Queueing Network Analysis of Compact Picking Systems

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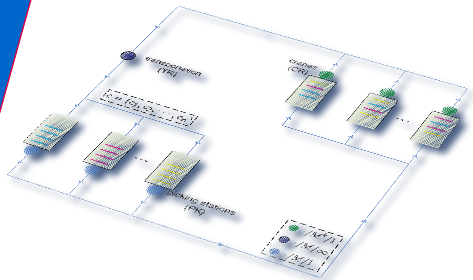

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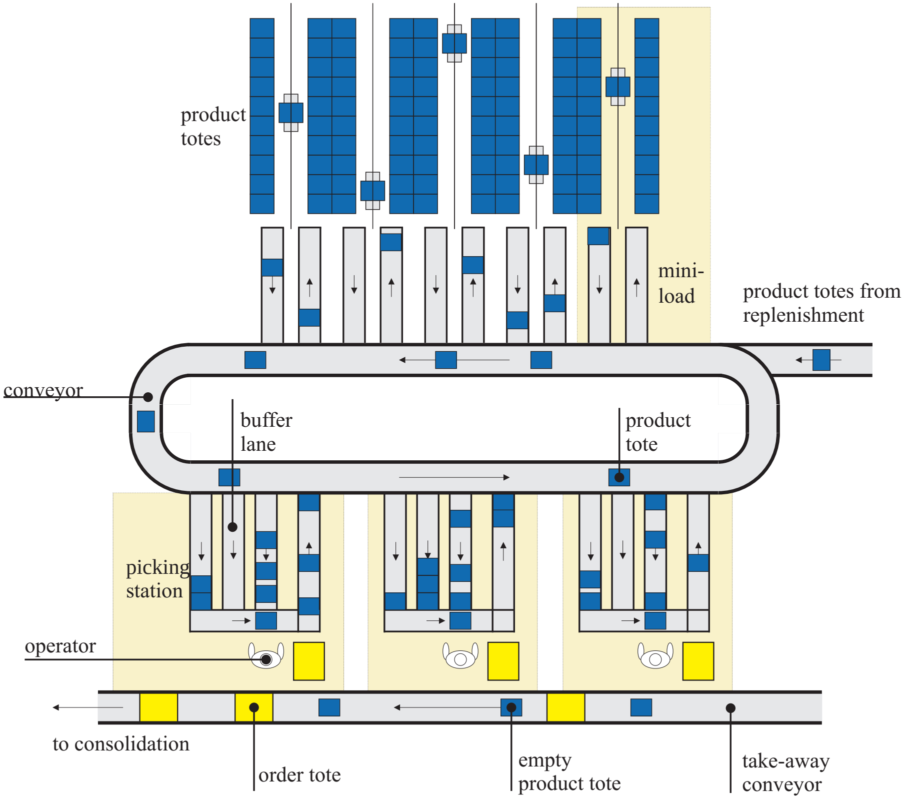
Outline

- Introduction.
- Modeling.
- Performance evaluation.
- Numerical experiments.
- Conclusion.

Introduction

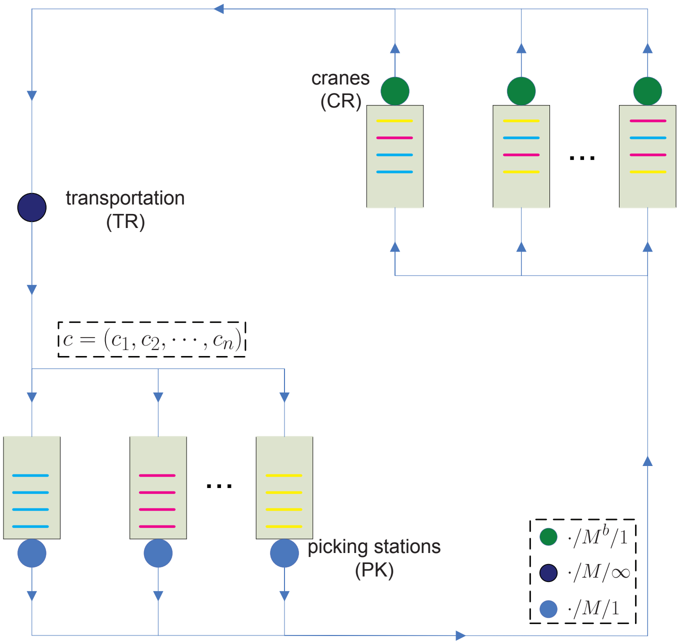
- Distribution centers (DCs) receive and warehouse items, pick and send items according to orders.
- Analytical approach is important (efficient, what-if scenarios etc.).
- Extensive research in sub-systems, especially AS/RS. Very few analytical approaches take into account the interplay of other systems in the DC.
- Queueing network models.

