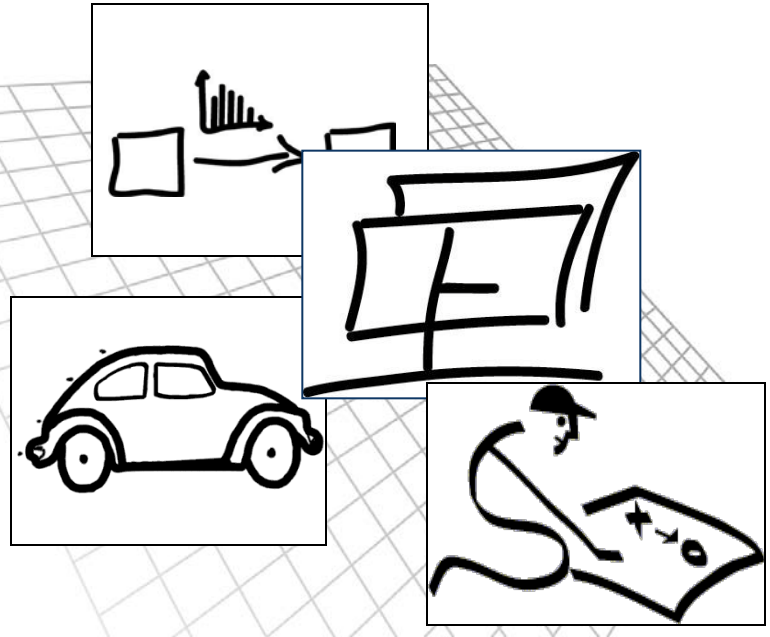

Blocking phenomena in Order-picking Systems

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Workshop
„Stochastic Models for Warehouseing Systems“
Eindhoven, October 30 2009

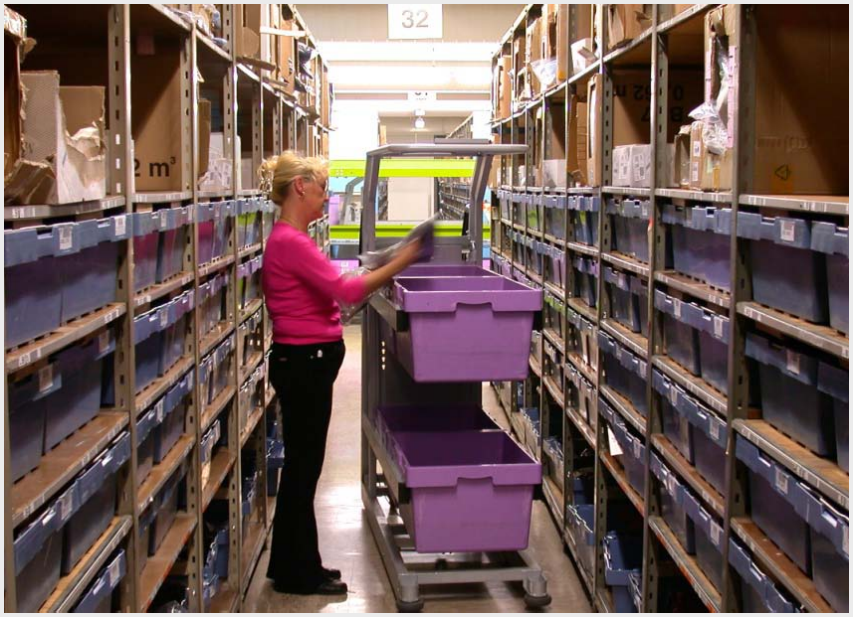
Institut für Fördertechnik
und Logistiksysteme
Universität Karlsruhe (TH)

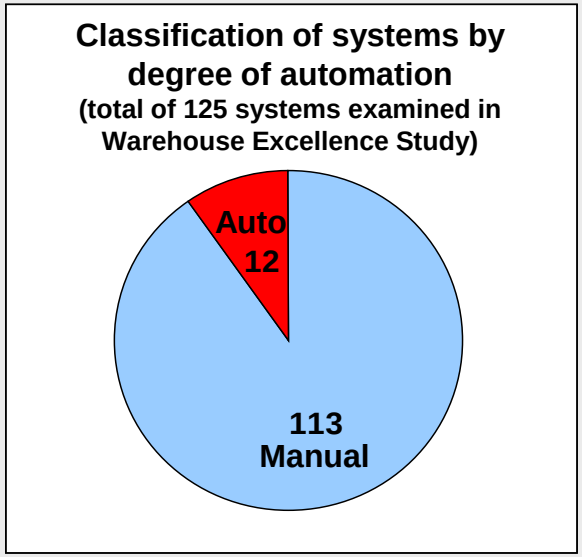
Agenda

- Problem Statement: Dynamic effects in manual order picking systems
  - Importance of manual order picking systems
  - Types of blocking
- Queueing Model of a manual order picking system
  - Parameter transformation
  - Existing algorithms to calculate characteristic values
- Modified algorithm and application to order picking systems
  - Weakness of existing approaches
  - Improvement of existing approaches
  - Performance of the modified algorithm
  - Application to exemplary order picking system
- Summary and next steps

Blocking in manual order picking systems

- **„Activities of an order picker are interrupted by another order picker“**
  - Order picker is waiting for another order picker
- Picker-to-part systems (manual systems) are commonly used in real distribution centers





