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Analytical Models for Design, Planning and Operation of Warehousing Systems

Sunderesh S. Heragu

Mary Lee & George F Duthie Chair in Engineering Logistics

Director, Logistics and Distribution Institute

Department of Industrial Engineering

University of Louisville

Louisville, KY

UNIVERSITY OF LOUISVILLE

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Credits

- National Science Foundation via several grants, particularly
  - CMMI9900039
  - CMMI9908437
  - CMMI 0522798
  - CMMI 0946713
- Collaborators -
  - Henk W. M. Zijm
  - Peter Schuur
  - Ronald J. Mantel
  - Saifallah Benjafaar
  - Robert J. Graves
- Charles J. Malmborg
- Ananth Krishnamurthy
- Byung-In Kim
- Gang Meng
- Jing Jia
- Ja-Shen Chen
- Xiao Cai
- Banu Ekren
- Savoye Logistics
- St. Onge Company
- Vanderlande Industries

Three decision levels

- **Strategic – long-term, design oriented – e.g., facility location, equipment selection, distribution network design**

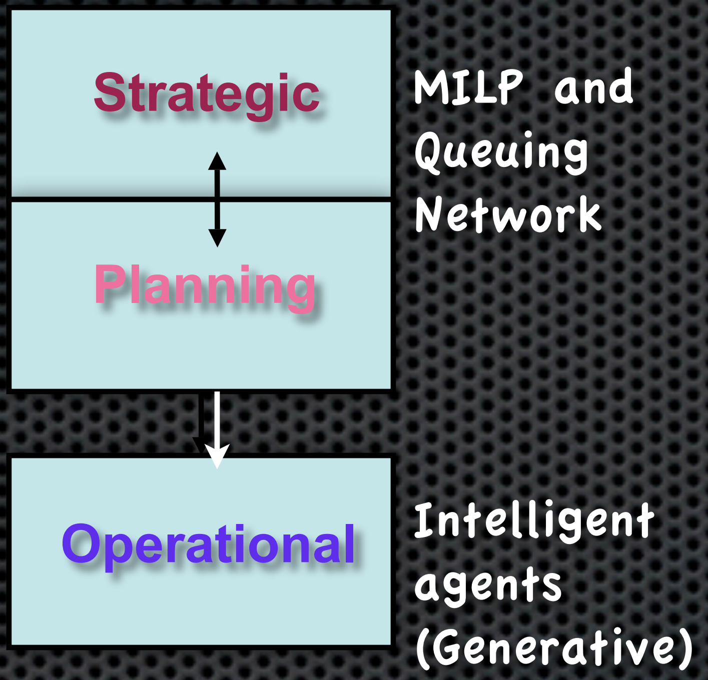
- **Planning – intermediate-term – e.g., aggregate planning, resource requirements planning**

- **Operational – short-term – e.g., daily work-force scheduling**

Three decision levels

Traditional view

Three decision levels

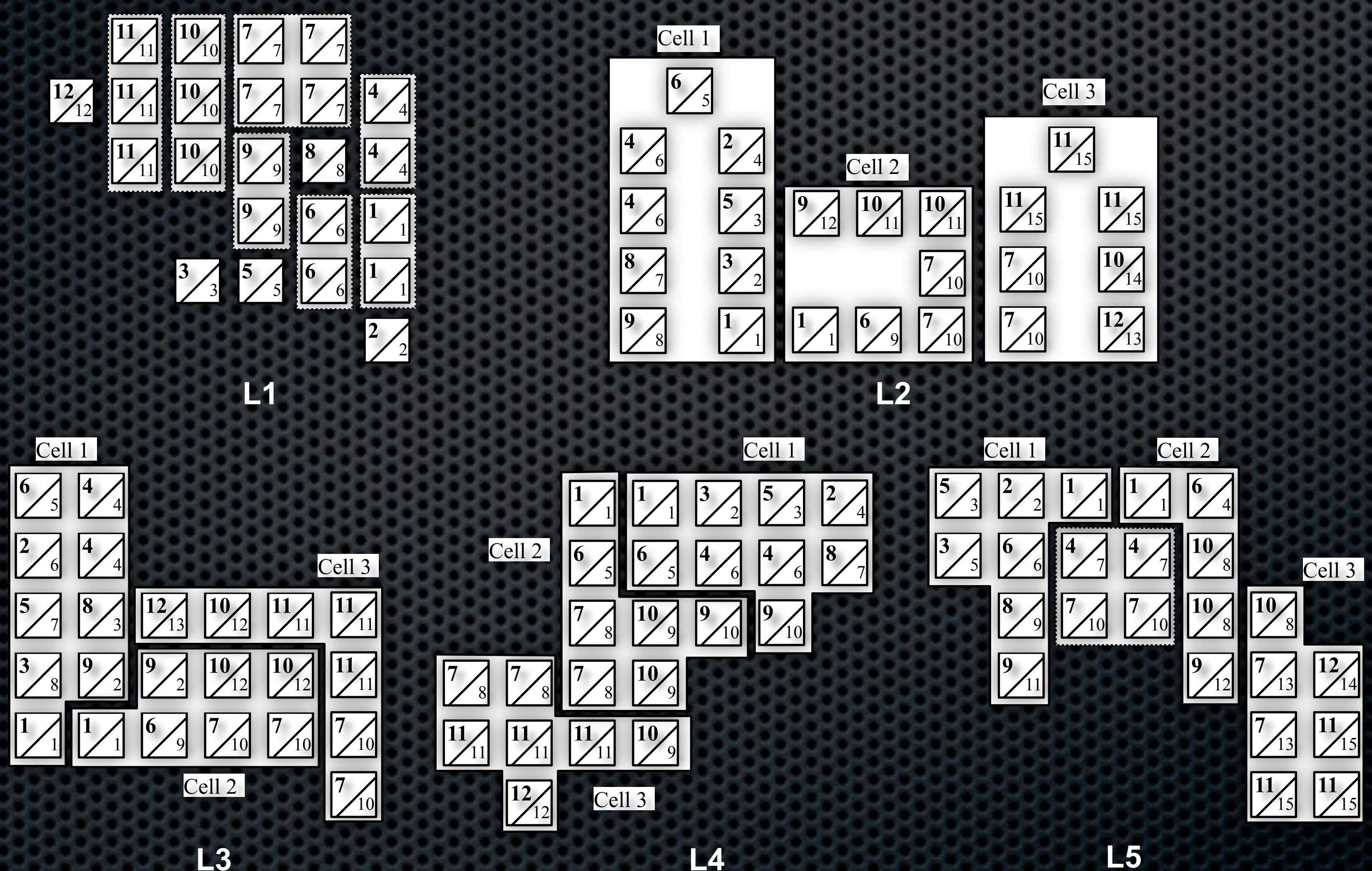


Modern view

Examples

- **Balancing Supply and Demand at Boston Coach – a midsize limousine operator (Forrester, 2005)**
- **Three levels of models developed by IBM Research and BCS (On-Demand Innovation Services)**
  - **Optimization algorithms developed for crew scheduling**
  - **Mid level analysis using stochastic models**
  - **Real-time assignment of drivers and limos to reservation requests using wireless technologies**
- **Results**
  - **Sales ↑10%**
  - **Productivity ↑ 20%**
  - **On-time pick up rating 99%**

Another Example



Design & Operational Criteria for Layout Evaluation

Criteria	Unit Cost	Layout					
		L0	L1	L2	L3	L4	L5
WIP (Planning)	5	29.67	17.87	24.16	20.91	17.88	25.71
MH (Design)	2	30.65	22.75	36.35	33.9	30	20.35
Lateness	10	8.09	0.12	2.58	0.05	0.12	4.49
Overall Cost		290.6	**136.05**	219.3	172.9	150.6	214.2

Criteria	Unit Cost	Layout					
		L0	L1	L2	L3	L4	L5
WIP	1	29.67	17.87	24.16	20.91	17.88	25.71
MH	10	30.65	22.75	36.35	33.9	30	20.35
Lateness	1	8.09	0.12	2.58	0.05	0.12	4.49
Overall Cost		344.3	245.49	390.2	360.0	318	**233.7**

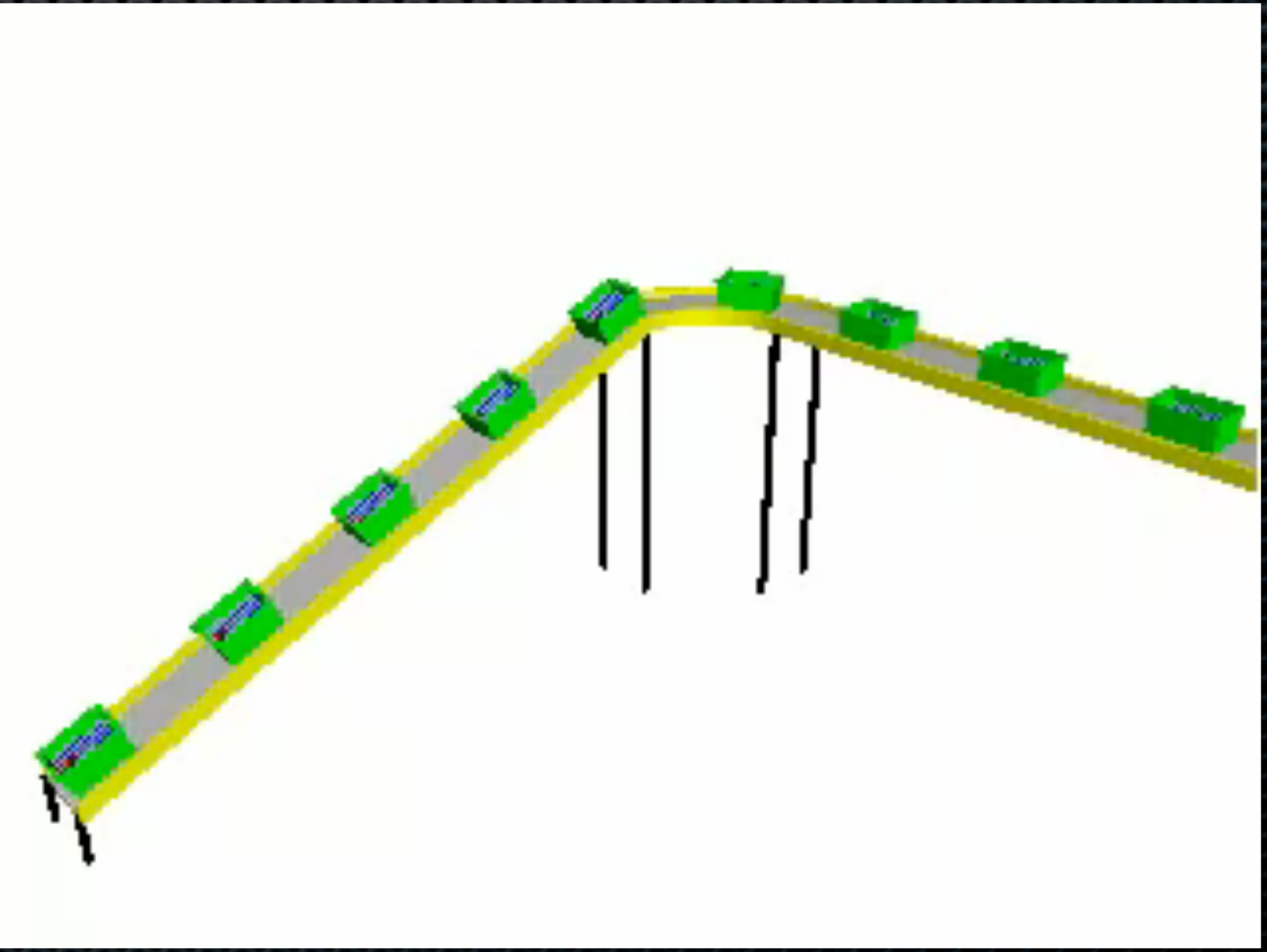
WH Design Prior work (sample)

