Vehicle Interference Effects in Warehousing Systems with Autonomous Vehicles

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Outline

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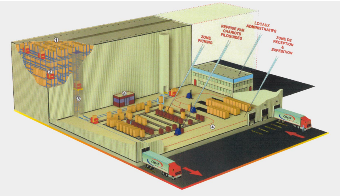
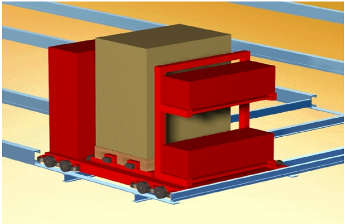
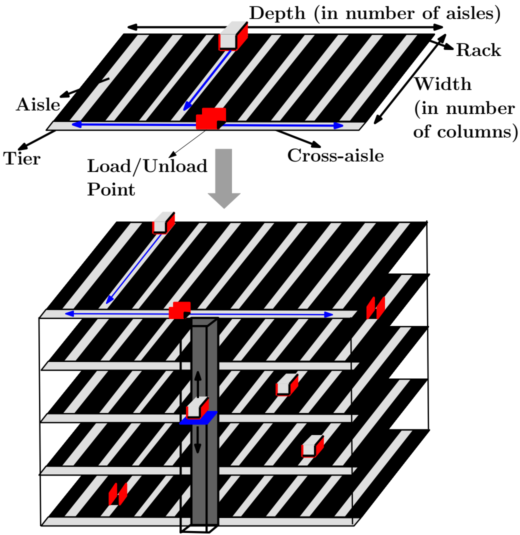
AVS/R System Overview

- AVS/RS: Uses autonomous vehicles instead of aisle-captive cranes
- System configuration
  - Rectilinear movement
  - Horizontal movement (x and y axes) by autonomous vehicles
  - Vertical movement (z axis) by lifts
  - Vehicles move between tiers using lifts
- Modular and adaptive design





Components of an AVS/R System



Design Parameters in AVS/RS

System Sizing Decisions

- Number of vehicles and lifts
- Depth/Width ratio
- Location of cross-aisle and load/unload points
- Number of zones

Operational Decisions

- Vehicle assignment rule
- Dwell point policy
- Command cycle
- Storage policy
- Transaction scheduling policy (FCFS, Random)

Key Performance Measures

- Transaction cycle time, Queue lengths, Throughput, Vehicle utilization

Outline

1. Autonomous Vehicle Technology
2. Focus of Current Research
3. Queuing Model
4. Numerical Results
5. Summary

Review of Analytical Models for AVS/RS

Author	Method
Malmborg (2002, 2003)	State equation based models
Kuo et al. (2004)	Probabilistic approach
Zhang et al. (2008)	Variance based approximations
Heragu and Srinivasan (2008)	Semi-open queuing networks
Roy et al. (2009)	Semi-open queuing networks

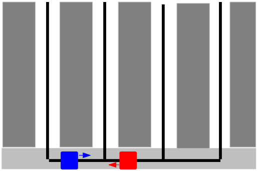
Objective of these models

- Model vehicle-lift interface and its effect on cycle times
- Quantify performance benefits of AVS/R systems

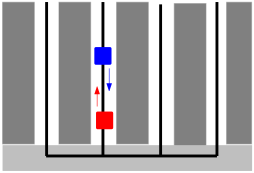
Limitation: These models does not account for possible vehicle interference and its effect on system performance

