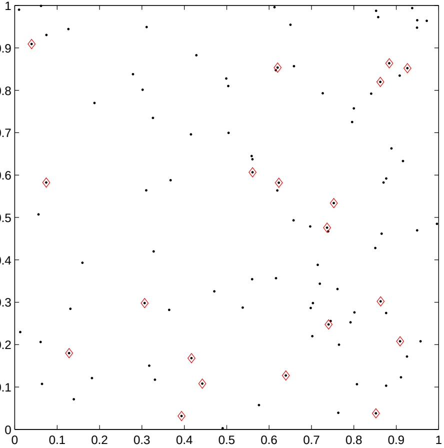

2D Poisson Voronoi Model

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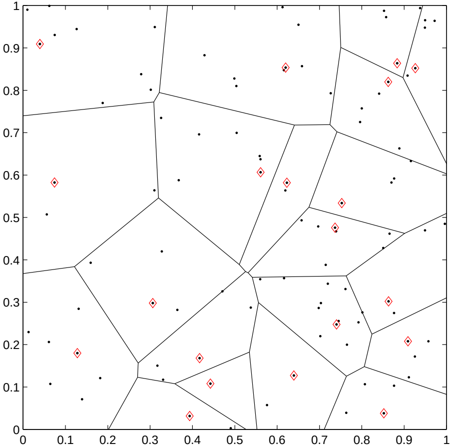

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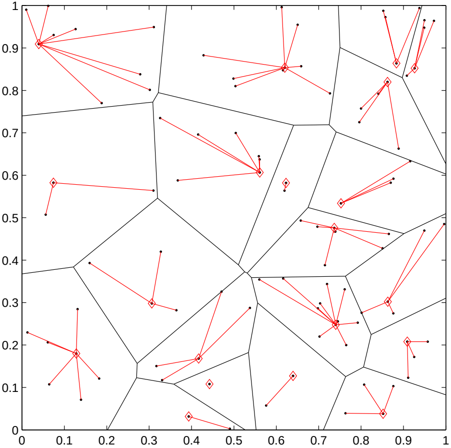

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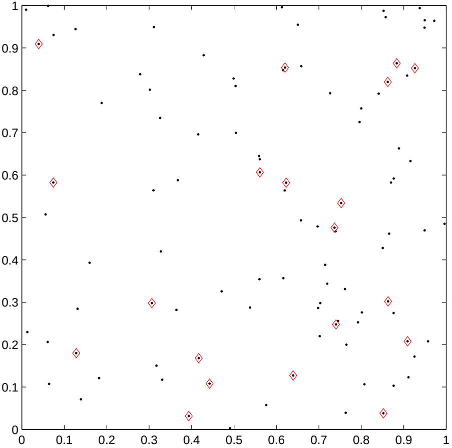

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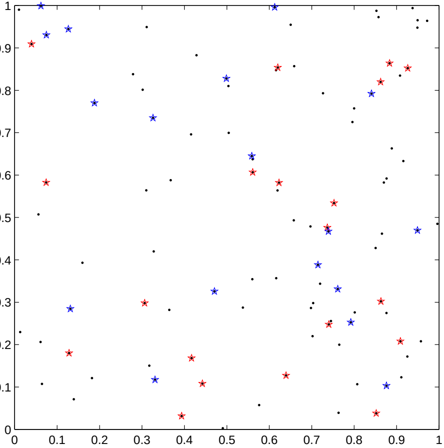

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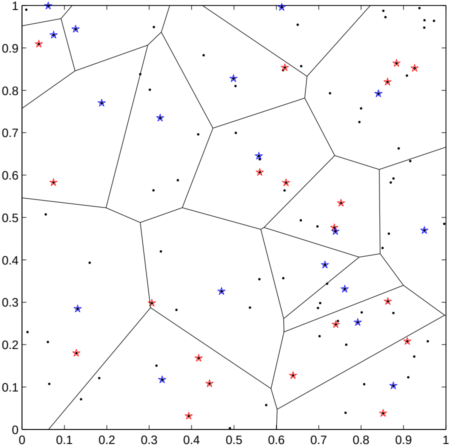

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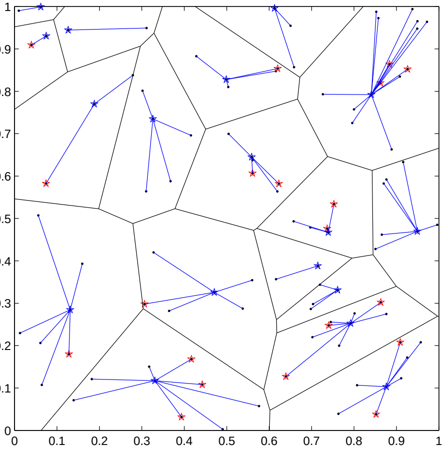


Lost Rounds

Running the algorithm with large r will limit the number of CHs and thus reduce costly data transmissions to BS.