Warehouse Excellence Studie, IFL, 2006-2009

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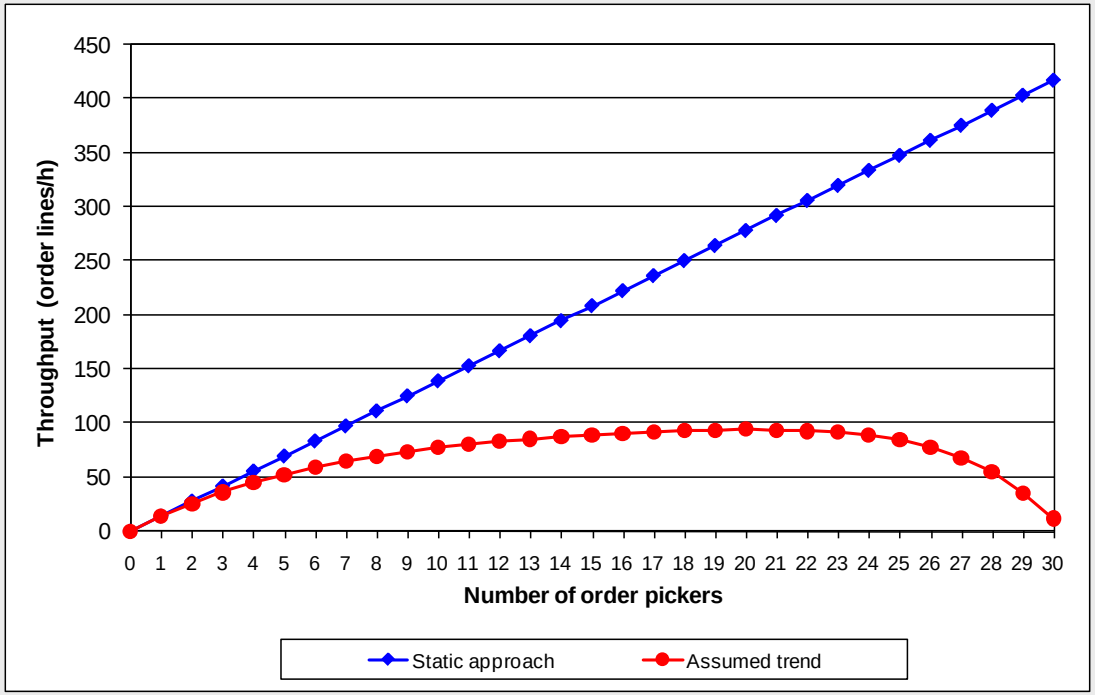
Reduced performance as a result of dynamic effects

- Blocking situations lead to dynamic effects (waiting, queueing)
- Dynamic effects reduce the total system throughput (order lines per time unit)



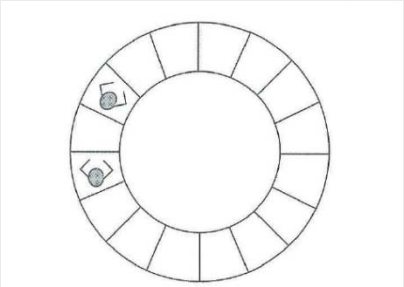
- DFG-funded research project to evaluate effects of blocking

Research approaches for manual order picking systems with blocking

- Static approach
  - Travel time models of Gudehus, Hall, Roodbergen und Vis, Sadowsky, etc.
  - Static calculation of the mean order lead time
  - No consideration of dynamic effects
  - Used in the rough planning phase of order picking systems

- Probability calculation (Ph.D. thesis of Ralf Lüning – 2005)
  - Calculation of the probability of two order pickers to access the same rack at the same time
  - Calculation of the probability of two order pickers meeting in one aisle

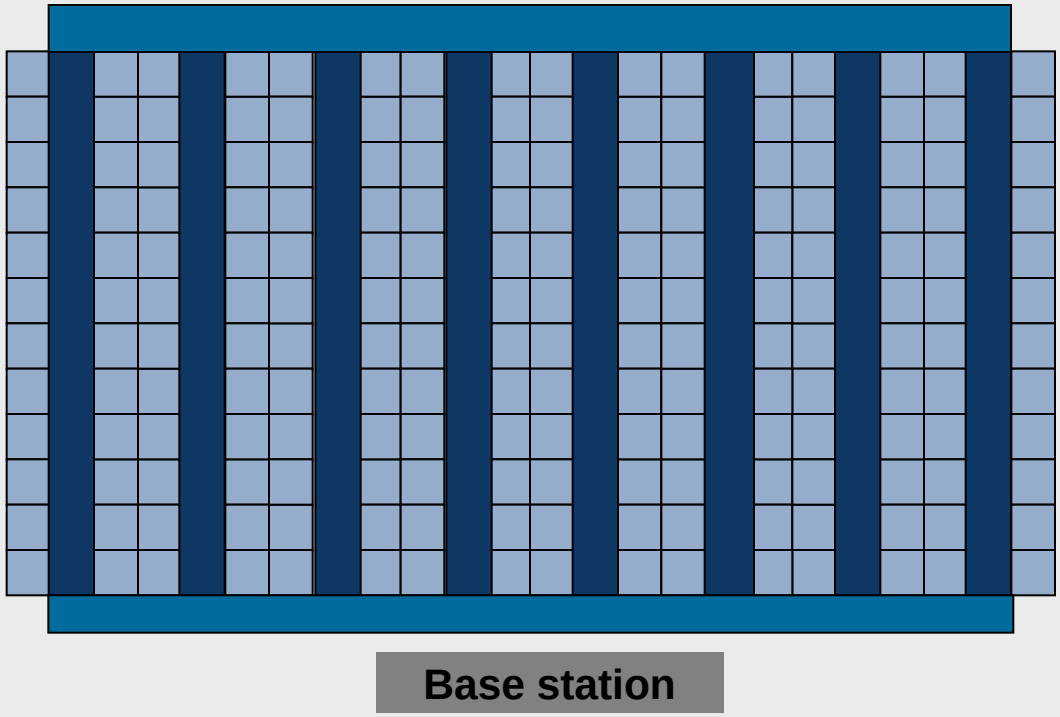
- Congestion Model (Gue, Meller, Skufca – 2006)
  - Transformation of an order picking system into a closed circle
  - Arbitrary repetition of random experiments („walking or picking“) and calculation of blocking ratios
  - Pick time : walk time either 1:1 or ∞ : 1 (infinite walking speed)



- Queueing Models
  - Modelling of one aisle as a queueing system (Shih, Pan 2008)
  - Modelling of elementary spaces as queueing systems (Furmans, Huber)

Limited resources of the order picking system lead to blocking

- Typical order picking system with a picker-to-part configuration



- Blocking occurs at the following ressources:

Base station | **Rack column** | **Cross-Aisle**

Queueing model of a manual order picking system

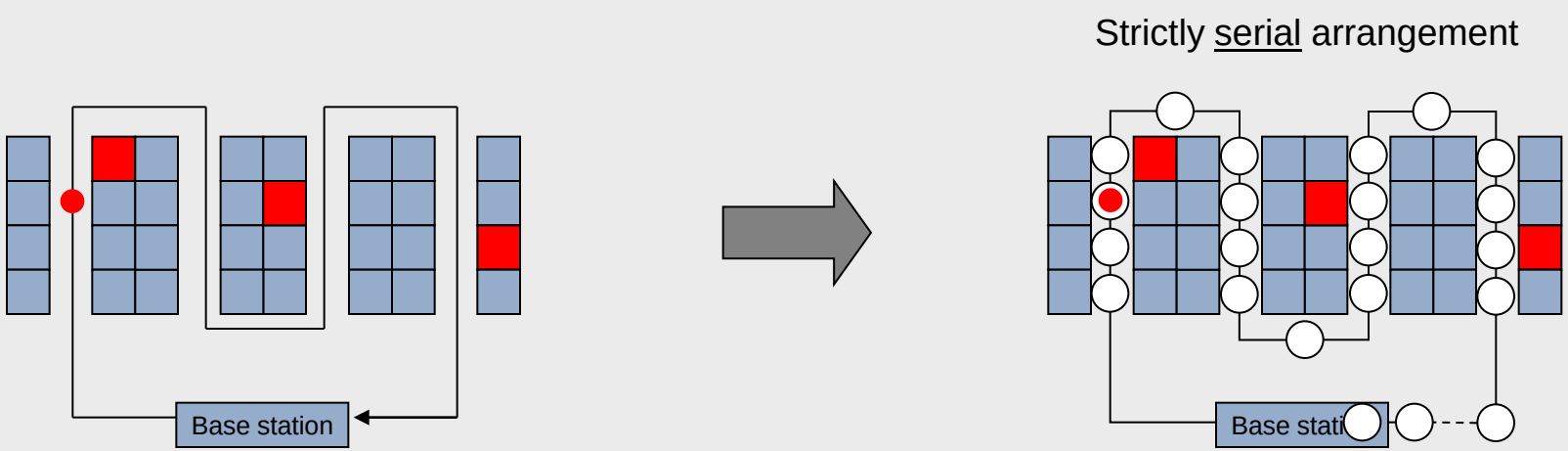
Manual order picking system	Queueing model
Fixed number of order pickers	k customers in a closed queueing network
Resources (Space in front of a rack column, space in a rack column, base station)	n single-server queueing systems
Blocking situations	Blocking after service, system capacities $m_i = 1$ (no buffer spaces)
Picking probability, picking and walking time, product specifications	Service rates μ_i, variabilities c^2_i
Routing strategies, order specifications	Transition probabilities q_{ij}

Closed queueing model of a manual order picking system

- Closed queueing networks are used to model order picking systems because:
  - In real systems, the number of order pickers is fix over a certain period of time
  - In closed queueing networks, the number of customers k is fix

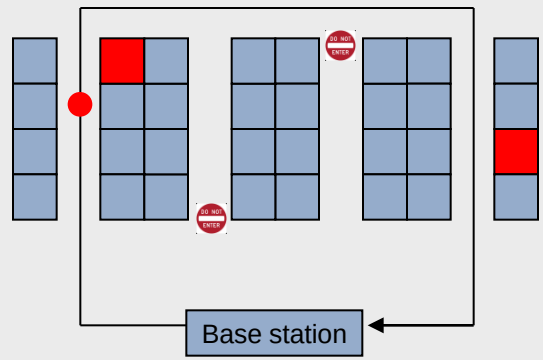
Transforming the routing strategy into transition probabilities

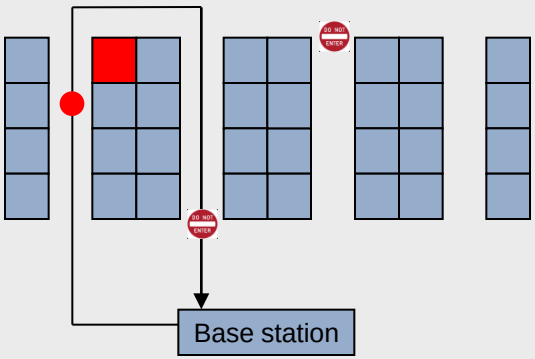
- Traversal (S-shape) routing strategy (without aisle skipping) as queueing model



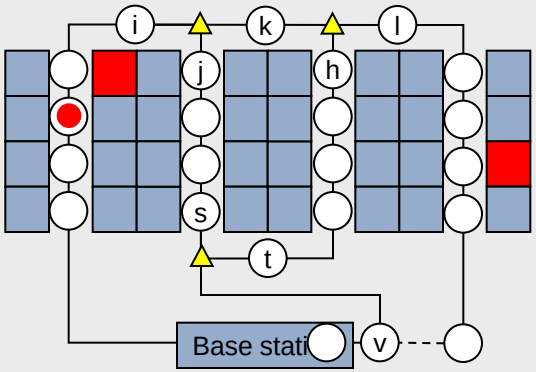
Transforming the routing strategy into transition probabilities (ctd.)

- Traversal (S-shape) routing strategy (with aisle skipping and one-way traffic) as a queueing model





Parallel arrangement



△ = Branching element

Transition probabilities:

$q_{i,j} = p_i$ $q_{k,h} = 0$ $q_{s,t} = p_s$

$q_{i,k} = 1-p_i$ $q_{k,l} = 1$ $q_{s,v} = 1-p_s$

Computation of characteristic values for the queueing model

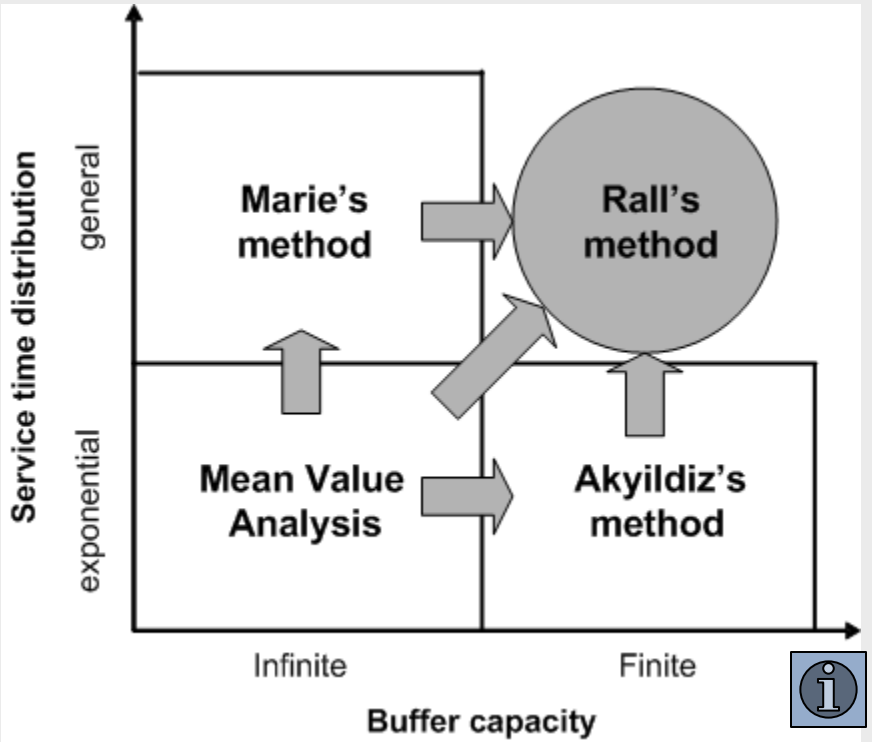
- Assumptions:
  - Closed queueing network (CQN) with n queueing systems and k customers
  - No buffer space in front of the queueing systems → finite queue capacities
  - General distribution of service times
    - Manual operations are usually not exponentially distributed