Types of Vehicle Interference

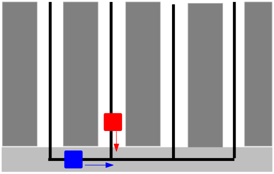
On the Cross-aisle



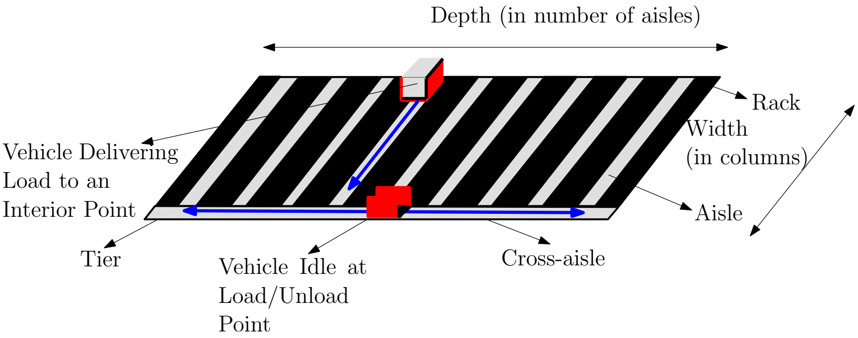
Within an Aisle



At an intersection of Aisle and Cross-aisle



Current Research: Analyze the Effect of Vehicle Interference



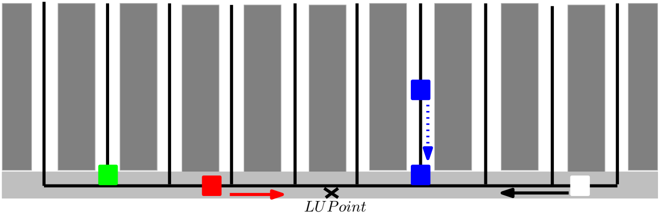
- **Is the effect of vehicle interference significant?**
- Efficient single tier systems form effective building blocks for multi-tier systems

Research Approach

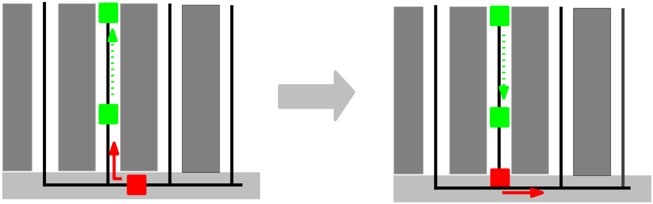
- Develop protocols for vehicle interference
- Develop a semi-open queuing network model of a single tier
- Solve the model using a decomposition based approach
- Validate the analytical model against simulations
- Analyze the effect of vehicle interference on performance

Protocols for Vehicle Interference

Each half of the cross-aisle has atmost one vehicle at any time t:

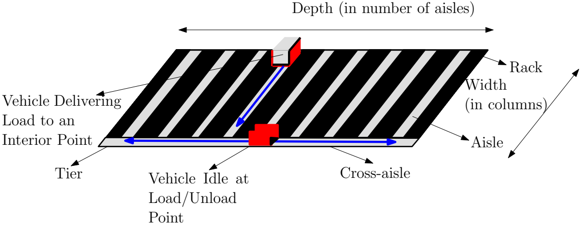


Vehicles within an aisle yield to other incoming vehicles:



Outline

Assumptions

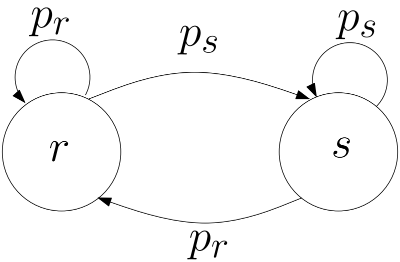


- System Design Assumptions
  - One load/unload point
  - Single command cycle
  - Random vehicle assignment
  - LU dwell point policy
  - Random storage policy
  - FCFS transaction scheduling
- Model Assumptions
  - Poisson arrivals

Description of Vehicle Classes

Vehicle Class Prior to Start of Service	Transaction Type	Vehicle Class After Start of Service
Store (s)	Retrieval	Retrieve (r)
Store (s)	Storage	Store (s)
Retrieve (r)	Retrieval	Retrieve (r)
Retrieve (r)	Storage	Store (s)