Compact Picking System



- Product/Order totes.
- “Products-to-man” by AS/RS (cranes).
- Picking station product tote pipeline.

The Queueing Network Model



- Crane as greedy bulk servers (b).
- Ignoring storing activities (effective processing times).
- Customers.
- Pipeline capacities c.
- Routing.

Complicating features

- Multi-class closed network.
- No product-form solution.

Mean Value Analysis

Population vector $\underline{c} = (c_1, c_2, \ldots, c_n)$; m cranes; Service rates: γ, μ and λ.

- Waiting times (Arrival Theorem)

$$\begin{aligned} W_{i,TR}(\underline{c}) &= 1/\gamma \\ W_{i,PK}(\underline{c}) &= (L_{i,PK}(\underline{c} - \underline{e}_i) + 1)/\mu \\ W_{i,CR}(\underline{c}) &= (L_{CR}(\underline{c} - \underline{e}_i) + 1)/(b\lambda) \quad \text{or} \\ &= (L_{CR}(\underline{c} - \underline{e}_i) + b)/(b\lambda) \end{aligned}$$

- Throughput (Little's law)

$$TH_i(\underline{c}) = \frac{c_i}{W_{i,TR}(\underline{c}) + W_{i,PK}(\underline{c}) + W_{i,CR}(\underline{c})}$$

- Queue lengths (Little's law)

$$\begin{aligned} L_{i,PK}(\underline{c}) &= TH_i(\underline{c}) W_{i,PK}(\underline{c}) \\ L_{i,CR}(\underline{c}) &= \frac{1}{m} TH_i(\underline{c}) W_{i,CR}(\underline{c}) \\ L_{CR}(\underline{c}) &= \sum_i L_{i,CR}(\underline{c}) \end{aligned}$$

Results Mean Value Analysis

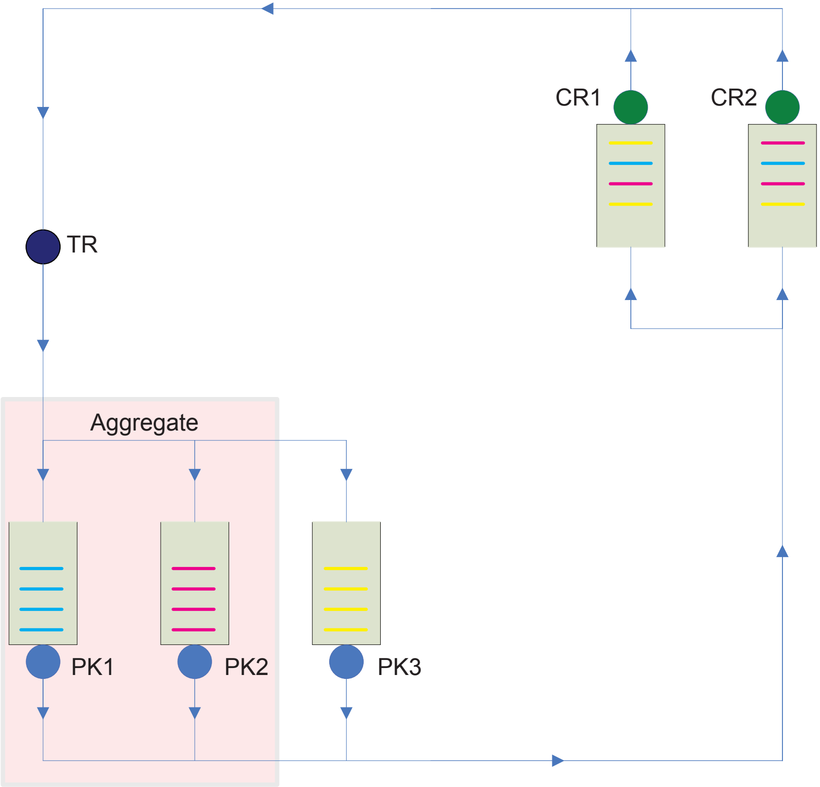
Mean pick time X sec
Mean transportation time XX sec
Mean retrieval time XX sec [a]
5 cranes
n picking stations