- Limited number of applicable calculation techniques:

Publication	Blocking Protocol			Service Time		Solution Approach												
						Exact					Approximate							
						Product Form				Sym	Cyclic				Arbitrary			
	BAS	BBS	RS	Exp	Gen	PFMSS	PFMVA	PFCA	Other	SSSR	ATA	NWD	VQCD	Other	MSS	AMVA	MEM	Other
Akyildiz (1987)	X			X		X												
Akyildiz (1988a)	X			X		X										X		
Akyildiz (1988c)	X				X										X			
Kouvatsos & Xenios (1989)			X		X												X	
Bouchouch et al. (1996)	X				X							X						
Rall (1998)	X				X										X			X

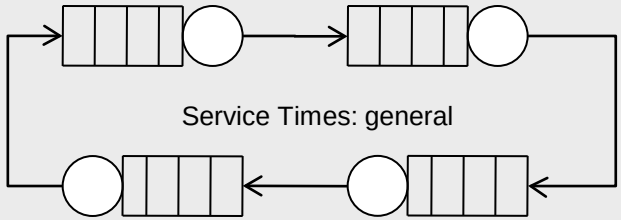
Computation of characteristic values for the queueing model – Rall‘s method

- Rall‘s method combines existing approaches into a new framework



Rall‘s method – Key Steps

- Throughput λ(K) in closed queueing networks with blocking and general service times



- **Step 1**: Assume exponential service times and apply method of Akyildiz
  - Blocking will lead to a loss in customer productivity → Reduction of K to K* (where K* ≤ K)
  - “λ(5) of the blocking network will be approx. equal to λ(4) in the equivalent non-blocking network”

- **Step 2**: Apply correction factor f on K* such that variabilities $c^2 \neq 1$ can be considered
  - $K^{**} = K^* \cdot f$
  - f is a linear function of the network occupancy level ω and the network variability c^2_R

- **Step 3**: Apply Marie’s method using K**

Computation of characteristic values for the queueing model – Rall‘s method

- **<u>Derivation of variability influence:</u>**

$$c^2{}_R = \frac{\sum_{i=1}^{N} e_i \cdot \rho^2{}_i \cdot \frac{1}{\sqrt{m_i}} \cdot \frac{\bar{k}_i}{M_i} \cdot c^2{}_{B,i}}{\sum_{i=1}^{N} e_i \cdot \rho^2{}_i \cdot \frac{1}{\sqrt{m_i}} \cdot \frac{\bar{k}_i}{M_i}}$$

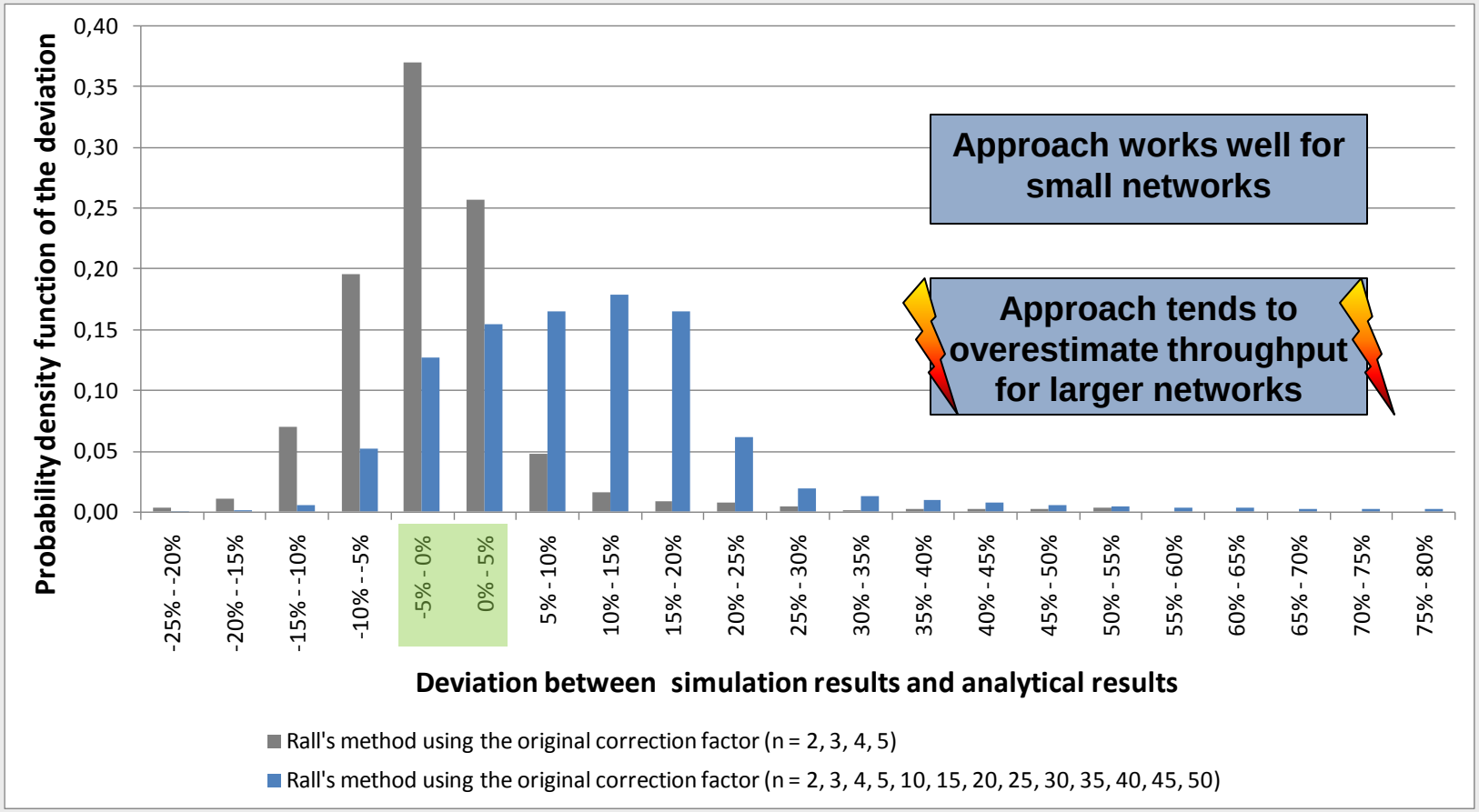
- **<u>Derivation of the correction factor f:</u>**
  - f = func(c_R^2; ω)