However, the larger r the more likely it is to have $X_i = 0$ for some i which may be interpreted as the complete loss of data in the entire network in this round.

To deal with lost rounds, suppose the network user is willing to accept a loss probability $\alpha > 0$. Since $P(X_i = 0) = (1 - 1/r)^n$, the admissible r-values are limited to integers $1 \leq r \leq r_\alpha$ where

$$r_\alpha = [(1 - \alpha^{1/n})^{-1}].$$

Alternative: modify LEACH so that each round results in the formation of at least one CH. Will cause a random time delay and a conditioning of the binomial distributions to be positive. However, here we stay with the original LEACH protocol and analyze its performance by using lost rounds as a means to optimizing and tuning the model parameters.

Renewal-Reward Analysis

Condition on fixed node locations $(\xi)_{1 \le j \le n}$ in planar region Λ. Suppose each round has constant length μ. Thus operation of the protocol consists of independent LEACH renewal cycles of length $r\mu$. With jth cycle associate

$$R_j = \text{total energy used by the system during entire cycle}$$

and put

$$R(t) = \text{total energy consumed by network up to time } t$$

Will see that the energy only depends on size and shape of clusters. Thus, each R_j is a sum $R_j = T_{j1} + \cdots + T_{jr}$ of identically distributed (dependent) random variables,

$$T_{ji} = \text{energy dissipated in the network during round } i \text{ of cycle } j.$$

Renewal-Reward Theorem

$$\frac{R(t)}{t} \longrightarrow \frac{E_\xi(R)}{r\mu} = \frac{1}{\mu} E_\xi(T) \quad P_\xi - a.s.$$

Thus, we take $E(T)$ to be the basic performance measure of the network.

Energy Dissipation Model

Suppose X clusters with $L_1, \ldots, L_X$ nonCH nodes in each. A cluster with CH located at ξ^{CH} and L nonCH nodes at locations $\xi_1, \ldots, \xi_L$, has power expenditure

$$2E_{\mathrm{elec}}L + E_{\mathrm{DA}}(L+1) + \mathcal{E}_{\mathrm{fs}}\chi^2, \quad \chi^2 = \sum_{l=1}^{L} |\xi^{CH} - \xi_l|^2,$$

per 1-bit message. Since $\sum_{m=1}^{X} L_m = n - X$,

$$T_{\mathrm{within}} = 2E_{\mathrm{elec}}(n-X) + nE_{\mathrm{DA}} + \mathcal{E}_{\mathrm{fs}} \sum_{m=1}^{X} \chi_m^2, \quad \chi_m^2 = \sum_{l=1}^{L_m} |\xi_m^{CH} - \xi_{lm}|^2$$

where ξ_m^{CH} and (ξ_{lm}) indicate locations of CH and nonCH nodes of the relevant cluster.

Energy Dissipation Model, cont

The energy required to transmit data from CHs to BS is, either (multipath shadowing)

$$T_{\mathrm{distBS}} = E_{\mathrm{elec}} X + \mathcal{E}_{\mathrm{mp}} \sum_{k=1}^{X} |\xi_k^{CH} - \xi_{\mathrm{BS}}|^4,$$

or (free-space transmission)

$$T_{\mathrm{nearBS}} = E_{\mathrm{elec}} X + \mathcal{E}_{\mathrm{fs}} \sum_{k=1}^{X} |\xi_k^{CH} - \xi_{\mathrm{BS}}|^2.$$