Throughput (totes per sec)

b	n	C	Simul	Approx	Approx	Error (%)	Error (%)
4	1	5	0,0613	0,0913	0,0662	48,94	7,99
		10	0,1008	0,1227	0,1084	21,74	7,55
		15	0,1195	0,1250	0,1231	4,63	3,04
	3	5	0,1621	0,2612	0,1817	61,10	12,07
		10	0,2710	0,3600	0,2898	32,84	6,93
		15	0,3356	0,3744	0,3466	11.56	3,27

[a] data censored due to company's policy

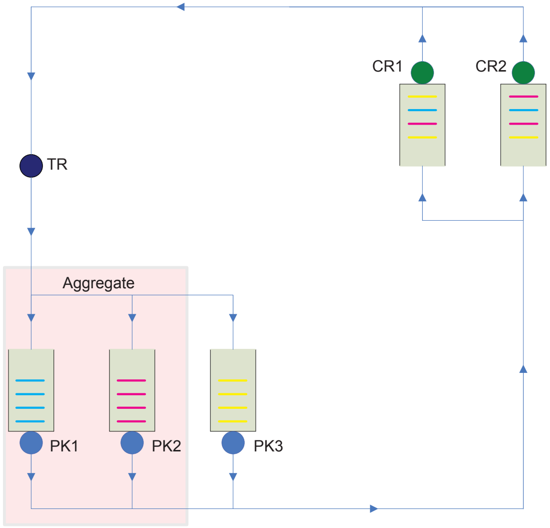
Aggregation Method: General Idea



The idea is to sequentially aggregate two nodes into a composite node with “properly” defined queue-dependent service rates; the service rate for queue size k is determined as the weighted average of the throughputs of the two nodes, given that there are k customers in the two nodes.

Class Aggregation

For $k = 1 \cdots (c_1 + c_2)$



(a.1) Compute the weight function

$$w_k^{(1)}(i) = \text{Prob}\{\ i \text{ in PK1} \mid k \text{ in PK1 and PK2}\},$$

$$i = \max(0, k - c_2) \cdots \min(k, c_1).$$

(a.2) Compute the service rates of CP1 by

$$\mu_k^{\text{CP1}} = \sum_{i=\max(0,k-c_2)}^{\min(c_1,k)} w_k^{(1)}(i)(\mu_i^{\text{PK1}} + \mu_{k-i}^{\text{PK2}}).$$

Weight Function

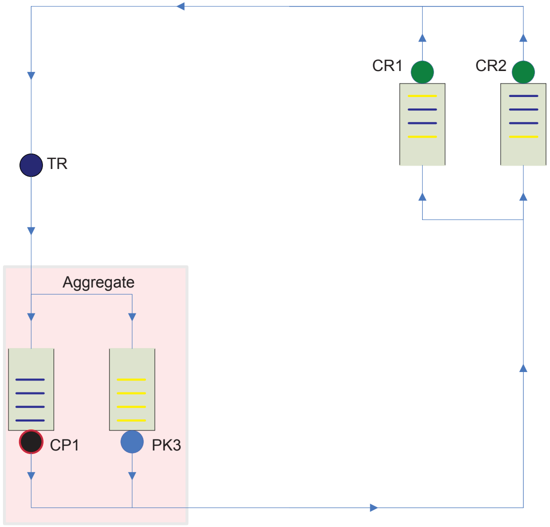
Based on:

- Assume the PF property holds well approximately;
- In a PF network, the cond. prob. do not depend on specifics of the rest, e.g. Prob$\{i, j | i + j = k\} \sim \frac{1}{\mu^i} \frac{1}{\mu^j} \frac{1}{(c_1 - i)!} \frac{1}{(c_2 - j)!}$,

we can approximate the weights using only the parameters of the two nodes in aggregation.