    If the occupancy level ω of the network is small, Akyildiz gives good approximations

    Variabilities in the network: larger f for lower variabilities / smaller f for higher variabilities
  - Formula: $f = 1 - \omega + \omega \cdot \frac{1}{\sqrt[3]{c^2{}_R}}$

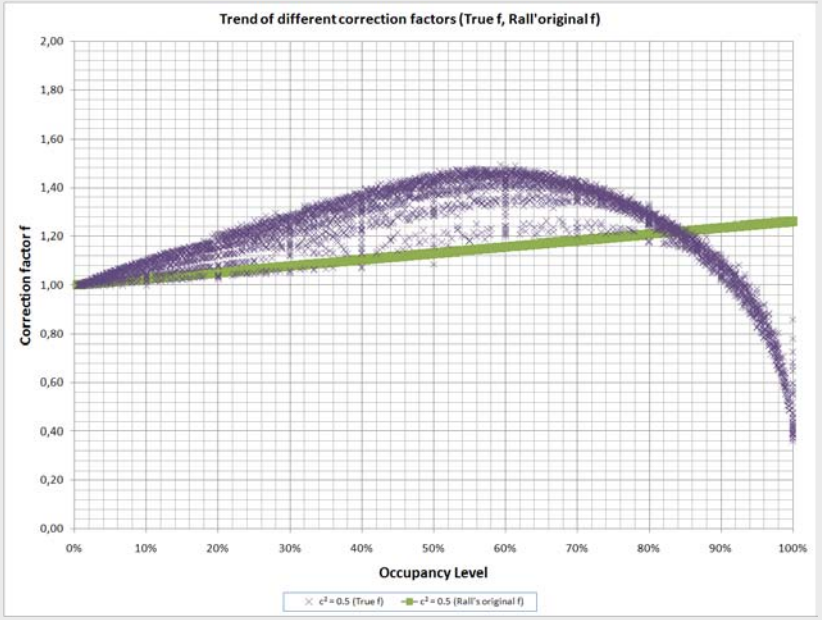
Computation of characteristic values for the queueing model

- Observation on the frameworks performance (using original correction factor)

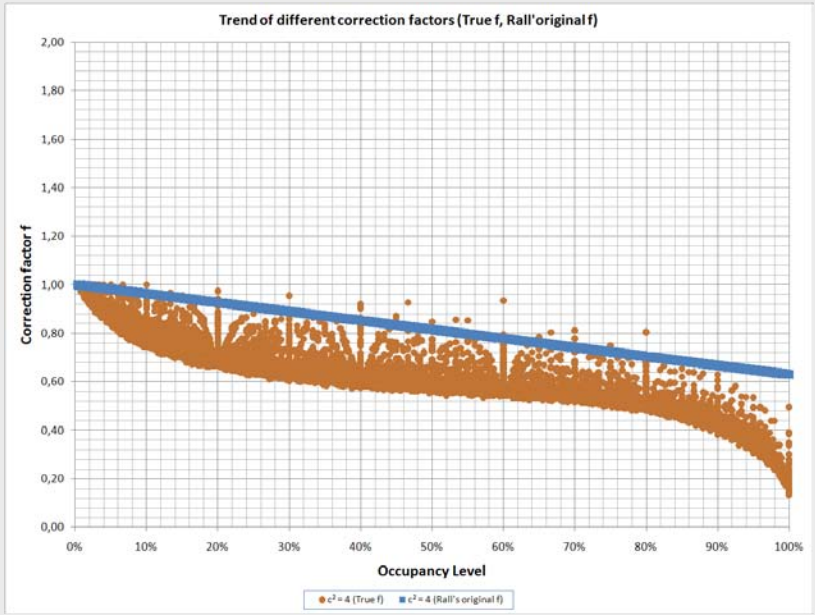


Computation of characteristic values for the queueing model

- For a better understanding of the true correction factor trends, a large set of experiments was analyzed
- Results show that the trend:
  - depends strongly on the service time variability
  - Is (in most cases) not linearly correlated to the occupancy level



Variability c² = 0.5



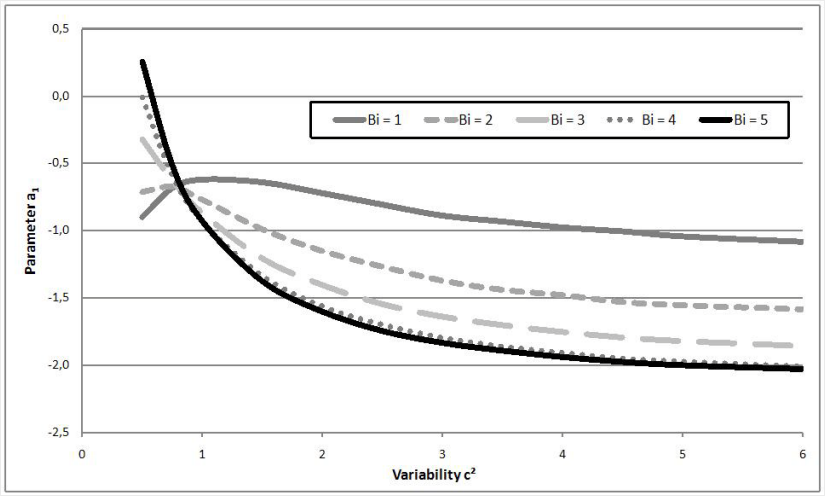
Variability c² = 4

Computation of characteristic values for the queueing model

- To determine a **new correction factor**, each variability scenario was analyzed separately and polynomial estimator functions were derived

$$f_{new} = a_0 + a_1 \cdot \omega + a_2(\omega)^2 + a_3(\omega)^3$$

- The coefficients a_i depend on variability c^2 and queue capacities B_i (example graph: a_1)
- For each variability c^2, a_i can be estimated using a linear function of form: $a_i = m_{a_i} \cdot B_i + b_{a_i}$



Computation of characteristic values for the queueing model

- The new correction factor can be estimated using:

$$f_{new} = \begin{cases} 1-\omega+\omega\cdot\dfrac{1}{\sqrt[3]{c^2{}_R}} & \text{for } n<10 \\ a_0 + a_1\cdot\omega + a_2(\omega)^2 + a_3(\omega)^3 & \text{for } n\geq 10 \end{cases}$$

where

$$a_0 = (-0.01\cdot(\ln c^2)^2 - 0.002)\cdot B_i + 1.04 - \frac{0.01\cdot\ln c^2}{c^2}$$