- **Strategic**
  - **WH aisle configuration - Gue and Meller (2009)**
  - **WH Design - Heragu et al. (2004)**
  - **Design conceptualization (Malmborg et al., 2008)**

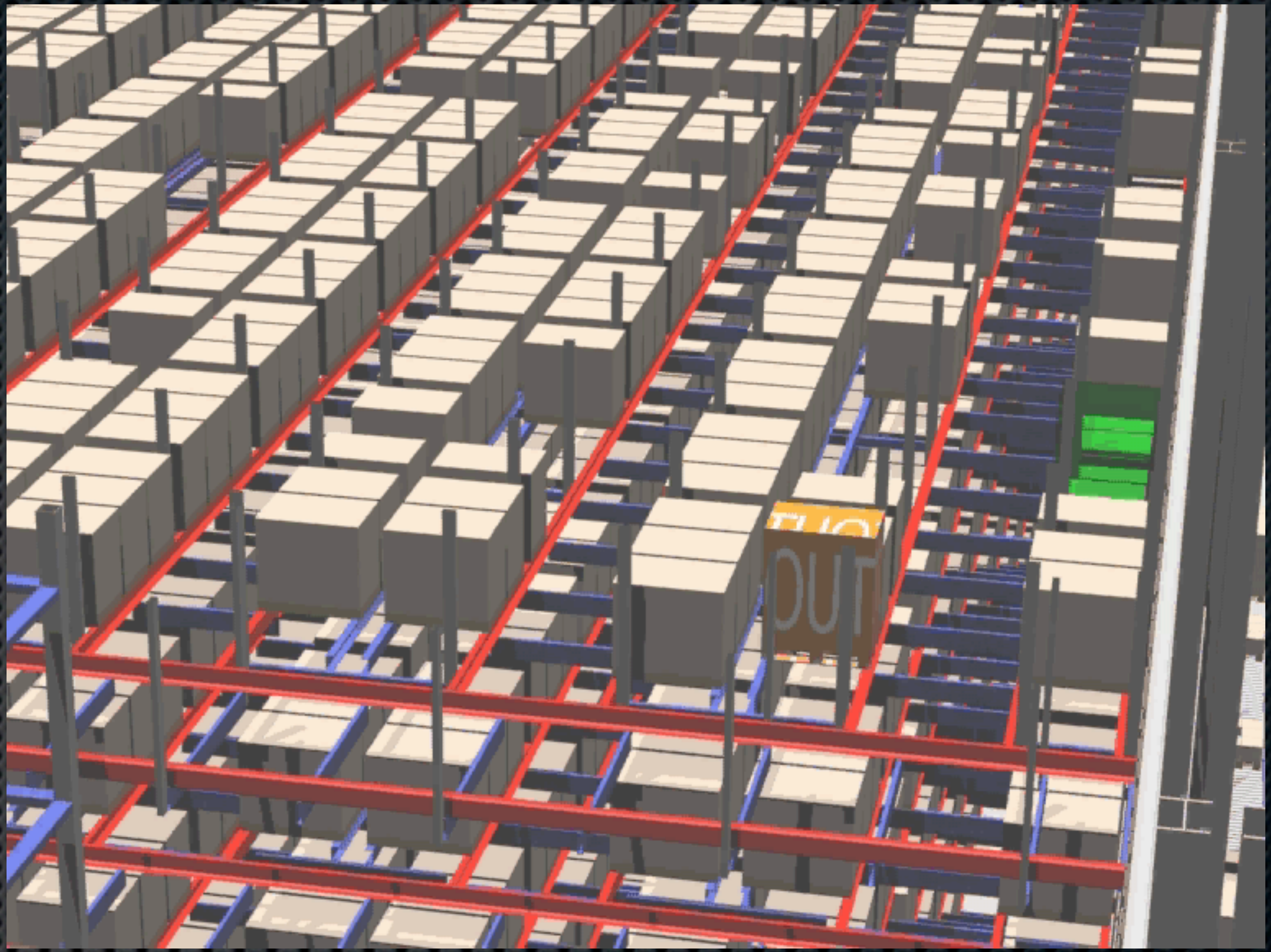
Automated Warehouse

AS/RS

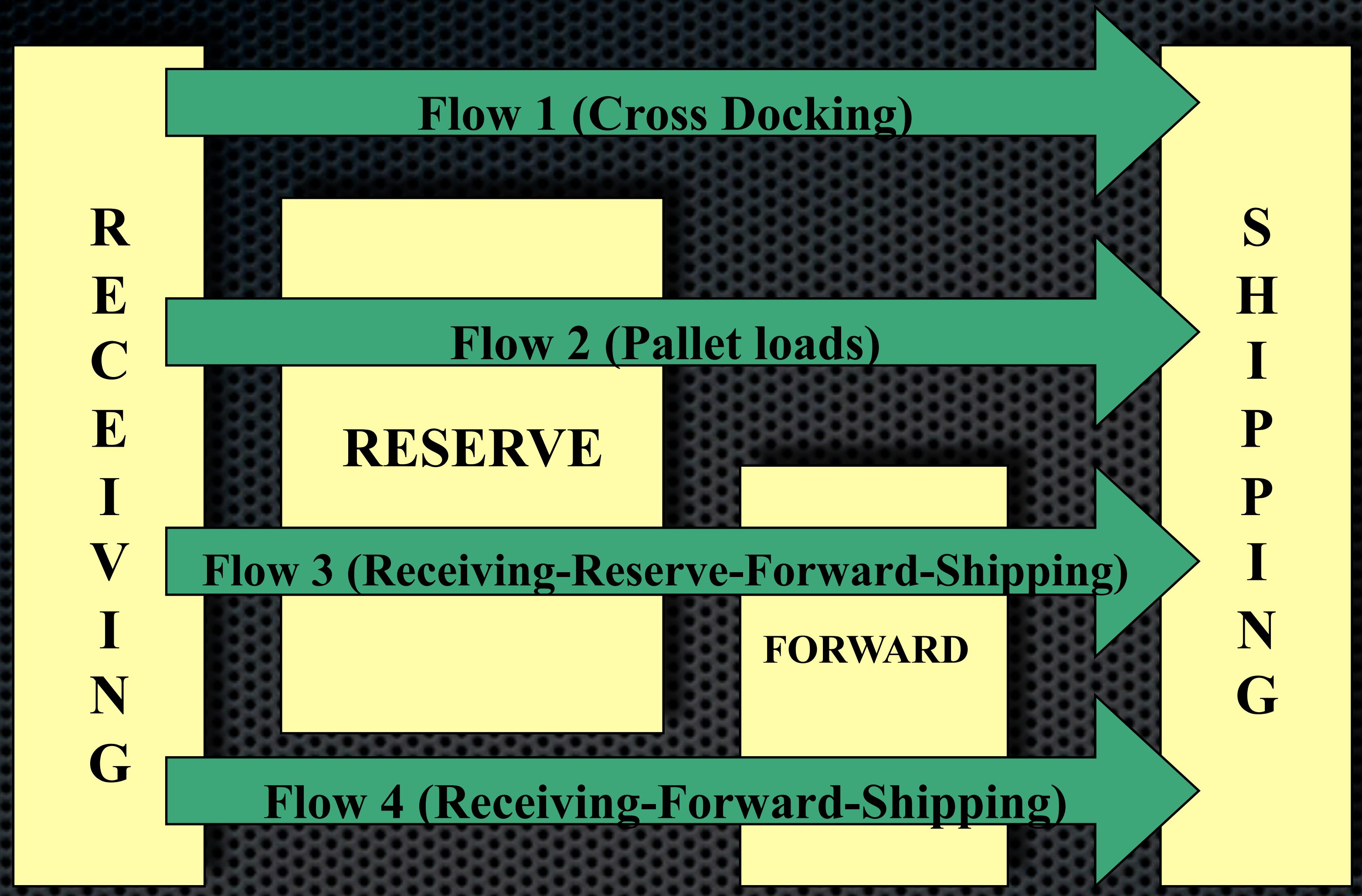
- Has been around for > 40 years
- Efficient for stable, high throughput environments
- Rigid design
- Not inexpensive



AVS/RS



- Relatively new
- 50 installations (all in Europe)
- Flexible design and Modular
- Uses a combination of lifts and vehicles for S/R



Source: Heragu et al. (2005)

Warehouse Design

Heragu et al. (2004)

Modeling Approaches

- **MILP Model**
- **Queuing Network Models**
  - **Open Queuing Network Models**
  - **Closed Queuing Network Models**
  - **Semi-Open Queuing Network Models**
- **Real-Time Decision Models**

MILP Model for W/H Design - Assumptions

- **Available total storage space is known**
- **Expected time a product spends on the shelves is known (referred to as the dwell time)**
- **Cost of handling each product in each flow is known**
- **Dwell time and storage cost have a linear relationship**
- **Annual product demand rates are known**
- **Storage policies and material handling equipment are known and these affect the unit handling and storage costs**

MILP Model

- Minimize storage and handling costs
- Subject to
  - Each storage space can only have one item
  - Assign each item to one of three areas so that space required does not exceed space available
  - Lower and upper bound on space constraints in each area - reserve, forward and crossdock must not be violated

i	number of products, $i=1,2,\ldots,n$,
j	type of material flow, $j=1, 2, 3, 4$,
λ_i	annual demand rate of product i in unit loads,
A_i	order cost for product i,
P_i	price per unit load of product i,
p_i	average percentage of time a unit load of product i spends in reserve area if product is assigned to material flow 3,
$q_{ij}=1$	when product i is assigned to material flow $j=1$, 2 or 4; $\lceil d_i \rceil+1$ when product i is assigned to flow $j=3$, where d_i is the ratio of the size of the unit load in reserve area to that in forward area and $\lceil d_i \rceil$ is the largest integer greater than or equal to d_i,
a,b,c	levels of space available in the vertical dimension in each functional area, $a=$cross-docking, $b=$reserve and $c=$forward,
r	inventory carrying cost rate,
H_{ij}	cost of handling a unit load of product i in material flow j,
C_{ij}	cost of storing a unit load of product i in material flow j per year,
S_i	space required for storing a unit load of product i,
TS	total available storage space,
Q_i	order quantity for product i (in unit loads),
T_i	dwell time (years) per unit load of product i,
LL_{CD}, UL_{CD}	lower and upper storage space limit for the cross-docking area,
LL_F, UL_F	lower and upper storage space limit for the forward area,
LL_R, UL_R	lower and upper storage space limit for the reserve area.