Aggregate Sequentially

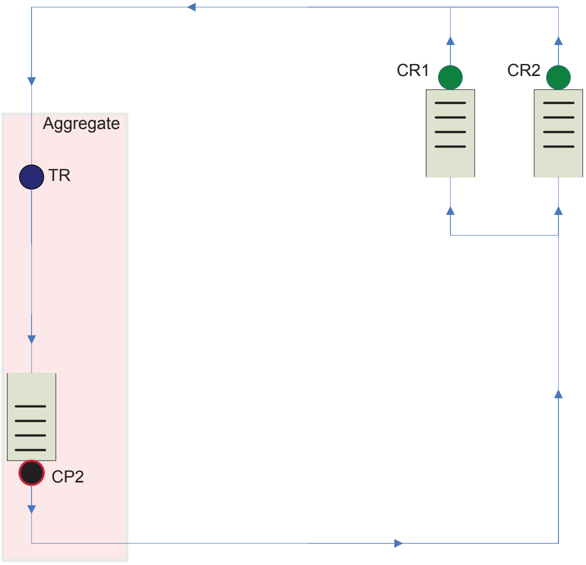
For $k = 1 \cdots (c_1 + c_2 + c_3)$



(b) Aggregate CP1 and PK3 as CP2. Compute $w_k^{(2)}(i)$ and μ_k^{CP2} similarly to the previous step, i.e., substitute PK1, PK2, c_1 and c_2 by CP1, PK3, $c_1 + c_2$ and c_3, respectively.

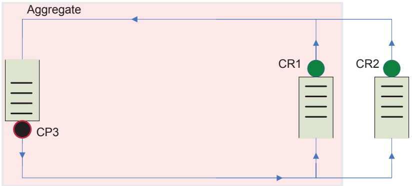
Single-Class Network

- $\mu_k^{\mathsf{CP3}} = \sum_{i=0}^{k} w_k^{(3)}(i)\mu_i^{\mathsf{CP2}}$.
- Prob$\{i, j | i + j = k\} \sim \frac{1}{j!\gamma^j} \prod_{l=1}^{i} \frac{1}{\mu_l^{\mathsf{CP2}}}$.



Bulk Server: Aggregation

For $k = 1 \cdots (c_1 + c_2 + c_3)$



(d.1) Compute

$$w_k^{(4)}(i, j) = \text{Prob}\{i \text{ in CP3 }, j \text{ in service in CR1} \mid k \text{in CP3 and CR1}\},$$
$$i = 0 \cdots (k-1),\ j = 1 \cdots (k-i) \wedge b,$$
$$i = k,\ j = 0.$$

(d.2) Compute the service rates of CP4 by

$$\mu_k^{\text{CP4}} = (1-p) \sum_{i,j} w_k^{(4)}(i, j) \mu_i^{\text{CP3}},$$

where p is the routing probability to CR1.

(e) Compute $w_{|c|}^{(5)}(i, j)$, which gives marginal distribution for CR2.

Bulk Server: Weight Function

Finite MC with states (i, j) where

- i number at CR1, $i = 0, 1, \ldots, k$
- j number in batch, $j = 0, 1, 2, \ldots, b$

and transition rates:

- $(i, j) \to (i - j, \min(i - j, b))$: μ^{CR1}
- $(i, j) \to (i + 1, j)$: $p\mu_{k-i}^{\text{CP3}}$

(Details on the boundary are omitted.)

Disaggregation

In the end of the aggregation phase, we may reverse the “aggregation path” and compute the performance of each node in the original network.