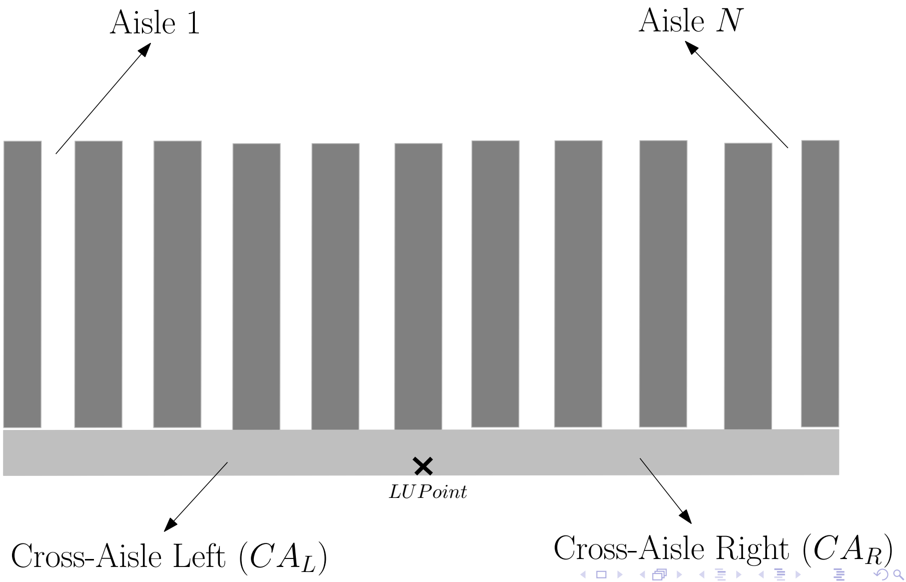
Vehicle Class Switching



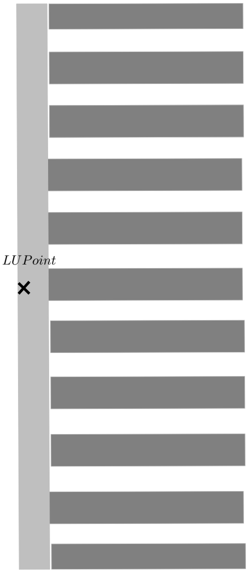
$$p_r = \frac{\lambda_r}{\lambda_r + \lambda_s}$$