Decision variables:

X_{ij}	1 if product i is assigned to flow type j; 0 otherwise,
α, β, γ	proportion of available space assigned to each functional area, $\alpha=$cross-docking, $\beta=$reserve and $\gamma=$forward.

Parameters and Variables

Model 1:

$$\min\ 2\sum_{i=1}^{n}\sum_{j=1}^{4} q_{ij}H_{ij}\lambda_i X_{ij} + \sum_{i=1}^{n}\sum_{j=1}^{4}\left(q_{ij}C_{ij}Q_i X_{ij}/2\right) \tag{1}$$

$$\sum_{j=1}^{4} X_{ij} = 1 \quad \forall i \tag{2}$$

$$\sum_{i=1}^{n}(Q_i S_i X_{i1}/2) \le a\alpha TS \tag{3}$$

$$\sum_{i=1}^{n}(Q_i S_i X_{i2}/2) + \sum_{i=1}^{n}(p_i Q_i S_i X_{i3}/2) \le b\beta TS \tag{4}$$

MILP Model

$$\sum_{i=1}^{n}((1-p_i)Q_iS_iX_{i3}/2)+\sum_{i=1}^{n}(Q_iS_iX_{i4}/2) \le c\gamma TS \quad (5)$$

$$\alpha+\beta+\gamma=1 \quad (6)$$

$$LL_{CD} < a\alpha\ TS < UL_{CD} \quad (7)$$

$$LL_R < b\beta\ TS < UL_R \quad (8)$$

$$LL_F < c\gamma\ TS < UL_F \quad (9)$$

$$\alpha, \beta, \gamma \ge 0 \quad (10)$$

$$X_{ij} = 0\ or\ 1 \quad \forall i, j. \quad (11)$$

MILP Model (cont.)

WH Operations Prior work (sample)

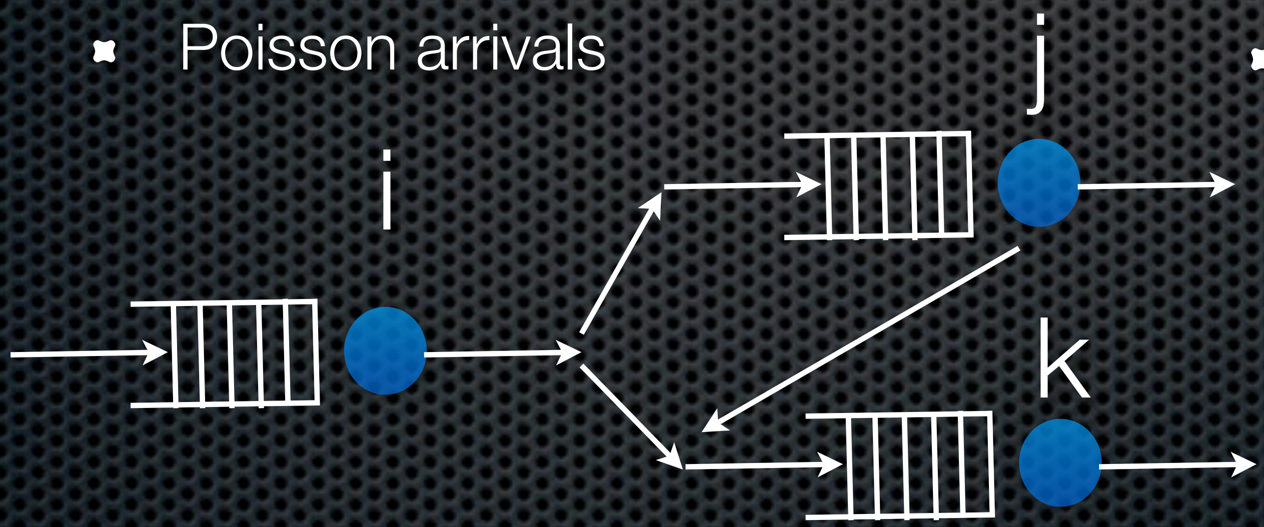
- **Planning**
  - **Storage assignment - Graves et al. (1976)**
  - **Travel time models - Bozer and White (1984)**
  - **Routing - de Koster and Roodbergen (2001)**
  - **Operational policy analysis - Krishnamurthy et al. (2008)**
  - **Order Oriented Slotting Policy - Schuur et al. (2009)**

Modeling Approaches

- Sample MILP Model
- Queuing Network Models
  - Open Queuing Network Models
  - Closed Queuing Network Models
  - Semi-Open Queuing Network Models
- Real-Time Decision Models

Open Queuing Network

- Jackson Network, Jackson (1957) and (1963)
- Product form solution exists for:
  - Poisson arrivals
  - Exponential service times
  - Known routing probabilities
- Three step approach:
  - Solve: $$\hat{\lambda}_k = \hat{\lambda}_{0k} + \sum_{j=1}^{n} \hat{\lambda}_j p_{jk}, \; k=1,2,\ldots,n$$
  - Analyze each station independently as M/M/m queues
  - Combine results



Approximate analysis of more general open networks

- **Seminal paper by Whitt (1983)**
- **Two limiting Assumptions: FCFS, infinite queue size, but …**
- **All we need to know about inter-arrival and service times are the first two moments!**
- **Reasonably accurate results, FAST!**
- **How does Whitt's Queuing Network Analyzer work?**
- **We simply solve two sets of equations simultaneously!**

Comprehensive Model

- **First two moments of external arrival rate for each product**
- **First two moments of service time for each processing operation**
- **Set-up Times**
- **Batch Size - Process as well as Transfer**
- **Machine/Material Handling Device Failures**
- **Empty Travel of Material Handling Device**

Approx. Analysis of Open Networks

- **Set up equations to determine scv of aggregate arrival at each station**

$$c_{aj}^2 = a_j + \sum_{i=1}^{n} c_{ai}^2 b_{ij}, \quad 1 \le j \le n$$

- **Merging:**

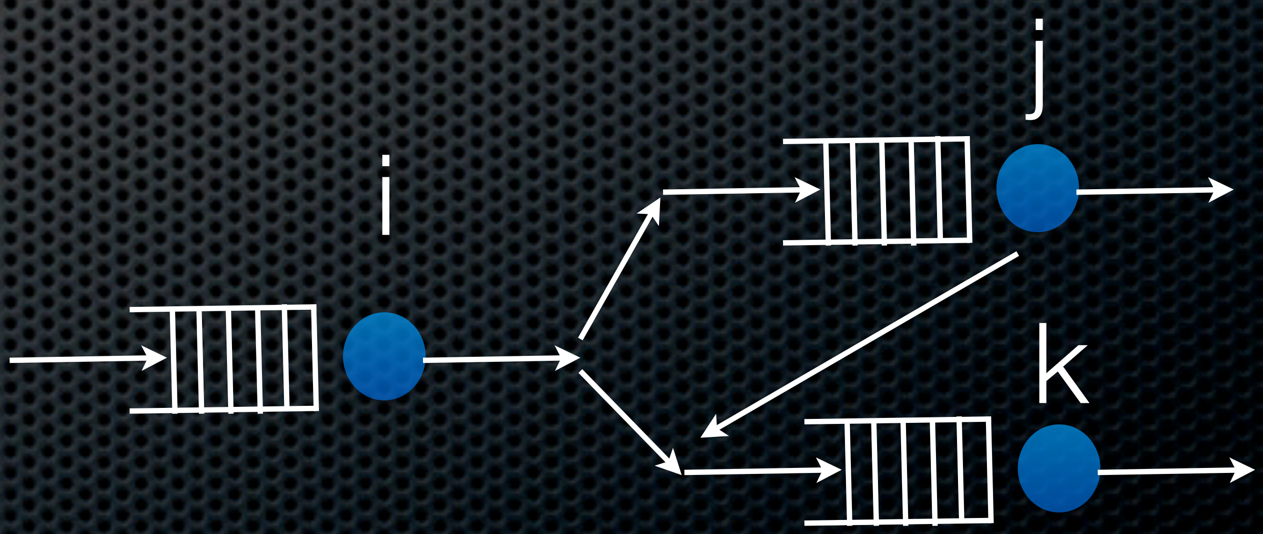
$$c_a^2 = w \sum_i \left(\frac{\lambda_i}{\sum_k \lambda_k} \right) c_i^2 + 1 - w$$

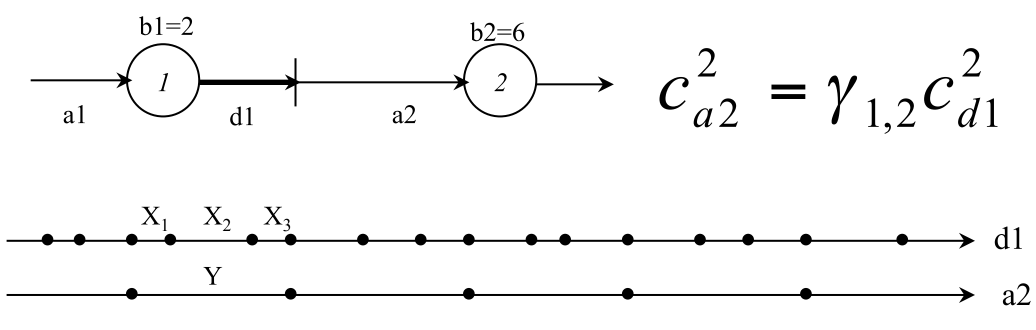
- **Splitting:**

$$c_i^2 = p_i c^2 + 1 - p_i$$

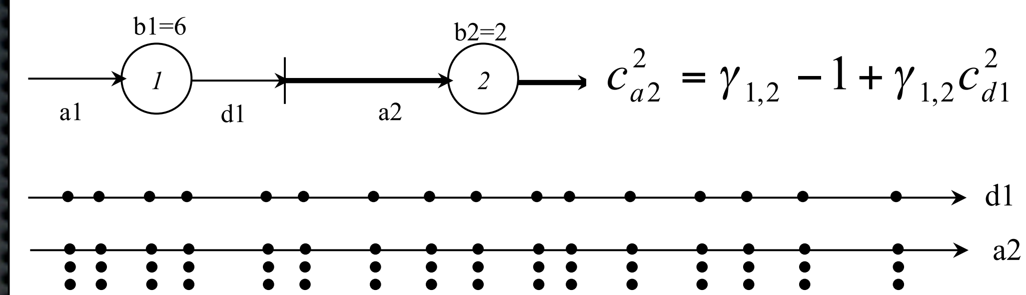
- **Departures:**

$$c_d^2 = 1 + (1-\rho^2)(c_a^2 - 1) + \frac{\rho^2}{\sqrt{m}}(c_s^2 - 1)$$





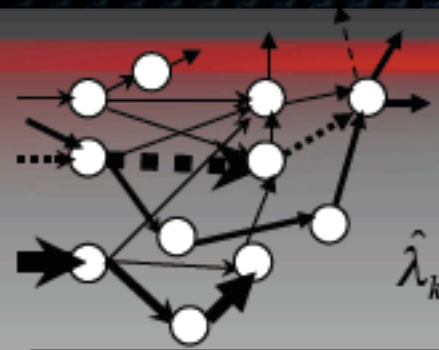
Batch: arrivals at machine 2 when relative batch size