Low-energy optimization

Proposition Consider the sensor network model with n nodes deployed uniformly in $[0, M]^2$ and with the BS in the point (uM, vM). Over many cycles of LEACH, the average energy dissipation per round and 1-bit message is given approximately by

$$\psi_n(r) = E_{\mathrm{elec}} n\Big(2 - \frac{1}{r}\Big) + E_{\mathrm{DA}} n + \pi^{-1}\mathcal{E}_{\mathrm{fs}} M^2 H_n(r) + C_M \frac{n}{r}, \quad 1 \le r \le n,$$

where

$$H_n(r) = \sum_{k=1}^{n} \frac{n-k}{k} \binom{n}{k} \Big(\frac{1}{r}\Big)^k \Big(1 - \frac{1}{r}\Big)^{n-k}$$

and

$$C_M = \begin{cases} \mathcal{E}_{\mathrm{mp}} M^4 \int_{[0,1]^2} ((x-u)^2 + (y-v)^2)^2 \, dxdy & \text{for } distBS, \\ \mathcal{E}_{\mathrm{fs}} M^2 \int_{[0,1]^2} ((x-u)^2 + (y-v)^2) \, dxdy & \text{for } nearBS. \end{cases}$$

To optimize LEACH for energy efficiency while accepting lost rounds at a rate of α, use

$$\tilde{r} = \arg\min\{\psi_n(r) : 1 \le r \le r_\alpha\}.$$

Approximation with infinite Voronoi diagram

Consider bivariate Poisson point process in the plane. Intensity λ_0 for blue points, intensity λ_1 for red points. Form the Voronoi tessellation of red points. By Foss-Zuev (1996), the functional on blue points in a typical (Palm distribution) Voronoi-cell Π,

$$S_f = \sum f(y_i) 1_{\{y_i \in \Pi\}},$$

has expected value

$$E(S_f) = \lambda_0 \int_{\mathbf{R}^2} f(y) e^{-\lambda_1 \pi |y|^2} \, dy$$

In particular,

$$E(\text{squared edge lengths}) = \lambda_0 \int_{\mathbf{R}^2} |y|^2 e^{-\lambda_1 \pi |y|^2} \, dy = \frac{\lambda_0}{\pi \lambda_1^2}.$$

Apply this with $\lambda_0 = (n-k)/M^2$ and $\lambda_1 = k/M^2$. Then

$$E(\chi_m^2 | X = k) \approx \frac{n-k}{\pi k^2} M^2, \quad 1 \leq m \leq k,$$

and

$$\mathbf{E}\Big(\sum_{m=1}^{X} \chi_m^2 \Big) \approx \mathbf{E}\Big(\frac{n-X}{X} \Big) \frac{M^2}{\pi} = H_n(r) \frac{M^2}{\pi},$$

Numerical illustration

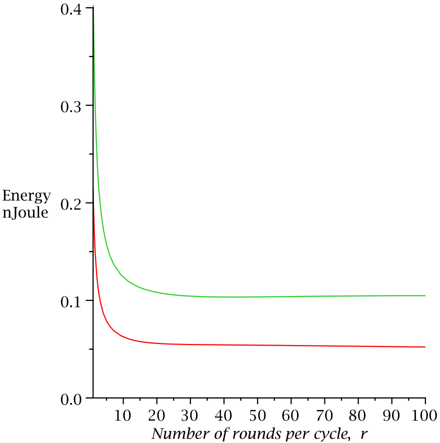


Figure 1. Energy loss per round as function of cycle length r, $M = 100$, distant base station $\xi_{BS} = (50, 175)$, lower curve: $n = 100$, upper curve: $n = 200$