$$p_s = \frac{\lambda_s}{\lambda_r + \lambda_s}$$

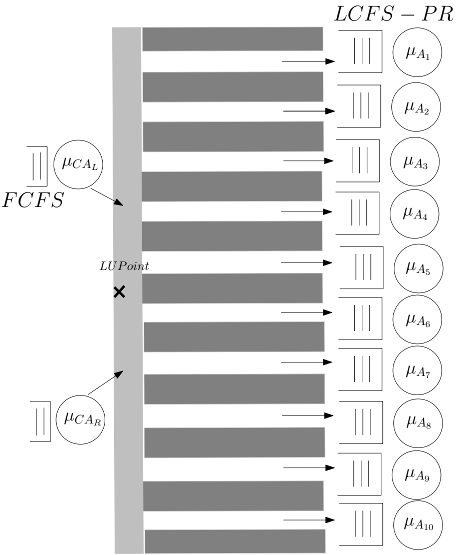
Nodes of the Queuing Model



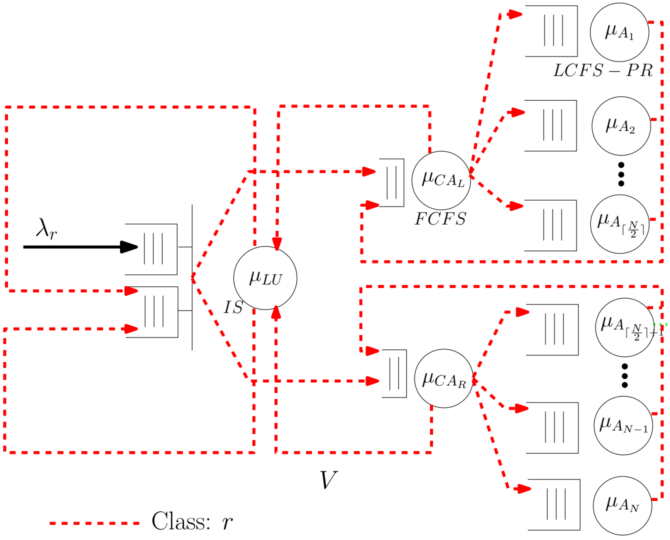
Nodes of the Queuing Model



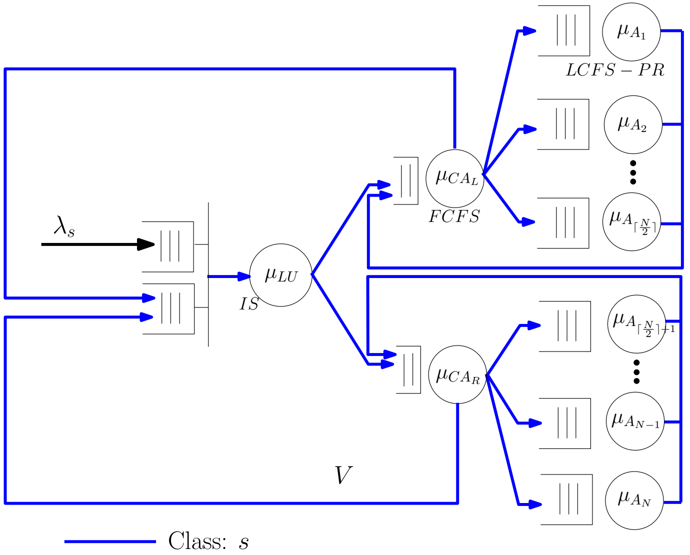
Nodes of the Queuing Model



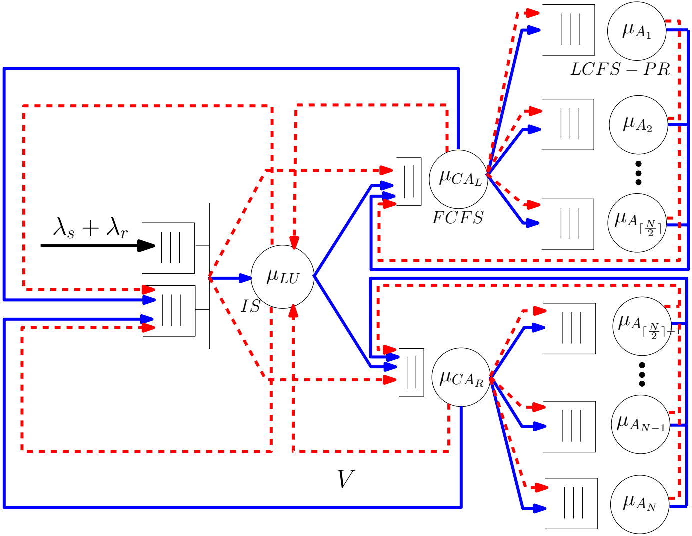
Queuing Model for a Single Tier



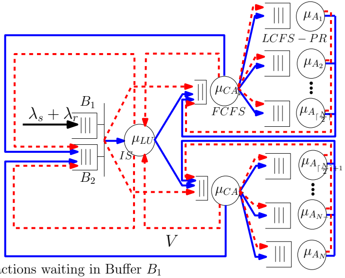
Queuing Model for a Single Tier



Queuing Model for a Single Tier

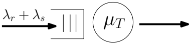


Queuing Model for a Single Tier: Decomposition

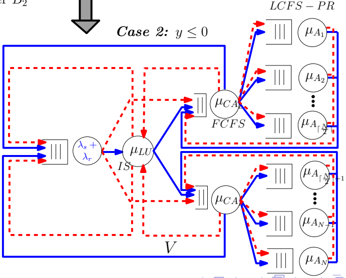


x_1=Number of transactions waiting in Buffer B_1
x_2=Number of idle vehicles in Buffer B_2
$y = x_1 - x_2$

Case 1: $y \geq 0$



Case 2: $y \leq 0$



Decomposition Based Approach for Solving the Model

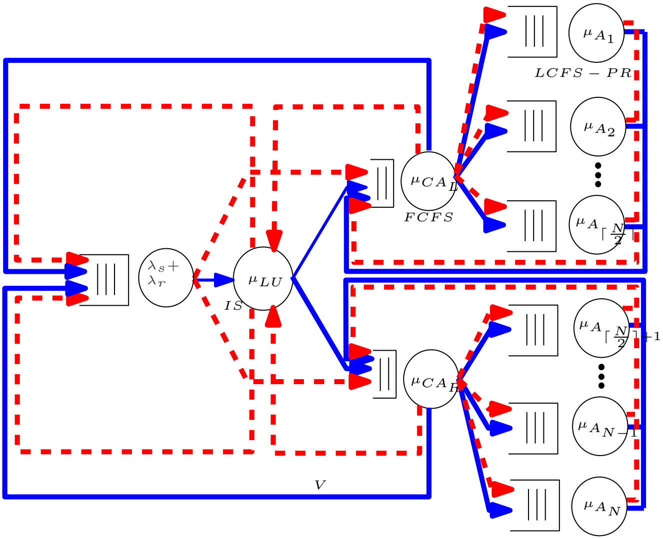
1. For case ($y \leq 0$): Solve the closed queuing network with two classes of vehicles: Store (s), and Retrieve (r) using an Approximate MVA (AMVA) algorithm