Burst: arrivals at machine 2 when relative batch size is greater than 1

Operations	First Moment	Second Moment
d1 // a2 → batch/bu	$\lambda_{a2} = \gamma_{1,2}\lambda_{d1}$	$c_{a2}^2 = \max\{0, \gamma_{1,2} - 1\} + \gamma_{1,2} c_{d1}^2$

Batch/Burst: The 4th Network Operation



$$\hat{\lambda}_k = \hat{\lambda}'_{0k} + \sum_{j=1}^{n} \hat{\lambda}_j p_{jk} \gamma_{jk} \qquad c_{ak}^2 = a_k + \sum_{i=1}^{n} c_{ai}^2 b_{ik}$$

Operation	First moment	Second Moment
departure	$\lambda_d = \lambda_a$	$c_d^2 = 1 + (1-\rho^2)(c_a^2 - 1) + \frac{\rho^2}{\sqrt{m}}(c_s^2 - 1)$
split	$\lambda_i = p_i \lambda_a$	$c_i^2 = p_i c^2 + 1 - p_i$
superposition	$\lambda_a = \sum_i \lambda_i$	$c_a^2 = w \sum_i \left(\frac{\lambda_i}{\sum_k \lambda_k} \right) c_i^2 + 1 - w$
batch/burst	$\lambda_{a2} = \gamma_{1,2} \lambda_{d1}$	$c_{a2}^2 = \max\{0, \gamma_{1,2} - 1\} + \gamma_{1,2} c_{d1}^2$

Parametric Decomposition: With Batching

C	M	Rho			Lq			Wq		
		FLQ	MPA	Simu.	FLQ	MPA	Simu.	FLQ	MPA	Simu.
1	1	0.635	0.860	0.858	1.90	7.58	7.38	1.01	4.48	4.38
1	2	0.876	0.876	0.886	10.18	11.07	11.41	2.82	3.09	3.22
1	3	0.724	0.724	0.704	2.95	3.32	3.36	1.27	1.48	1.56
1	4	0.953	0.953	0.935	27.88	32.18	30.68	8.83	10.24	9.85
1	5	0.990	0.990	0.981	181.8	215.1	134.4	45.20	53.53	33.55
2	1	0.988	0.763	0.749	92.72	3.99	3.83	55.60	2.31	2.21
2	2	0.950	0.950	0.944	21.77	22.61	22.04	11.90	12.38	12.07
2	3	0.895	0.895	0.877	13.49	14.80	13.28	5.35	5.92	5.32
2	4	0.795	0.796	0.784	7.23	7.72	6.86	2.39	2.57	2.27
3	1	0.931	0.931	0.929	15.47	18.54	19.04	4.93	5.97	6.14
3	2	0.967	0.967	0.945	38.31	44.56	21.36	16.24	18.95	9.05
3	3	0.800	0.800	0.807	4.11	4.63	4.23	2.65	3.07	2.70
3	4	0.978	0.978	0.964	72.99	83.89	39.33	19.73	22.72	10.60

Results: MPA - A PD tool

Modeling Approaches

- **Sample MILP Model**
- **Queuing Network Models**
  - **Open Queuing Network Models**
  - **Closed Queuing Network Models**
  - **Semi-Open Queuing Network Models**
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Closed Queuing Networks

- **Gordon-Newell networks (1967)**
  - **Product form solution exists for closed network with finite population**
  - **Buzen’s algorithm (1973) to compute normalization constant in above product form solution**
- **BCMP Networks (Baskett et al. 1975)**
  - **Product form solution exists for special cases of more general network**

Closed Queuing Network

- **Mean Value Analysis**
  - **Based on arrival theorem (the distr. seen by an arriving job is the same as that seen by an observer seeing the same system with this job removed)**
  - **Iterative approach**

$$W_{ij} = \frac{1}{\mu_i^j} + \left(\frac{N^j - 1}{N^j}\right)\left(\frac{L_{ij}}{\mu_i^j}\right) + \sum_{r \neq j} \frac{L_{ir}}{\mu_i^r} \quad i = 1, 2, \ldots, m,\ j = 1, 2, \ldots, r$$

$$X_j = \frac{N^j}{\left(\sum_{i=1}^{m} v_{ij} W_{ij}\right)}$$

$$L_{ij} = X_j (v_{ij} W_{ij})$$

Modeling Approaches

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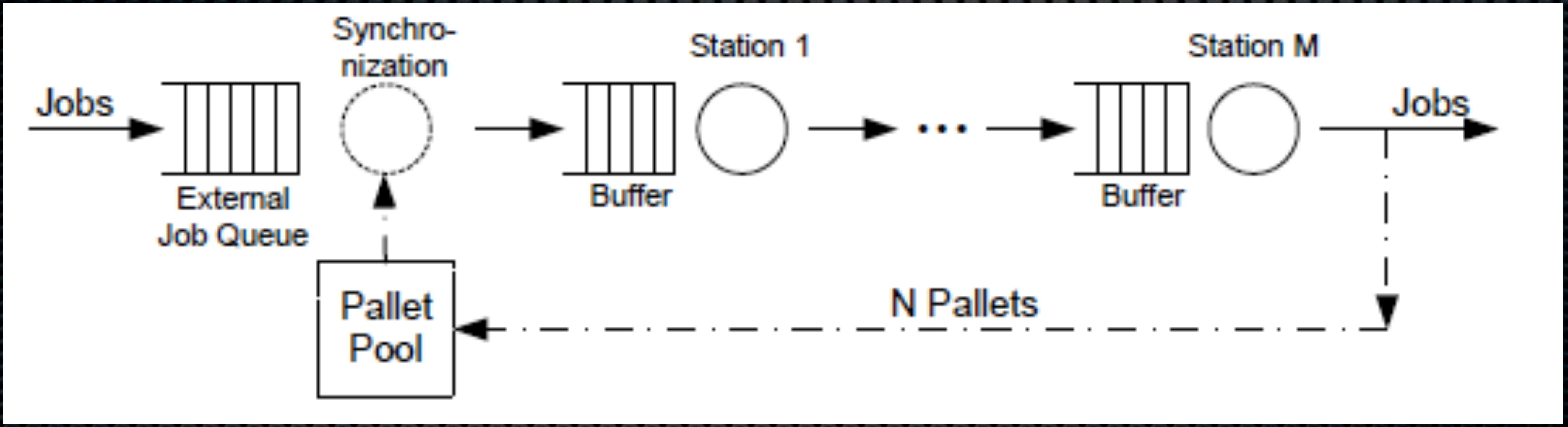
Semi-Open Queuing Network (SOQN)



- **Stability condition (Dallery 1990, Buitenhek et al., 2000) — arrival rate < max. throughput rate of the network**

Why SOQN?

Comparison with open and closed queuing models

λ =10 μ_1=12 μ_2=13 μ_3=15 μ_4=14 μ_5=13.5 N=15 (for SOQN)

L_n			W_s		
SOQN	OQN	CQN	SOQN	OQN	CQN
13.16	15.69	15	151.17	94.14	86.28

Matrix Geometric Method

- **Invented by Marcel F. Neuts, 1980**
- **Efforts abound on development and applications, e.g., Takahashi (1981), Latouche (1999), Alfa (2002)**
- **Follow a standard procedure to solve the generator matrix**

Matrix Geometric Method

$$
Q = \begin{array}{c} \\ (0,k) \\ (1,k) \\ (2,k) \\ \vdots \end{array}
\left[\begin{array}{cccc} (0,k) & (1,k) & (2,k) & \cdots \\ B_0 & C & 0 & \cdots \\ A & B & C & \cdots \\ 0 & A & B & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{array}\right],
$$

where $B_0 = \begin{bmatrix} -\lambda & & & \\ \mu_2 & -(\mu_2+\lambda) & & \\ & \ddots & \ddots & \\ & & \mu_2 & -(\mu_2+\lambda) \end{bmatrix}$, $C = \lambda I$, $A = \begin{bmatrix} 0 & \mu_1 & & & \\ & 0 & \mu_1 & & \\ & & \ddots & \ddots & \\ & & & 0 & \mu_1 \\ & & & & 0 \end{bmatrix}$,

and $B = \begin{bmatrix} -(\mu_1+\lambda) & & & & \\ \mu_2 & -(\mu_1+\mu_2+\lambda) & & & \\ & \ddots & \ddots & & \\ & & \mu_2 & -(\mu_1+\mu_2+\lambda) & \\ & & & \mu_2 & -(\mu_2+\lambda) \end{bmatrix}$.