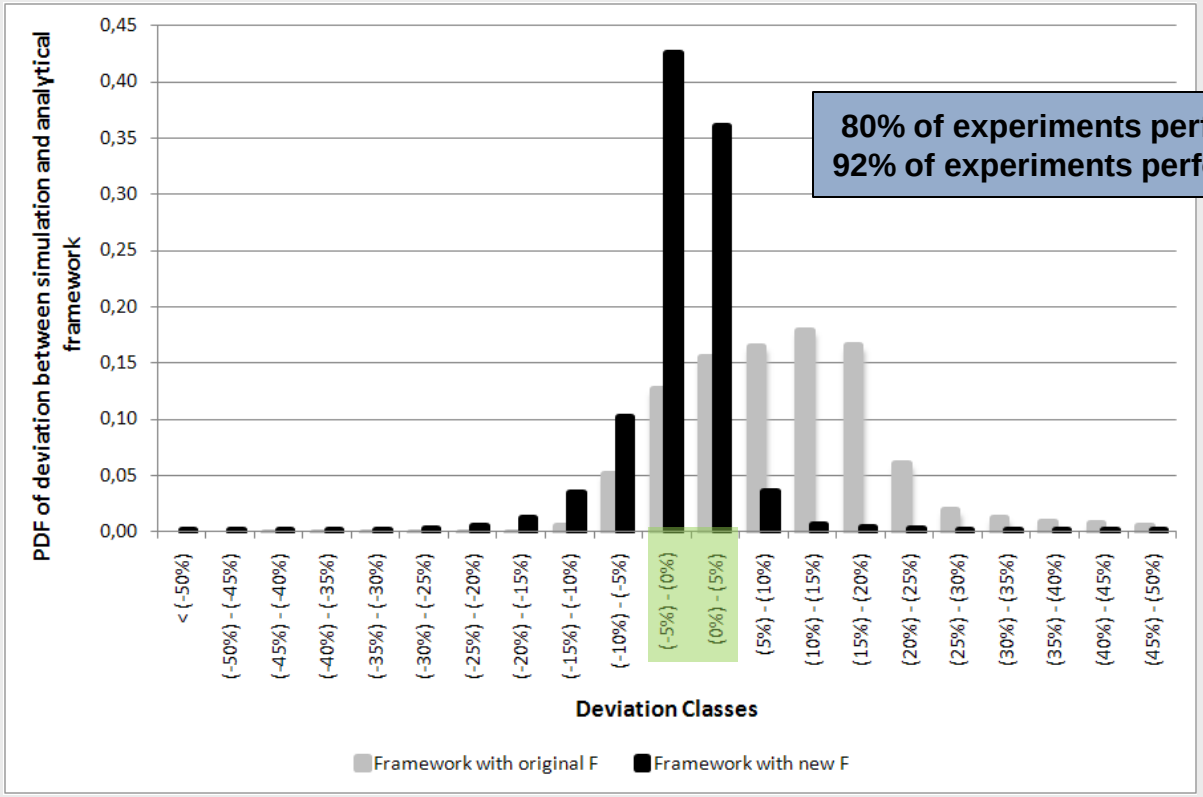
$$a_1 = (0.14\cdot c^4 - 0.24)\cdot B_i - 1.29 + \frac{2.43}{c^2} - \frac{1.71}{\sqrt[3]{c^2}}$$

$$a_2 = (0.57 - \frac{0.35}{\sqrt[3]{c^2}})\cdot B_i + 1.23 + \frac{0.99}{c^4}$$

$$a_4 = (-0.33 + \frac{0.21}{c^2})\cdot B_i - 1.02 - \frac{1.17}{\sqrt[3]{c^2}}$$

Computation of characteristic values for the queueing model

- With the new correction factor, the accuracy of the analytical approach can be improved

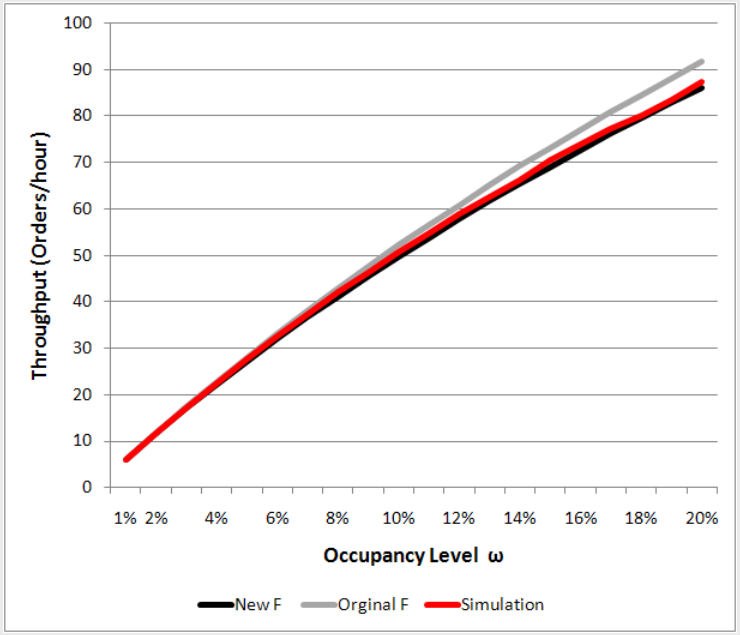


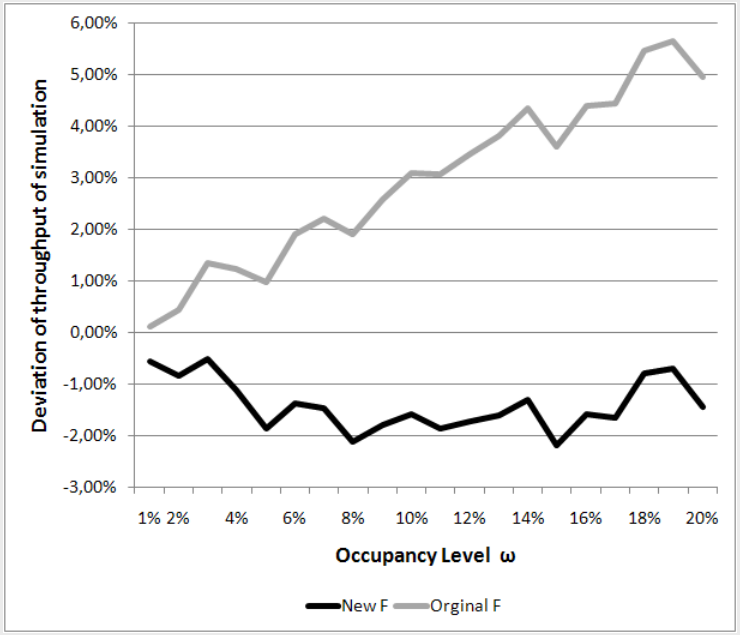
80% of experiments perform within +/- 5% error range
92% of experiments perform within +/- 10% error range