2. For case ($y \geq 0$): Solve the open queue as an M/G/1 queue

3. Link results from the above two cases and obtain the steady state distribution of the vehicles and transactions in the original semi-open queuing network

4. Obtain the performance measures (cycle time, vehicle distribution in the network and vehicle utilization)

CQN for case $(y \leq 0)$



Solution for case ($y \leq 0$)

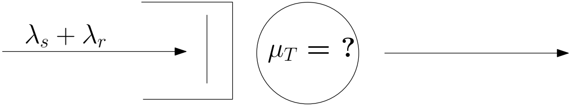
Node Characteristics:

- Aisle nodes ($Q_1, \ldots, Q_N$): LCFS-PR with exponential service times ($\mu_{A_1} = \mu_{A_2} = \ldots = \mu_{A_N}$), where N is the number of aisles
- Cross-aisle nodes (Q_{N+1} and Q_{N+2}): FCFS with uniform service times ($\mu_{CA_L} = \mu_{CA_R}$ and CV of 0.58)
- LU nodes (Q_{N+3}): IS with exponential service times
- Wait for transaction node (Q_{N+4}): FCFS node with exponential service time

Therefore, the network is non-product form (Baskett et al. (1975)) and an Approximate MVA algorithm (Lazowska et al. (1984)) is used to obtain conditional measures.

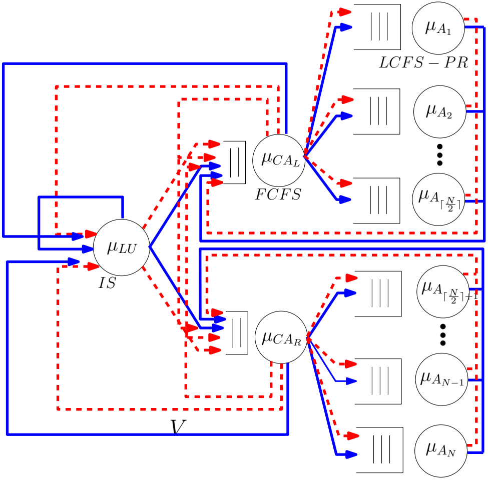
Solution for case ($y \geq 0$)

- When $y \geq 0$, arriving transactions wait for a vehicle



- Approach
  - Solve as an open queue with a single server station
- Challenges
  - Determine μ_T^{-1}(average service time) and coefficient of variation (CV) of the service time

Determining μ_T for case $(y \geq 0)$



Set μ_T to be the throughput of the closed queuing network with V vehicles

Solution for case ($y \geq 0$)

- CV of service time:
  - From simulation studies, the CV of the service time is 0.6-0.8
  - We estimate the CV by analyzing the vehicle distribution in a reduced closed queuing network
- Solution of open queue:
  - Open queue is analyzed as an M/G/1 queue
  - Determine $\pi(i|y \geq 0)$ by analyzing an M/G/1 queue with service rate μ_T

Unconditional Probabilities

- For Case 1 ($y \leq 0$):

  $\pi(y = i) = \sum_{q:|Q_{N+4}|=-i} \pi_q(|Q_1|, |Q_2|, \ldots, |Q_{N+4}||y \leq 0)\pi(y \leq 0)$ $\forall i = 0, .., -V$ where $|Q_m|$ denote the number of vehicles at node m

- For Case 2 ($y \geq 0$):

  $\pi(y = i) = \pi(i|y \geq 0)\pi(y \geq 0)\forall i = 0, .., \infty$

Two unknowns $\pi(y \geq 0)$ and $\pi(y \leq 0)$

1. $\pi(y = 0)$ is common to both cases
2. $\sum_{k=-V}^{\infty} \pi(y = k) = 1$

Performance Measures

- Using $\pi(y = i)$, we can obtain the following performance measures:
    - Vehicle utilization
    - Average number of transactions waiting for service
    - Expected storage cycle time and retrieval cycle time