- **Construct a generator matrix for the Markov Process (often in the form of quasi-birth-death process);**
- **Identify repetitive structure**
- **The stationary probability vectors form a matrix geometric series,** $\pi_{n+1} = \pi_n R,$ **for n ≥ some K**
- **Solve for the stationary probabilities**

Results

Table 1: Mean queue lengths in a five-station Markovian SOQN

	EJQ	EL1	EL2	EL3	EL4	EL5
Simulation	29.84	6.48	5.66	2.96	2.73	2.77
95% CI	±3.61	±0.03	±0.02	±0.03	±0.01	±0.01
Our Method	31.90	6.48	5.67	2.97	2.74	2.79
% error	6.9	0	0.18	0.34	0.37	0.72
Buitenhek	39.12	6.56	5.68	2.97	2.75	2.79
% error	31.10	1.23	0.35	0.34	0.73	0.72

Table 2: Comparison of three methods

	EJQ	EL1	EL2	EL3	EL4	EL5
Simulation	16.93	2.21	1.66	2.61	1.70	1.14
95% CI	±1.16	±0.01	±0.01	±0.03	±0.01	±0.02
Our method	18.72	2.09	1.70	2.62	1.67	1.23
% error	11	-5	2	0	-2	8
Buitenhek	20.3	2.1	1.71	2.68	1.71	1.15
% error	20	-5	4	3	1	1
Dallery	22.77	2.15	1.74	2.66	1.72	1.18
% error	34	-3	5	2	1	4

More Results

Table 3: A multi-class, general pallet SOQN experiment

Overall	**EJQ**	**EL1**	**EL2**	**EL3**	**EL4**	**EL5**	**EL6**
Simulation	18.33	2.07	4.11	2.43	4.02	1.82	5.31
95% CI	±0.74	±0.02	±0.01	±0.02	±0.02	±0.02	±0.03
Our Method	16.43	2.04	4.09	2.35	3.93	1.80	5.38
% error	-10	-1	0	-3	-2	-1	1
Buitenhek's	10.73	2.05	3.94	2.33	3.74	1.83	5.02
% error	-41	-1	-4	-4	-7	1	-5

Class 1	**EJQ**	**EL1**	**EL2**	**EL3**	**EL4**	**EL5**	**EL6**
Simulation	2.89	0.40	1.23	N/A	N/A	0.53	0.99
95% CI	±0.11	±0.01	±0.02	N/A	N/A	±0.01	±0.02
Our Method	2.59	0.40	1.42	N/A	N/A	0.54	1.21
% error	-10	0	15	N/A	N/A	2	22
Buitenhek's	1.69	0.40	1.08	N/A	N/A	0.55	0.98
% error	42	0	-12	N/A	N/A	4	-1

Design Conceptualization

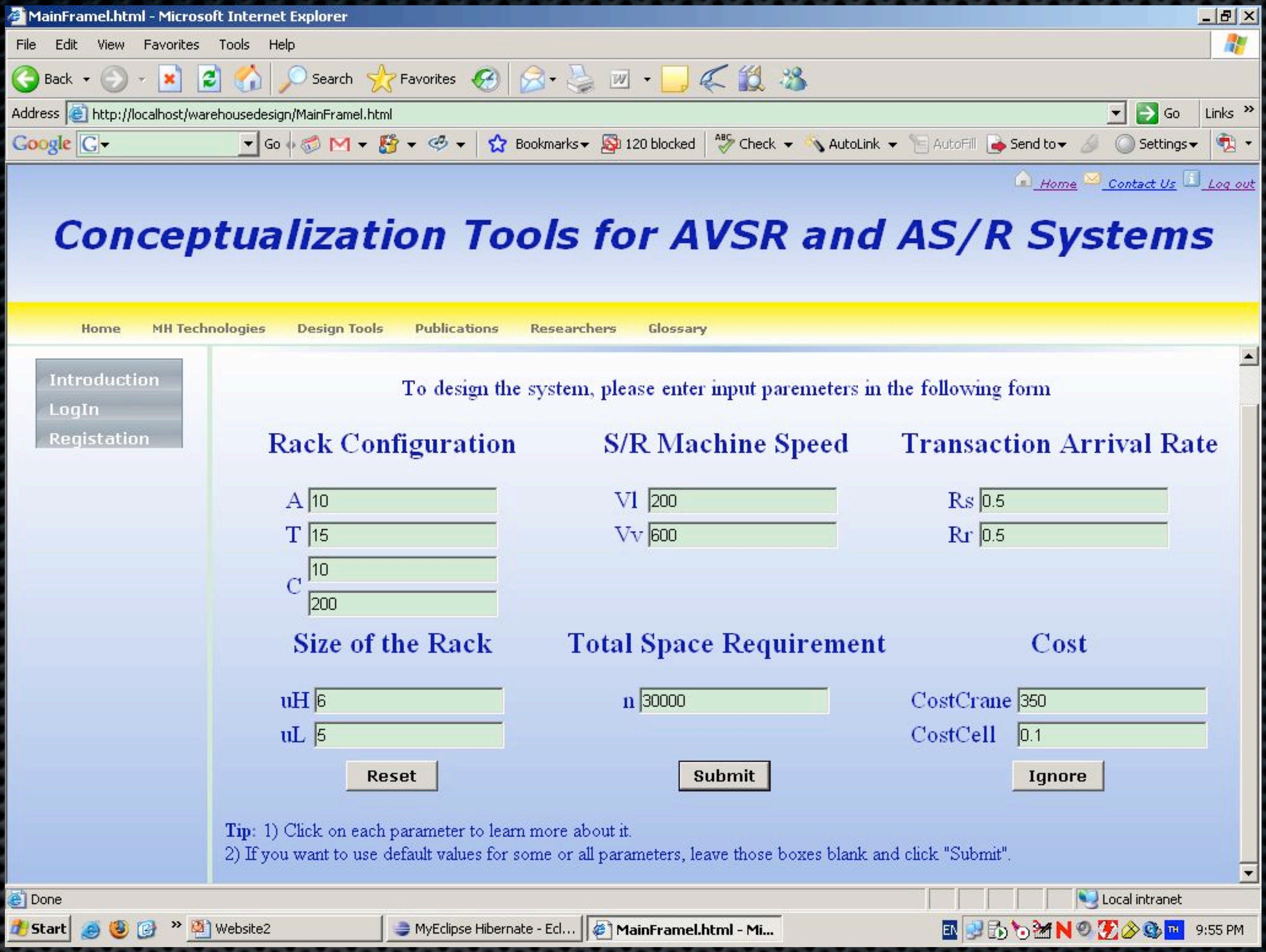
- What is it?
  - Sizing
  - Technology choice
  - Configuration
- How is it done?
  - Rules of thumb
  - Experience with past projects
  - Simulation used to verify one or two designs

- Is the AVS/RS or AS/RS better for a given scenario?
- For a given warehouse application, how should the reserve area be configured? How many aisles, columns and levels are required?
- How many autonomous devices (cranes, lifts and vehicles) are required to meet the requirements of throughput capacity, cycle times, and S/R device utilizations?

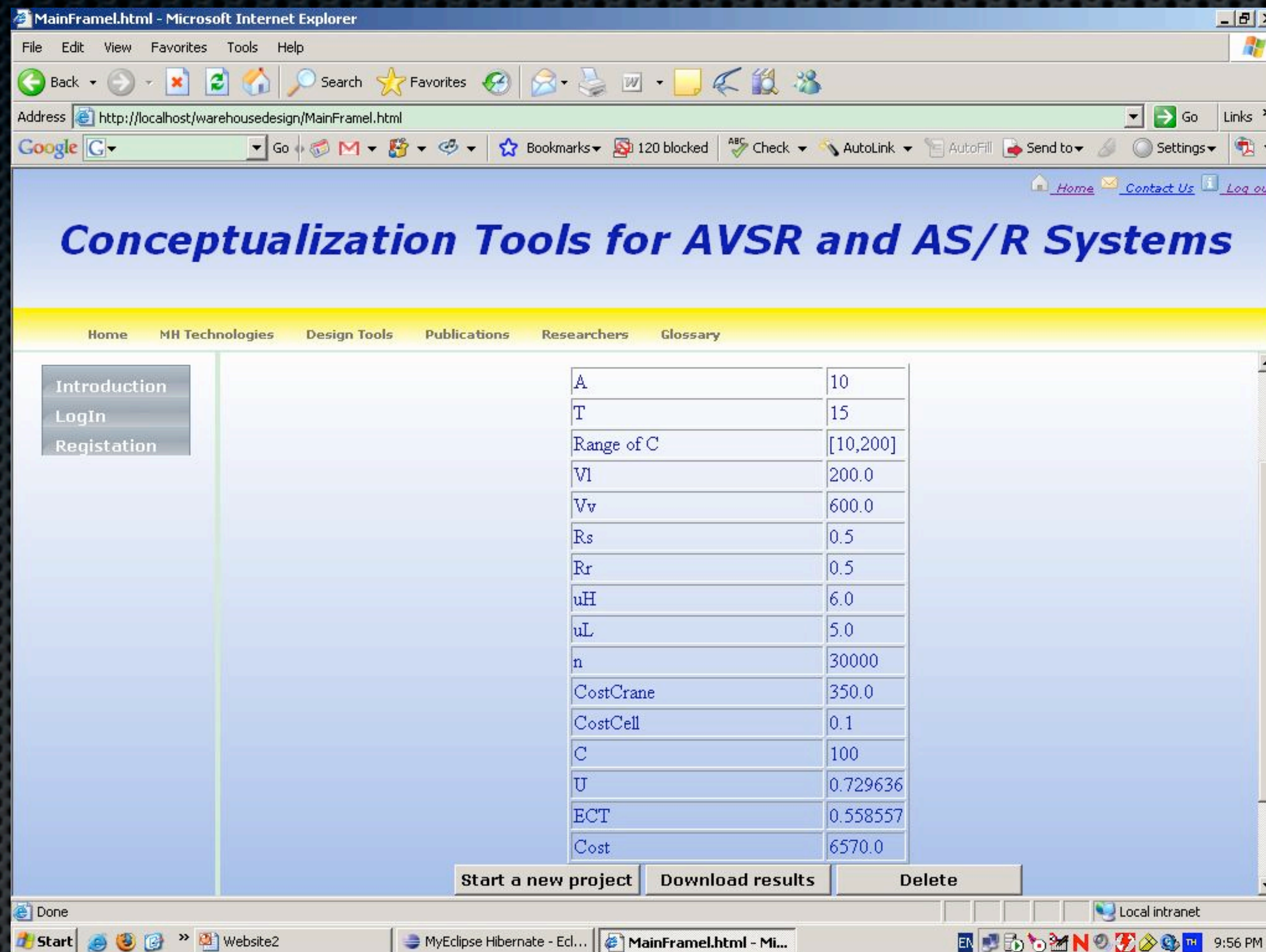
Sample Design Questions

- Should the high-bay area be an integrated entity, or should it be divided into zones (based on aisles, columns or tiers)? If it is the latter, how should the automated devices be allocated to the different zones?
- How many input/output (I/O) locations should a high bay area have and what are the optimal locations for a given rack configuration and performance requirement?
- Should the high bay area have allowances for intermediate buffers, and if so, how large should they be and where should they be located? What reductions in cycle times are obtained by introducing such buffers?
- Where should S/R devices idle after processing storage or retrieval transactions? Should they dwell at the point of service completion, in the high-bay area for a storage, and at the I/O points for retrievals?