Numerical illustration, cont

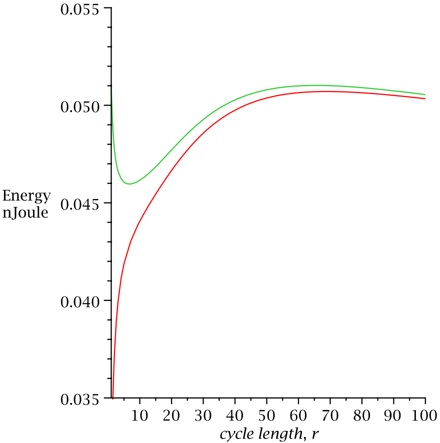
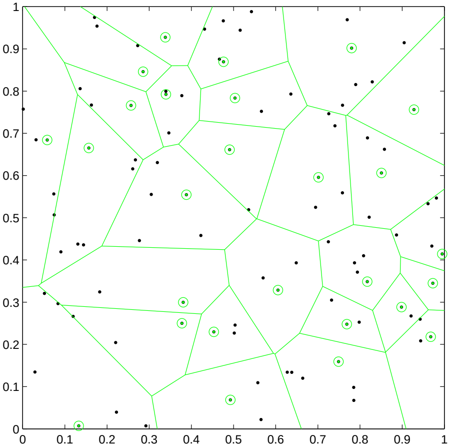
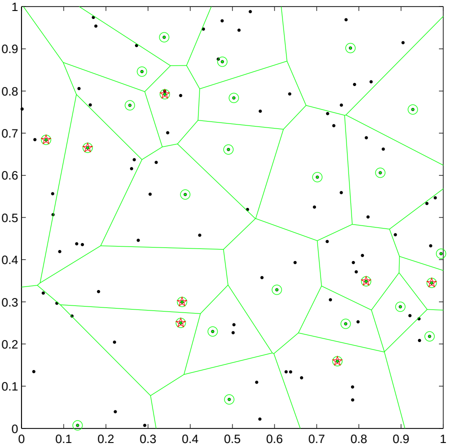


Figure 2. Energy loss per round for nearby base station at a corner point (upper curver) or middle point (lower curve)

2-level LEACH



2-level LEACH



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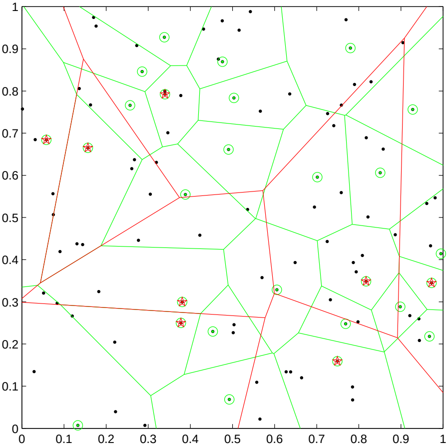

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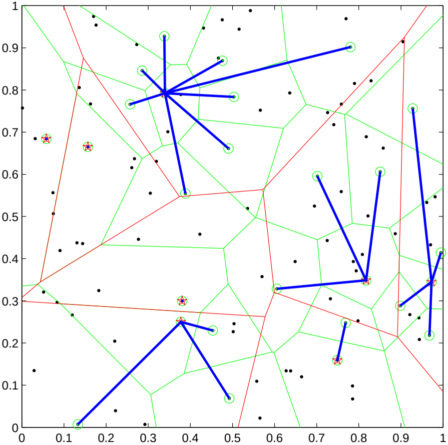
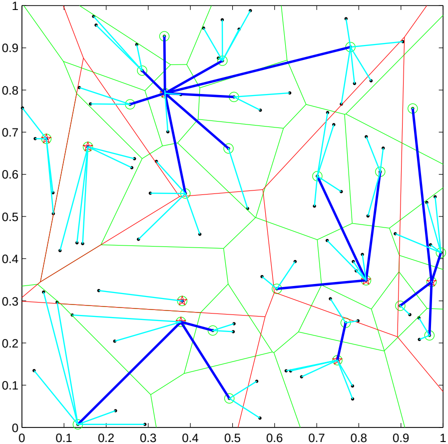


2-level LEACH



multi-level LEACH

Fix m parameters $r_1, \ldots, r_m$.

Round 1; the n nodes make m decisions. First, as in 1-level LEACH, pick $X_{1,1}$ CHs according to $\mathrm{Bin}(N, 1/r_1)$. These CHs announce with probability $1/r_2$ their willingness to be CH-heads (2-CH). Given $X_{1,1}$, the resulting number, $X_{2,1}$, of 2-CH nodes is $\mathrm{Bin}(X_{1,1}, 1/r_2)$. Repeat m times to get in the final step $X_{m,1}$ m-CH nodes. Network now enters steady state phase: data sent from nonCH to CH to 2-CH and so on, until the m-CH nodes complete the round by sending data to BS.