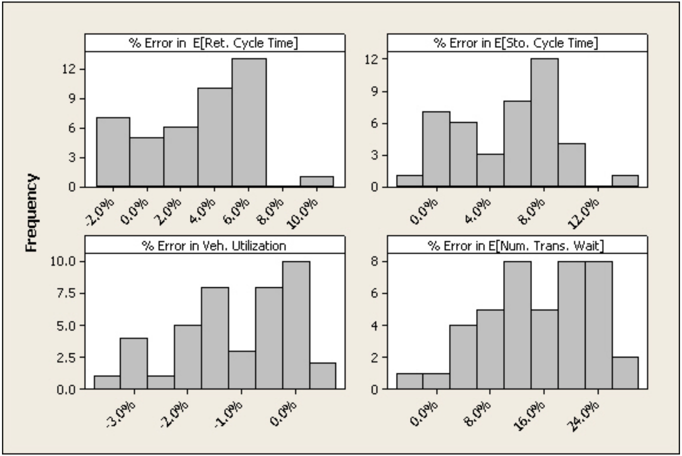
Outline

Model Validation against Simulation

- Design Parameters
  - Vehicles = 3,5
  - $\frac{D}{W} = 0.5, 1.5$
  - $\lambda_s + \lambda_r = 45 - 100$ *pall./hr* in increments of 5 *pall./hr*
  - Number of storage locations=7300
- Analyzed 40 cases where vehicle utilizations range between 60% to 90%
- Simulation: Modeled using AUTOMOD path mover system (15 replications for each scenario, 96000 transactions per run)

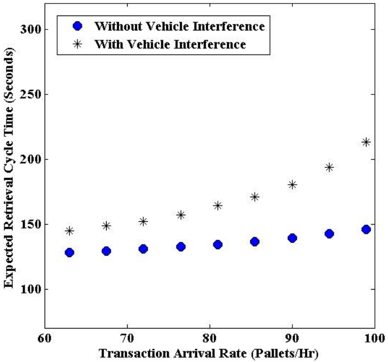
Model Validation:Results

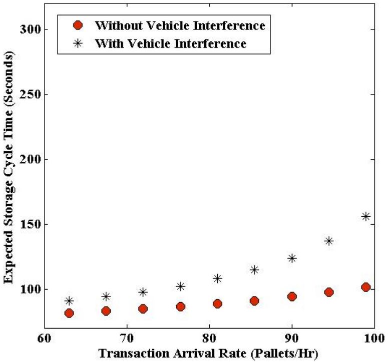
$\%Error = \frac{A-S}{S}$, where S=Simulation Value and A=Analytical Value



Effect of Vehicle Interference on Cycle Times: $\frac{D}{W} = 1.5$

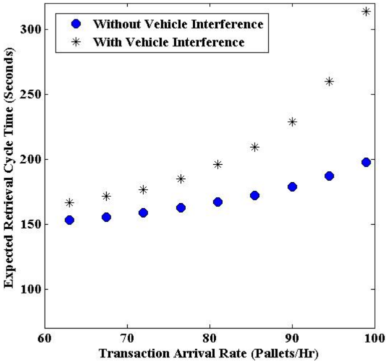
Tier Configuration:
7300 Locations, 45 Aisles, 81 Columns, 5 Vehicles

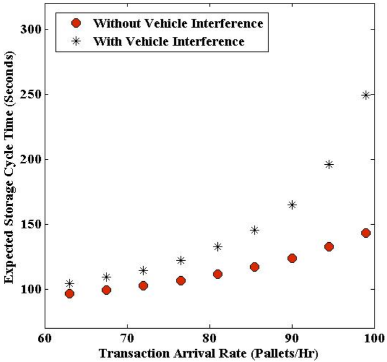




Effect of Vehicle Interference on Cycle Times: $\frac{D}{W} = 0.5$

Tier Configuration:
7300 Locations, 27 Aisles, 135 Columns, 5 Vehicles





Outline

Summary and Next Steps

Conclusions:

- Developed analytical model of single tier with vehicle interference
- Vehicle interference increase cycle times

Next Steps:

- Refine analytical model and validate against detailed simulations
- Use analytical model to obtain design insights
- Model to account for lift interactions in multi-tier systems

Thank You!

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Questions or Comments?

CQN to determine CV for the Open Queue