More Design Questions



Online Design Conceptualization Tool


MainFramel.html - Microsoft Internet Explorer
File Edit View Favorites Tools Help
Back Search Favorites
Address http://localhost/warehousedesign/MainFramel.html
Go Links
Google Go Bookmarks 120 blocked Check AutoLink AutoFill Send to Settings
Home Contact Us Log out
Conceptualization Tools for AVSR and AS/R Systems
Home MH Technologies Design Tools Publications Researchers Glossary
Introduction
LogIn
Registation
A 10
T 15
Range of C [10,200]
Vl 200.0
Vv 600.0
Rs 0.5
Rr 0.5
uH 6.0
uL 5.0
n 30000
CostCrane 350.0
CostCell 0.1
C 100
U 0.729636
ECT 0.558557
Cost 6570.0
Start a new project
Download results
Delete
Done
Local intranet
Start
Website2
MyEclipse Hibernate - Ecl...
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Results

Modeling Approaches

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Real-time WH Operations Prior work (sample)

- **Operational**
  - **Kim et al., (2002, 2004, 2005)**
  - **Graves et al., (2008)**

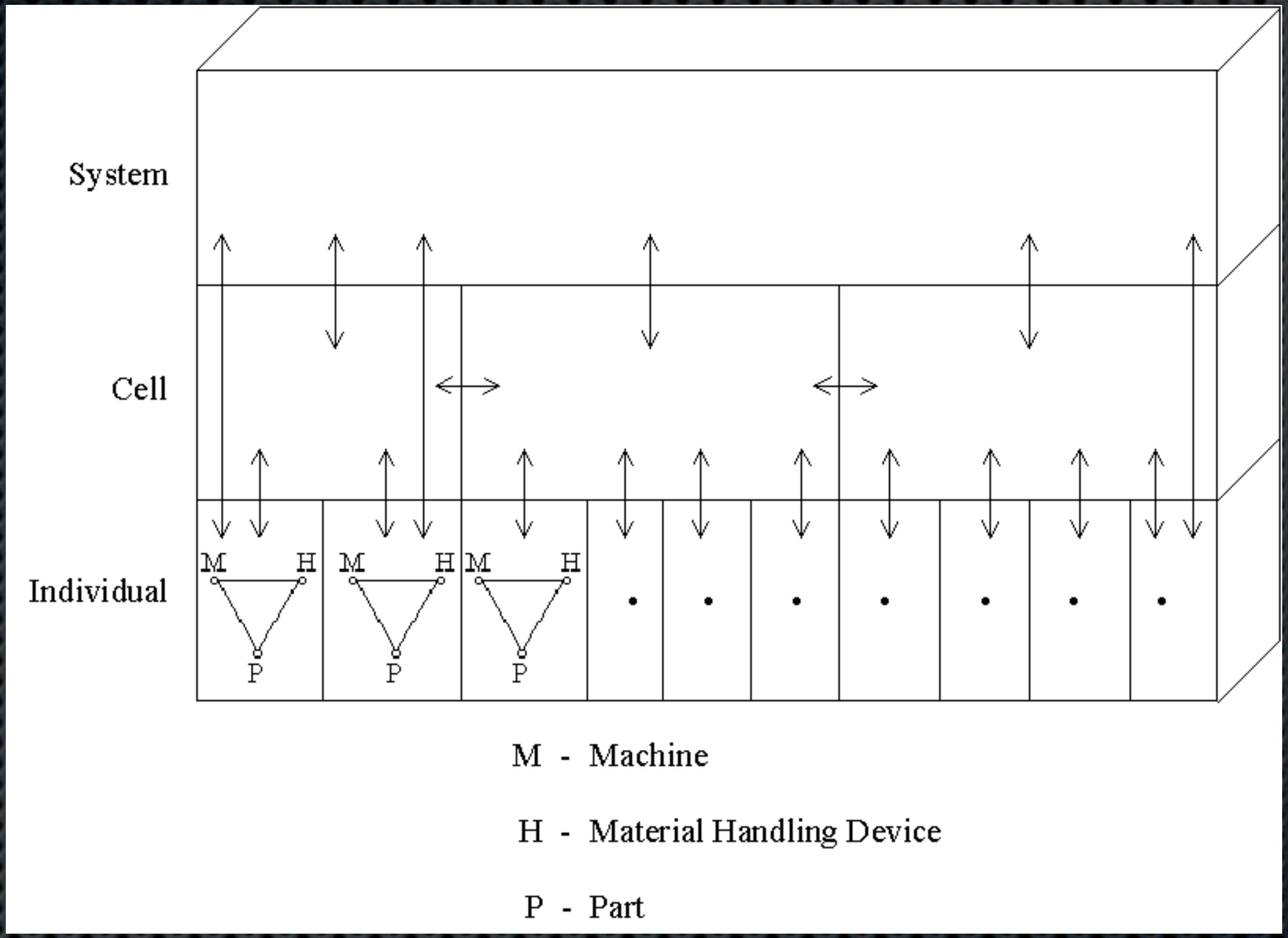
Holonic Modeling Framework

- **Holons behave partly as ‘wholes’ and wholly as ‘parts’ (Vamos, 1983)**
- **Holons make autonomous decisions**
- **Higher level controllers**
- **Guidelines and system wide constraints**

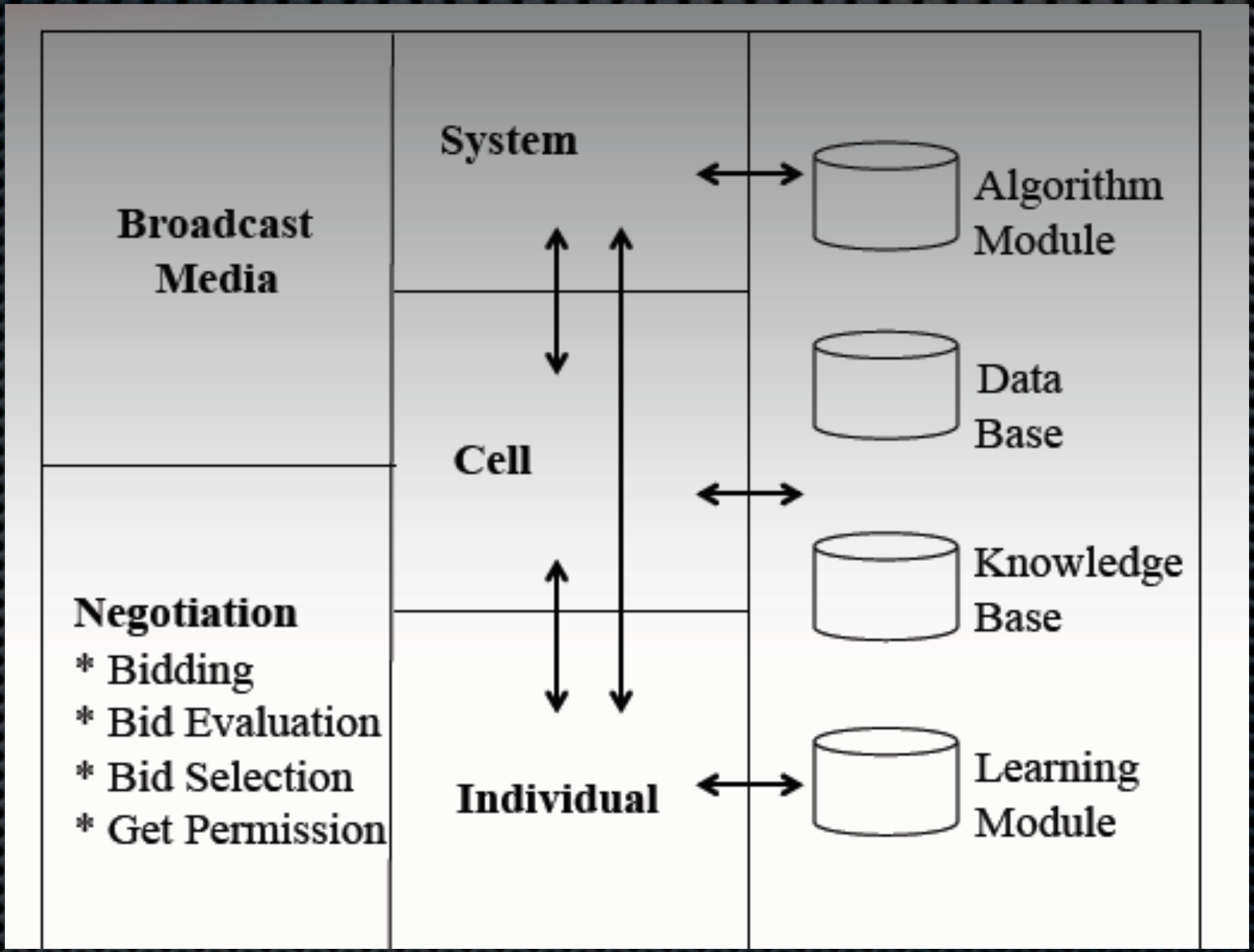
Intelligent Agents

- **Intelligent agents representing**
  - **Entities and resources**
  - **Functioning cooperatively**
  - **Accomplishing individual, cell-wide and system-wide goals**