Round 2: CHs of levels 1 to $m-2$ remain the same. Only the (m-1)-CH nodes select new set of m-CHs among the $X_{m-1,1} - X_{m,1}$ available candidates. Selection probability changes to $1/(r_m - 1)$ and gives $X_{m,2}$ mCH nodes in charge of BS transmission in round 2. After first sub-cycle of r_m rounds, all $\sum_{j=1}^{r_m} X_{m,j} = X_{m-1,1}$ CHs on level $m-1$ are used up, will be replaced beginning of round $r_m + 1$. New selection probability is $1/(r_{m-1} - 1)$. After $r_m r_{m-1}$ rounds go one level down in the hierarchy and make the appropriate update.

multi-level LEACH

Remark

The subsequent decisions and updates of CHs at the various levels follow the path of a contour around the branches of a regular rooted tree of depth m where all tree-nodes at distance j from the root have r_j branches. Finally, after a total of $r_1 \cdots r_m$ rounds the system has completed a full cycle in which each node has been a CH on each level exactly once.

multi-level LEACH

Theorem Network with n nodes located uniformly in $[0, M]^2$. The BS is at (uM, vM), and the energy model of HBC applies. For the m-level LEACH model with parameters $\mathbf{r} = (r_1, \ldots, r_m)$, the average energy dissipation per round and 1-bit message is given approximately by

$$\begin{aligned}\psi_n^{(m)}(\mathbf{r}) \quad = \quad & 2E_{\mathrm{elec}}\Big(1 - \frac{1}{r_1 \cdots r_m}\Big)n + E_{\mathrm{elec}} \sum_{j=1}^{m} \frac{n}{r_1 \cdots r_j} \\ & + E_{\mathrm{DA}}\Big(1 + \sum_{j=1}^{m-1} \frac{1}{r_1 \cdots r_j}\Big)n + \pi^{-1}\mathcal{E}_{\mathrm{fs}} M^2\, H_n^{(m)}(\mathbf{r}) + C_M \frac{n}{r_1 \cdots r_m},\end{aligned}$$

where

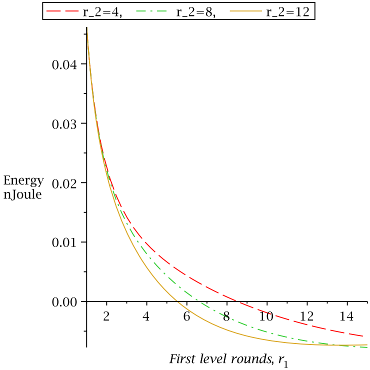
$$H_n^{(m)}(\mathbf{r}) = \sum_{j=1}^{m} \frac{r_j - 1}{r_1 \cdots r_j - 1} H_n(r_1 \cdots r_j), \quad H_n^{(1)} = H_n.$$

For a given acceptable loss rate α, optimal performance of m-LEACH is achieved by minimizing $\psi_n^{(m)}(\mathbf{r})$ over all m-tuples $\mathbf{r} = (r_1, \ldots, r_m)$ of integers such that

$$r_1 \cdots r_m \leq r_\alpha.$$

Comparison 1-level and 2-level LEACH

$$\psi_n^{(2)}(r_1, r_2) - \psi_n(r_1 \cdot r_2)$$
$$= 4.2 \cdot 10^{-4}\left(0.55\frac{n}{r_1} + \pi^{-1} H_n(r_1) - \pi^{-1}\frac{(r_1-1)r_2}{r_1 \cdot r_2 - 1} H_n(r_1 \cdot r_2)\right).$$

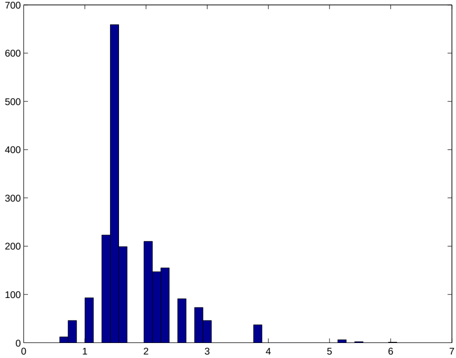


$n = 200$ ($r_{0.01} = 43$, $r_{0.05} = 67$)

Remarks

1) A referee pointed out possible overlap with work by Banerjee and Lahiri, SAMSI 2007-08. In [2] (slide presentation) the performance measure for energy optimization of LEACH appears to be more complex and relate to protocol with random number of rounds per cycle (?). Limiting energy is discussed in context of spatially rescaled systems.

2) Typical energy distribution, given node locations:



$n = 100$, $r = 5$, nmb of runs 2000