Numerical Experiments: Design

Design of Experiments

- Fixed configuration: Transportation time 1.
- Variables:

Variable	Short Description	Domain
n	Number of pick stations.	$\{1, 3, 5\}$
m	Number of cranes.	$\{1, 3, 5\}$
b	Batch size.	$\{1, 2, 4, 8\}$
μ^{PK}	Service rate of PK.	$\{1/2, 1, 2\}$
r^{PB}	Ratio between population and batch size.	$\{1, 2, 4\}$
r^{MU}	Ratio between service rates of PK and CR.	$\{1/4, 1/2, 1, 2, 4\}$
δ^{PK}	Mode of perturbation for PK.	$\{\circledcirc\circledcirc, \circledcirc+, +\circledcirc, ++, +-\}$
δ^{CR}	Mode of perturbation for CR.	$\{\circledcirc\circledcirc, \circledcirc+, +\circledcirc, ++, +-\}$

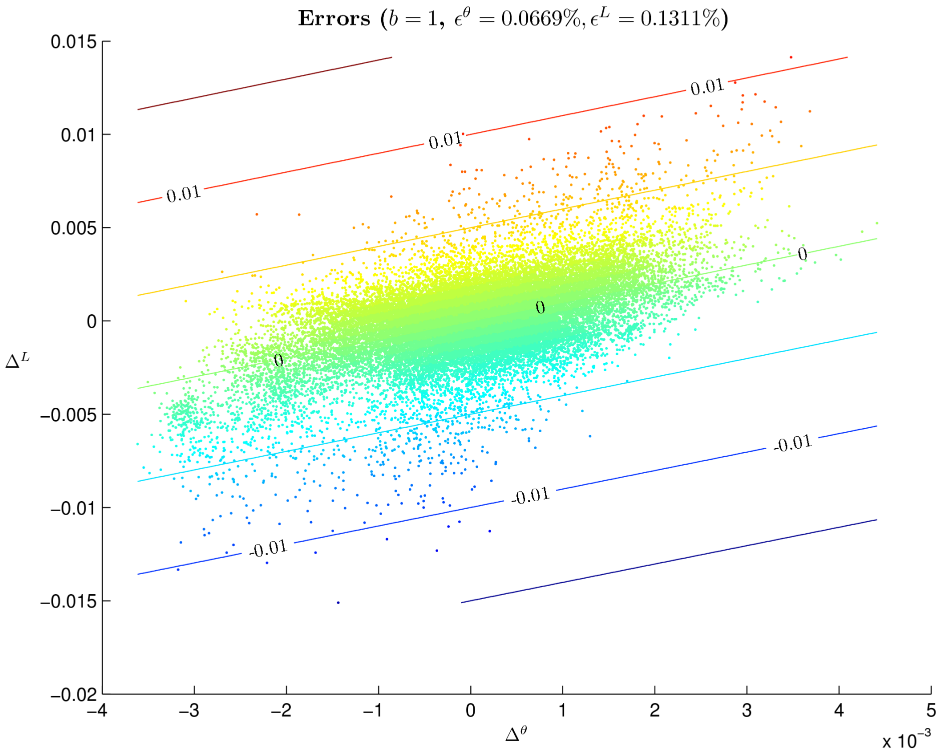
- Response: Throughput (θ), utilization (ρ), and average number in node (L).
- In total 20790 distinct cases.

Numerical Experiments: Results

All numbers ($F \pm \mathbf{E}N$) are percentage. An entry of $\Delta(\alpha\%) = q$ is interpreted as: $\alpha\%$ of the observed relative errors are smaller than $q\%$.