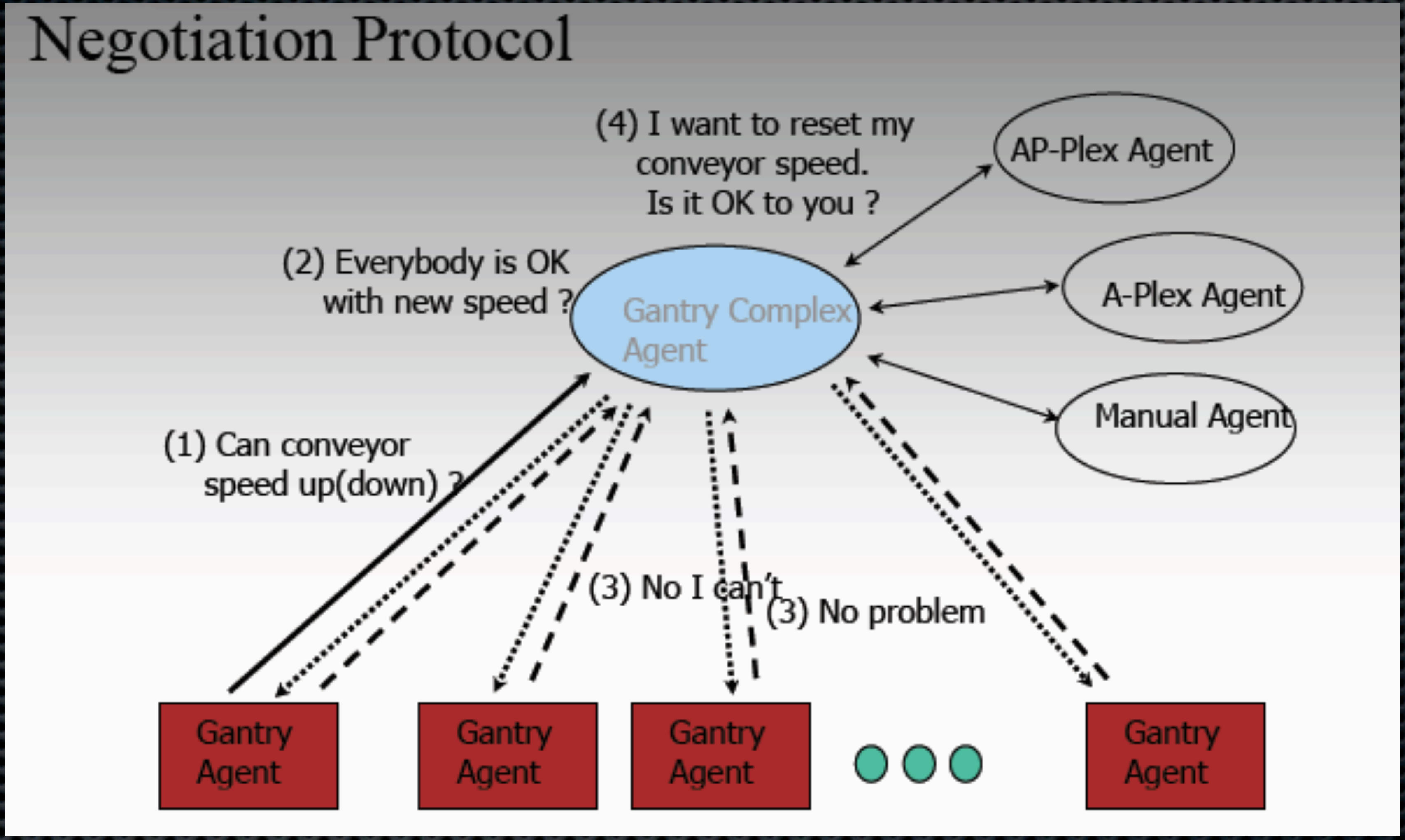
Holonic Framework



Decision Making Modules



Dynamic Conveyor Speed Adjustment

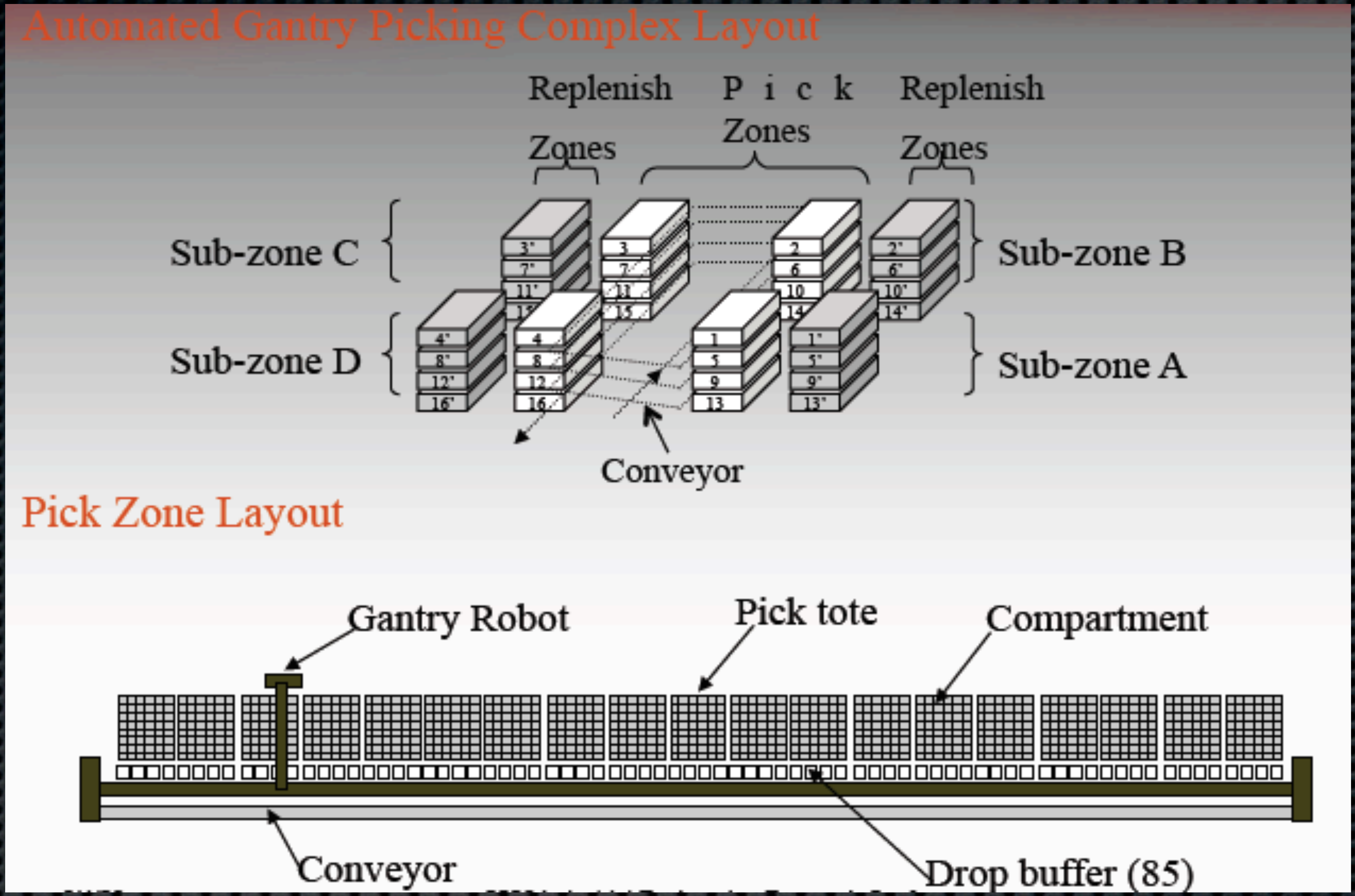


Characteristics of an IA approach

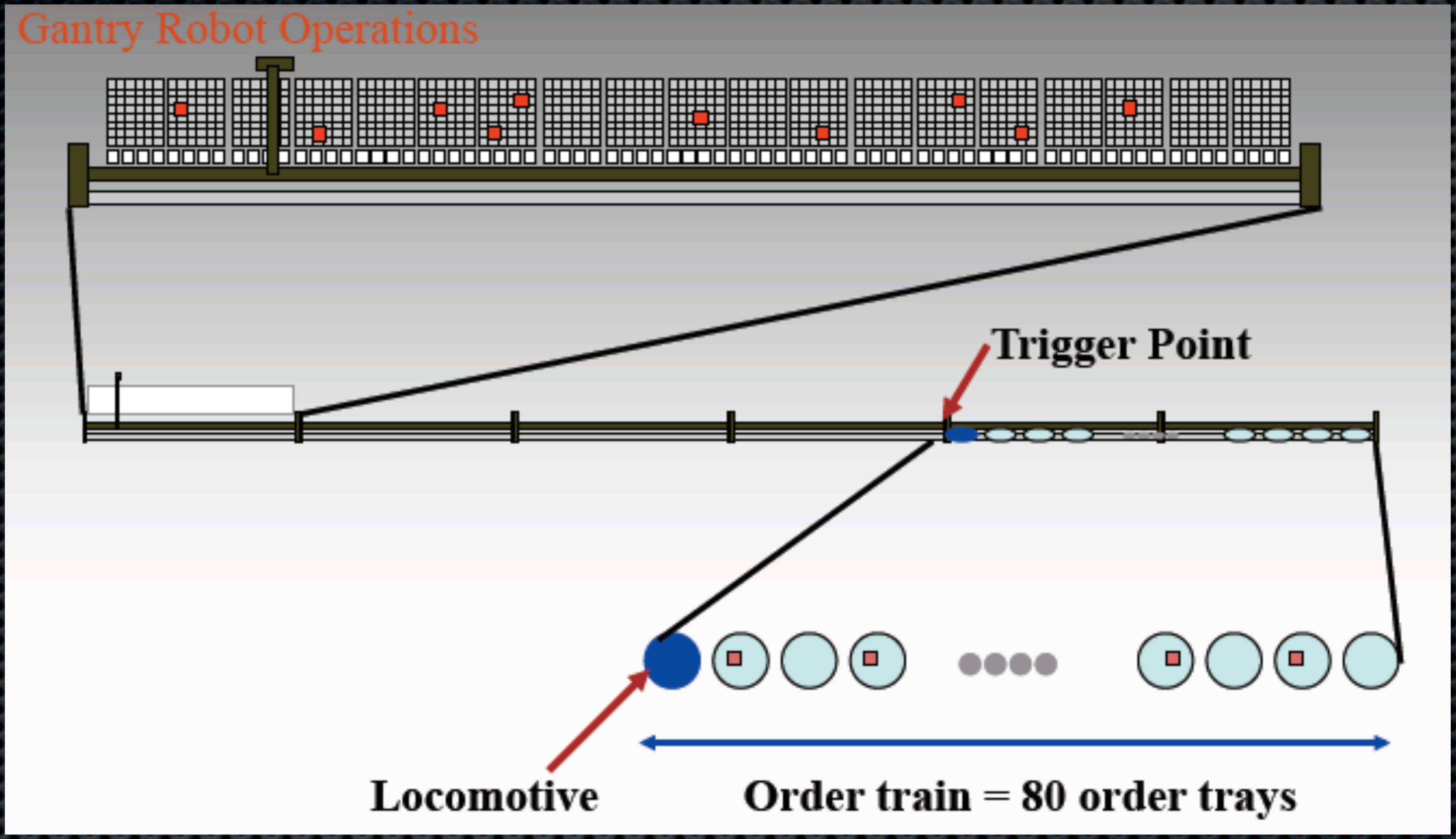
- **Flexibility and Reconfigurability**
- **Robustness against External and Internal Disturbances**
- **Scalability**
- **Timeliness**
- **Globally Optimized Solution**
- **Predictability**
- **Stability**