Applying the new method to order picking systems

- In order picking systems, only a limited number of order pickers will work → small ω
- Analytical framework works more accurately for small ω:
  - 90% of experiments are within +/- 5% error range
  - 98% of experiments are within +/- 10% error range

- Example: Order picking system with 10 aisles and 10 rack columns per aisle





Summary and next steps

- In manual order picking systems, blocking can lead to dynamic effects
  - reduced order throughput / “less picks per man hour”
  - static calculations can produce wrong cost estimations and performance forecasts

- A queueing model approach for the calculation of performance figures of such systems was developed
  - existing methods were modified → improved quality of results

- Future research activities:
  - Find more correlations between input parameters to derive better formulas, truly understand the correction factor
  - Find ways to reduce computation time for very large networks
  - Continue to test and modify the method specifically for order picking systems
    - Fast-mover / slow-mover allocation
    - Alternate routing strategies

Related Problems

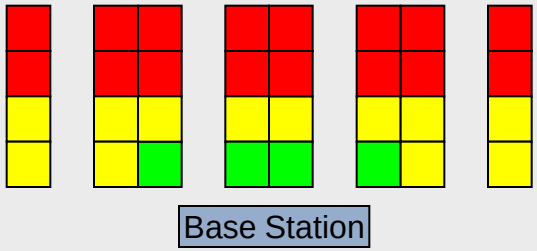
Thank you for your attention!

Discussion ...

BACKUP-SLIDES

Blocking in manual order picking systems

- Blocking situations can occur in many order picking systems
- Reasons for blocking situations:
  - **High space utilization**
    - Cost pressure (competition , energy)
    - Continuous enlargement of warehouses (size of assortment, lead times)
  - **Organizational strategies**
    - Fast mover / average mover / slow mover allocation of items
    - “Return” routing policy



Research approaches for manual order picking systems with blocking

- Static approach
  - Travel time models of Gudehus, Hall, Roodbergen und Vis, Sadowsky, etc.
  - Static calculation of the mean order lead time
  - No consideration of dynamic effects
  - Used in the rough planning phase of order picking systems

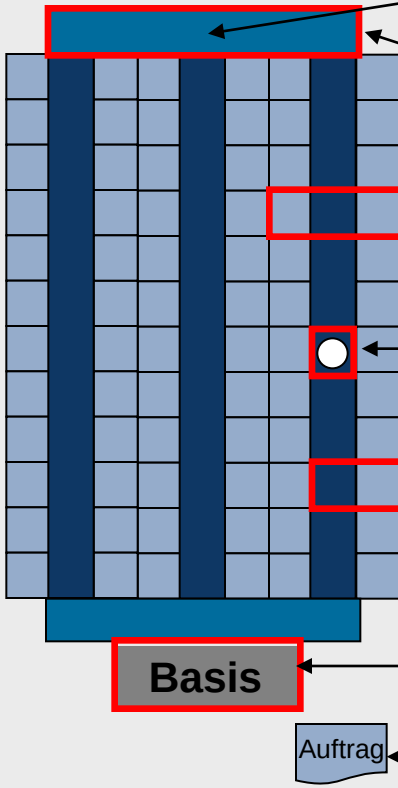
- Probability calculation (Ph.D. thesis of Ralf Lüning – 2005)
  - Calculation of the probability of two order pickers to access the same rack at the same time
  - Calculation of the probability of two order pickers meeting in one aisle

- Congestion Model (Gue, Meller, Skufca – 2006)
  - Transformation of an order picking system into a closed circle
  - Arbitrary repetition of random experiments („walking or picking“) and calculation of blocking ratios

- Queueing theory
  - Consideration of stochastic effects
  - Modelling of one aisle as a queueing system (Shih, Pan 2008)
  - Modelling of elementary spaces as queueing systems (Furmans, Huber)

Parameter eines manuellen Kommissioniersystems

- Beispielhaftes manuelles Kommissioniersystem mit statischer Artikelbereitstellung

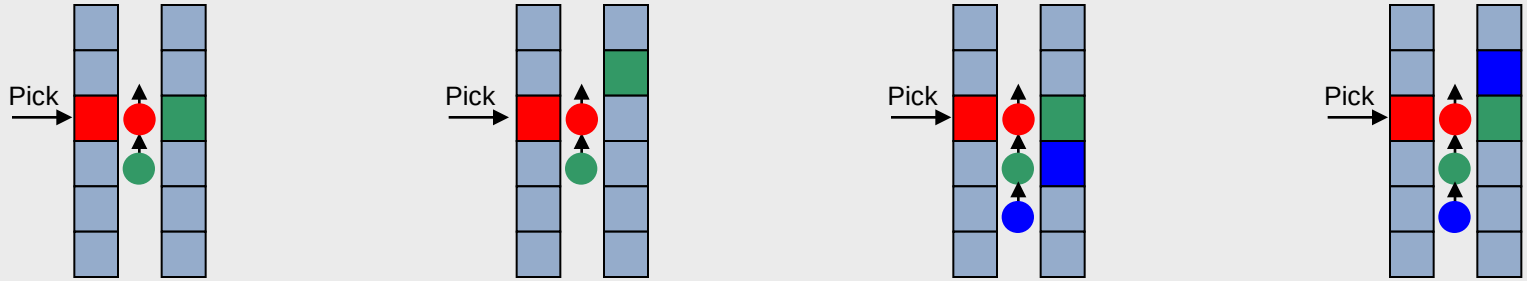


- Laufstrategien der Kommissionierer (Entscheidungspunkte)
- Stirngänge zur Verbindung von zwei Gängen
- Gegenüberliegende Regalspalten mit zugehörigem Einzelplatz im Gang
- Kommissionierer
- Weg-, Tot- und Greifzeiten an jeder Regalspalte
- Basiszeit
- Anzahl Entnahmepositionen, Entnahmeorte, Entnahmemengen

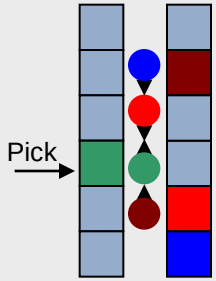
Types of blocking

- The space in front of any resource is defined as an „elementary space“
- **Elementary space can be occupied by only one order picker at a time**

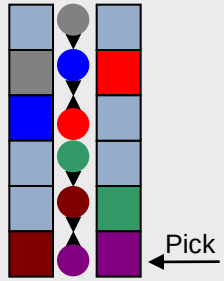
- Blocking with same direction of movement