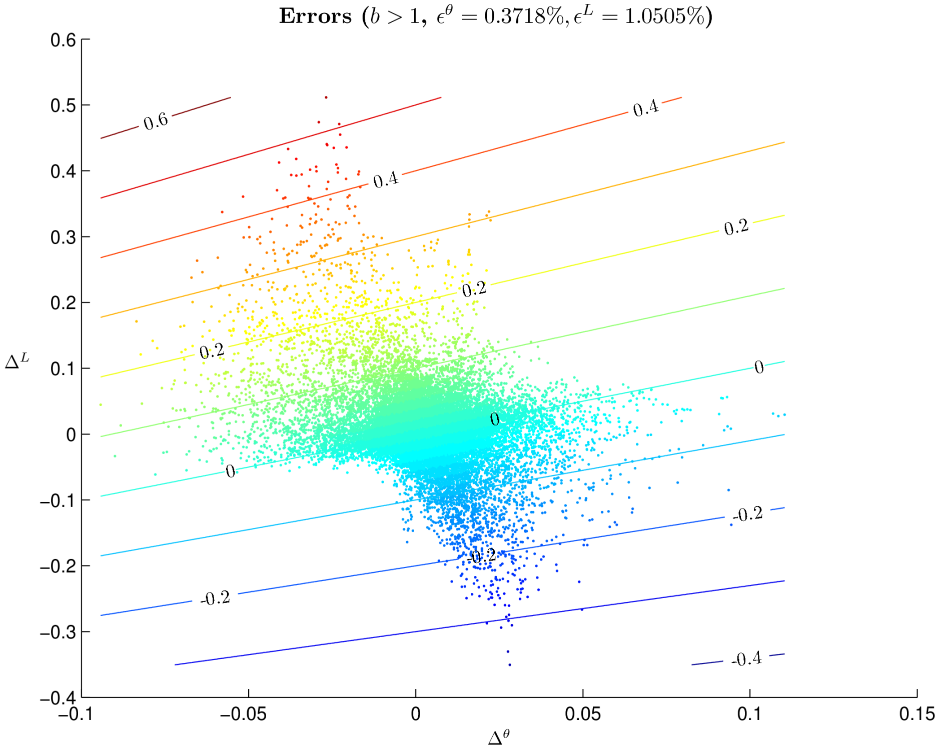
Slice	Value	θ			ρ			L		
		ϵ	Δ(2.5%)	Δ(97.5%)	ϵ	Δ(2.5%)	Δ(97.5%)	ϵ	Δ(2.5%)	Δ(97.5%)
b	>1	3.72E−1	-1.10E+0	1.69E+0	4.60E−1	-1.31E+0	2.41E+0	1.05E+0	-3.41E+0	5.36E+0
b	1	6.69E−2	-2.09E−1	1.82E−1	9.03E−2	-2.55E−1	2.45E−1	1.31E−1	-4.51E−1	3.91E−1
	2	2.09E−1	-3.99E−1	9.34E−1	2.66E−1	-4.57E−1	1.21E+0	3.92E−1	-9.72E−1	1.66E+0
	4	3.56E−1	-1.15E+0	1.58E+0	4.67E−1	-1.33E+0	2.31E+0	8.88E−1	-2.43E+0	4.39E+0
	8	5.51E−1	-2.27E+0	2.58E+0	6.46E−1	-2.54E+0	3.46E+0	1.87E+0	-7.51E+0	9.76E+0
Node	TR	1.91E−1	-3.85E−1	9.81E−1	∅	∅	∅	1.96E−1	-3.85E−1	9.91E−1
	PK	5.70E−1	-2.23E+0	2.53E+0	5.67E−1	-2.23E+0	2.54E+0	1.83E+0	-6.44E+0	9.04E+0
	CR	2.22E−1	-4.28E−1	1.03E+0	3.52E−1	-3.58E−1	2.22E+0	5.03E−1	-1.72E+0	1.49E+0

Exact Cases: $b = 1$



The approximation is exact. The actual coverage of the simulation 99% confidence interval are 99.53%, 98.34% and 98.60% for θ, ρ and L respectively. Points are colored by the inferred relative error of sojourn time $\Delta^W = (1+\Delta^L)/(1+\Delta^\theta) - 1$, using Little's law.

Inexact Cases: $b > 1$



General Observations

- Overall the approximation works extremely well;
- For PK nodes, the accuracy becomes (slightly) less when the cranes act as bottle necks (could be fixed by MVA or open network model).

Conclusion

- Model compact picking systems as closed queueing network.
- Efficient and accurate method.
- Generalization.