Intelligent Agents

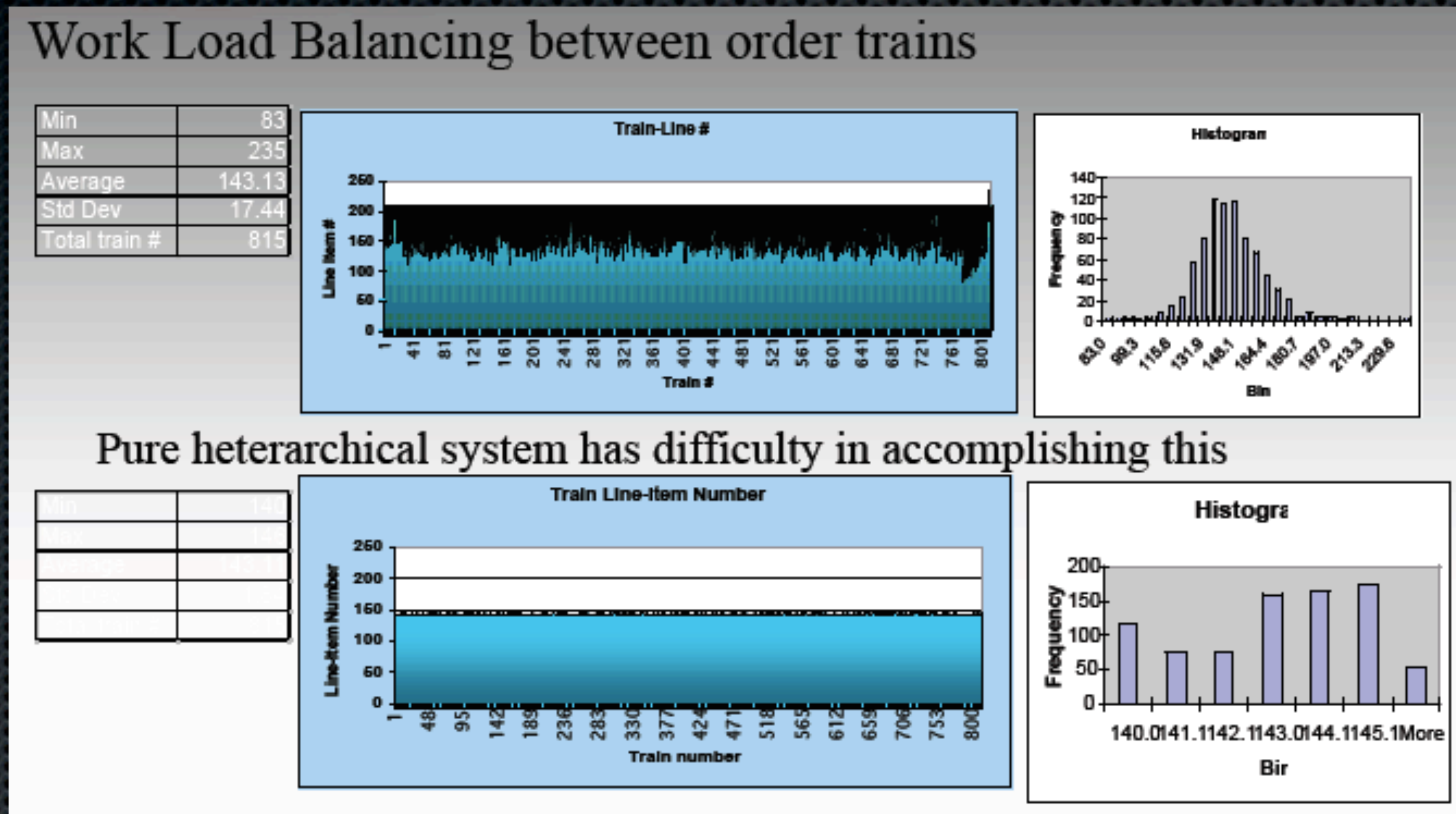
- **A High Speed Automated Warehouse**
- **Warehouse: designed to handle more than 8000 product types**
- **Gantry Picking Complex (GPC): 16 pick zones**
- **Storage Capacity: 437,760 SKUs**
- **Order characteristics: 65,000 orders (117,000 line items)/day**
- **OAPS (Order Analysis and Planning System)**
- **FSS (Finite Scheduling System)**



Automated CD Picking System



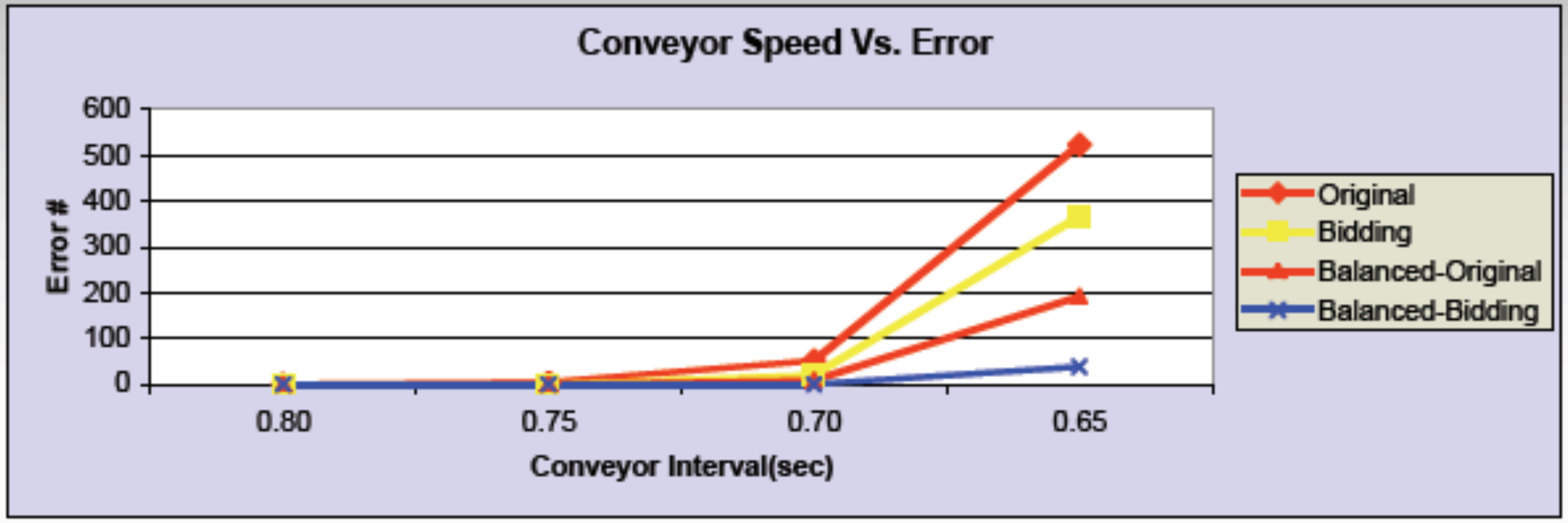
Order Picking



IA vs Heterarchical System

Simulation Results with various conveyor speeds

Conveyor Interval	Original Error	Bidding Error	Balanced Original Error	Balanced Bidding Error
0.80	0	0	0	0
0.75	4	0	0	0
0.70	51	20	9	0
0.65	523	367	191	39



Throughput improvement in the system by balancing workload and using bidding, 12.5 %(0.10/0.80) easily seen

$$\text{Objective: } \min \; MaxTSP$$

Linear Objective

$$\text{constraints: } \sum_{j \ge i}^{N_p+1} X_{pij} \le q_{pi} \quad \text{for each } i, p \qquad (1)$$

Capacity Constraint

$$\sum_{p=1}^{16} \sum_{i \in L_k} \sum_{j \ge i}^{N_p+1} X_{pij} = O_k \quad \text{for each } k \qquad (2)$$

Order fulfillment

$$MaxTSP \ge \sum_{i=1}^{N_p-1} \sum_{j=i}^{N_p} C_{pij} X_{pij} \quad \text{for each } p \qquad (3)$$

Maximum movement time

$$\sum_{j=i+1}^{N_p+1} X_{pij} \le 1 \quad \text{for each } i, p \qquad (4)$$

$$\sum_{i=0}^{j-1} X_{pij} \le 1 \quad \text{for each } j, p \qquad (5)$$

$$\sum_{i=0}^{j-1} X_{pij} = \sum_{k=j+1}^{N_p+1} X_{pjk} \quad \text{for each } j, p \qquad (6)$$

Arrival = Departure ≤ 1
Cycling prevention

$$\sum_{j=1}^{N_p} X_{p0j} = 1 \quad \text{for each } p \qquad (7)$$

Start from source

$$\sum_{i=1}^{N_p} X_{piN_p+1} = 1 \quad \text{for each } p \qquad (8)$$

End at sink

$$\sum_{j \ge i}^{N_p+1} X_{pij} + M(1 - Z_{pii}) \ge 1 \quad \text{for each } i, p \qquad (9)$$

$$\sum_{j \ge i}^{N_p+1} X_{pij} - M(Z_{pii}) < 1 \quad \text{for each } i, p \qquad (10)$$

$$X_{pii} = \sum_{j \ge i}^{N_p+1} X_{pij} - Z_{pii} \quad \text{for each } i, p \qquad (11)$$

Sub tour Elimination

$$X_{pij} = \text{non negative integer} \qquad (12)$$

Model in IA based Control System

- Chromosome String: feasible solution for the problem
- Letter in a String: compartment ID for each line item
- String length: number of line items in an order train
- Candidate Numbers for a letter: candidate compartments
- Objective Fitness Function: Max (movement time for 16 gantry robots)

10	2	801	90	37	90	……………….	567

10 25 180	2	79 180 801	90 421	37 85 256 782	90 421	……………….	101 567

- Initial Population: Random number from a candidate compartment list
- Next generation: 20% best fit remain, 78% mating, 2% mutation
- Implemented in C++

Embedded GA in IA system

Advanced Hierarchical vs. Heterarchical vs. Hybrid

Conveyor Feed Interval	0.65 sec			0.66 sec		
Architecture	Advanced Hier.	Pure Heter.	Hybrid	Advanced Hier.	Pure Heter.	Hybrid
Pick Errors	0	328	0	0	171	0
Mean Utilization (%)	90.60	90.76	91.00	89.20	89.48	89.62
Standard Deviation	0.37	0.57	0.62	0.38	0.57	0.66

Gantry Robots Breakdown

Conveyor Feed Interval	0.75 sec		
Architecture	Advanced Hier.	Pure Heter.	Hybrid
Pick Errors	5985	2807	2779
Mean Utilization (%)	75.28	76.11	76.13
Standard Deviation	6.52	8.32	8.19

* The Models are implemented in G2 (Gensym Corporation), C++

Results with IA system

Problem of Current Practice

- Long cycle time (24 hrs)
- Need to fix orders for 2days
- Lack of responsiveness

Realization of a short cycle time (e.g. 2 hrs)

- Need efficient replenishment plan
- Responsiveness AND efficiency

Cycles per day	1	5	10	20
No. of repl. totes	272	777	1228	1748

Replenishment Planning

	Avg totes repl/day		Picking gantry util. level (%)			
	Current Logic	New Logic	Current Logic		New Logic	
			Avg.	Stdev	Avg.	Stdev
1 cycle / day	272	156	69.4	1.4	68.4	0.2
5 cycle / day	777	150	70.0	2.0	68.9	0.6
10 cycles / day	1228	159	70.2	1.5	67.3	0.6
20 cycles / day	1748	168	70.4	1.3	69.0	0.6

Short cycle time, but no loss of productivity

Benefits of a real-time IA based approach

- **Intelligent agent based hybrid model for actual industrial problem**
- **Hybrid model outperforms pure hierarchical and heterarchical models**
- **Hybrid Scheduling and Control System Architecture for Robustness and Global Optimization**
- **Guidelines for designing intelligent agent based production/warehousing planning, scheduling, and control systems**

Heragu's Ten Principles in WH Design & Operations

- Variability of most kinds is bad, especially the controllable ones
- All are necessary conditions, none are sufficient
- The Universe is a holonic system
- Divide and conquer
- Always look for middle of the road solutions
- Every decision has pros and cons
- Aim for a lot size of one
- Just because you get better at solving problems, don't make them more complicated
- Think long term
- Work with reason, not excuses

Thank you!

Questions & Comments?