- Blocking with opposite direction of movement

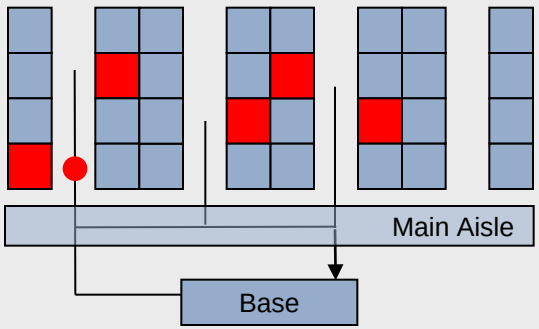


Same **and** opposite direction of movement

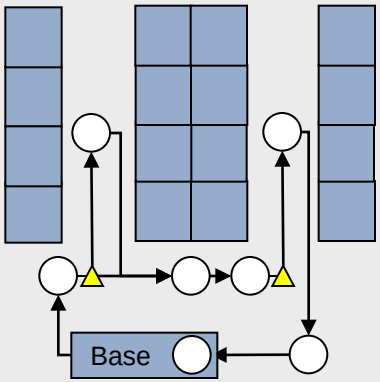


Queueing model of a manual order picking system

- Return routing strategy (with single or multiple aisles visits) as a queueing model



Parallel arrangement



„One-way traffic by aisle reservation“

If one order picker is within aisle i, all remaining order pickers with picks in i have to wait in front of the aisle i

Computation of characteristic values for the queueing model – Akyildiz

- **1st step: Determine the size of the statespace for the blocking network (N nodes, K customers)**
  - Determine the capacity vectors Z_i for each queueing system

$$a_i(k) = \begin{cases} 1 & \text{for} \quad k = 0, 1, ..., (B_i + \sum_{j=1 \& p_{ji}>0}^{N} m_j) \\ 0 & \text{otherwise} \end{cases}$$

$$Z_i = \begin{bmatrix} a_i(0) \\ a_i(1) \\ \vdots \\ a_i(K) \end{bmatrix}$$

  - Determine the resulting vector Z‘ by using the convolution operation

$$Z' = Z_1 \otimes Z_2 \otimes \ldots \otimes \mathbb{Z}_N$$

    - (K+1)st element of Z‘ corresponds to the number of states in the blocking network for K customers (= Z_{Block})

- **2nd step: Transformation into an equivalent product-form network**
  - Find K*, so that $Z_{non\text{-}Block} = \binom{N + K^* - 1}{N - 1} \approx Z_{Block}$

- **3rd step: Calculate $\lambda_{Non\text{-}Block}(K^*)$ ($\approx \lambda_{Block}(K)$) using the mean value analysis**

Akyildiz – Example 1 (Capacities $m_i > 1$)

- Inputs: Closed networks with 3 stations / Service times $t_1 = 0{,}6$, $t_2 = 0{,}8$, $t_3 = 1$ / Capacities $m_1 = 6$, $m_2 = 5$, $m_3 = 8$ / Visit Ratios $e_i = 1$ / Maximum number of customers $K = 19$
- This example represents „Example 1“ of Akyildiz (1988, page 68)

- 1st step: Create vectors:
  - $a_i(k) = 1$ for $k = 0,1,2,\ldots\ldots,m_i + 1$
  - $a_i(k) = 0$ for $k = m_i + 2,\ldots\ldots, K$

Vector element	Station 1	Station 2	Station 3
0	1	1	1
1	1	1	1
2	1	1	1
3	1	1	1
4	1	1	1
5	1	1	1
6	1	1	1
7	1	0	1
8	0	0	1
9	0	0	1
10	0	0	0
11	0	0	0
12	0	0	0
13	0	0	0
14	0	0	0
15	0	0	0
16	0	0	0
17	0	0	0
18	0	0	0
19	0	0	0

Akyildiz – Example 1 – Convolution algorithm

- 2nd step: perform convolution operation

Vector element	Station 1		Station 2		Station 3		Vector element	Number of states
0	1		1		1		0	1
1	1		1		1		1	3
2	1		1		1		2	6
3	1		1		1		3	10
4	1		1		1		4	15
5	1		1		1		5	21
6	1		1		1		6	28
7	1		0		1		7	35
8	0		0		1		8	41
9	0	⊗	0	⊗	1	=	9	46
10	0		0		0		10	49
11	0		0		0		11	50
12	0		0		0		12	49
13	0		0		0		13	46
14	0		0		0		14	41
15	0		0		0		15	35
16	0		0		0		16	28
17	0		0		0		17	21
18	0		0		0		18	15
19	0		0		0		19	10
							20	6
							21	3
							22	1
							23	0

- For 19 customers, the blocking network has 10 states

Akyildiz – Example 1 – Use the binomial coefficient

- Number of states in the non-blocking network $Z^* = \binom{N+K^*-1}{N-1}$

- **Find K*, so that Z* approx. equals the kth element of the resulting vector**

Vector element	Number of states
0	1
1	3
2	6
3	10
4	15
5	21
6	28
7	35
8	41
9	46
10	49
11	50
12	49
13	46
14	41
15	35
16	28
17	21
18	15
19	10
20	6
21	3
22	1
23	0

→ $Z^* = \binom{N+K^*-1}{N-1} = \binom{3+K^*-1}{3-1} \approx 10$

→ Using K* = 3 → Z* = 10

→ The throughput of the blocking network with 19 customers approx. equals the throughput of the non-blocking network with 3 customers

Akyildiz – Example 1 – Calculate the throughput

- Calculation of the throughput in a non-blocking network with the respective number of customers K* (here: 3 instead of 19 customers, see preceding slide)

Akyildiz – Convolution operation: general observations

- k^{th} step: $a^{k\otimes}$

$$\begin{matrix} \dots \\ x \\ x \\ x \\ x \\ x \\ x \\ x \\ \dots \end{matrix} \otimes \begin{matrix} 1 \\ 1 \\ 1 \end{matrix} = \begin{matrix} \dots \\ y \\ y \\ y \\ y \\ y \\ \boxed{z} \\ y \\ y \\ y \\ y \\ y \\ \dots \end{matrix}$$

→ With each convolution the maximum entry z „shifts down one element“ in the resulting vector

→ After k convolutions, the maximum entry (z) is given for the (k+2) element of the vector

- In a closed queueing network with N stations the convolution operations needs to be performed (N-1) times.
- When the capacity of each station m_i is 1, all vectors will have 3 elements and the maximum entry of the resulting vector will always be given for the (N-1+2) = (N+1) element of the vector (see example above)
- The (N+1) element of the vector represents the number of states for N customers
- The maximum number of customers in a system with N stations and no waiting rooms is N
- **The maximum entry of the resulting vector will be given for the (N+1) element representing the maximum number of customers N**

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