Layout methods for order picking areas

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An example of order picking

How to become more efficient?

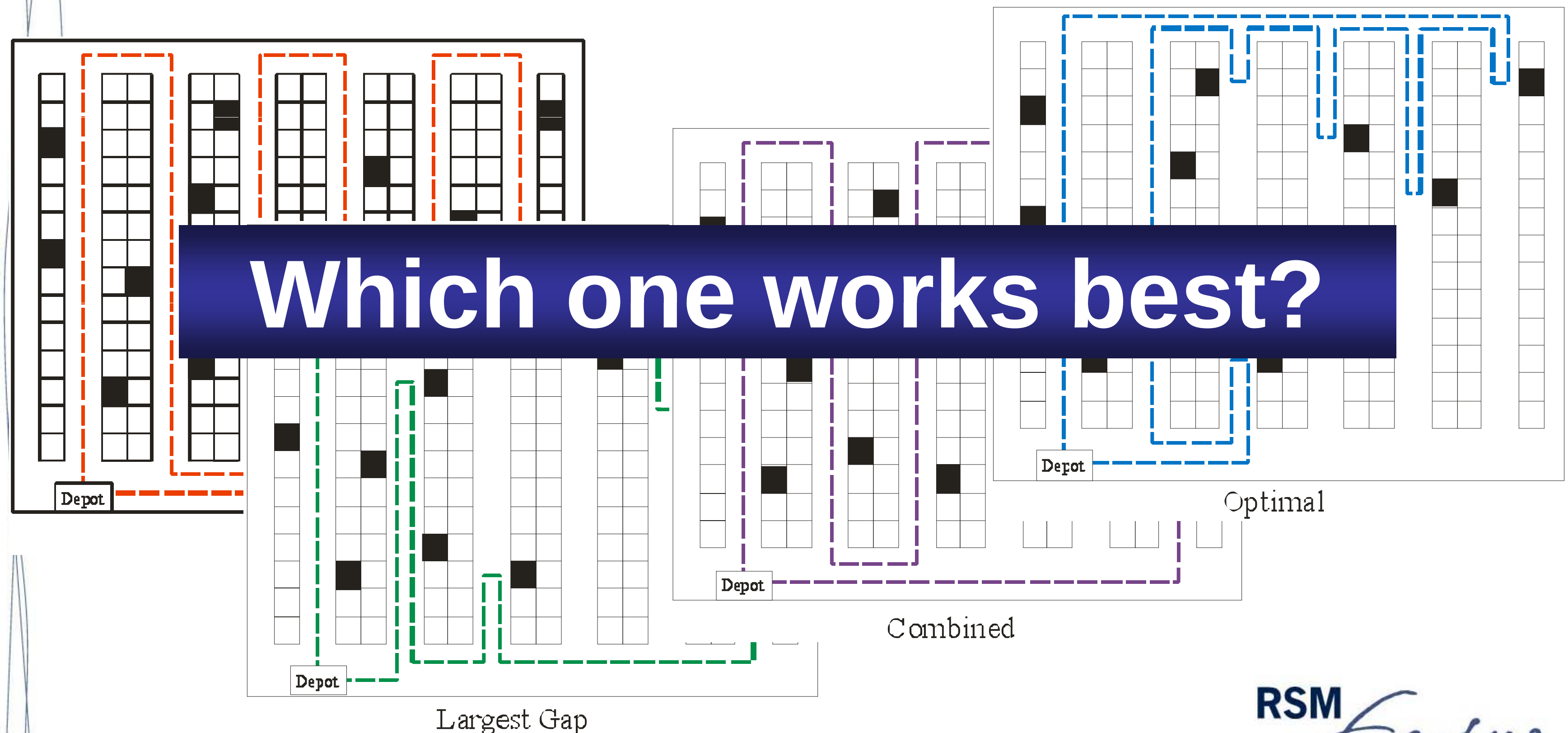
- The two main factors that influence efficiency:

1. operating policies
   - routing method
   - storage assignment method
   - and more.

2. layout
   - number of aisles
   - number of blocks
   - aisle length
   - depot location

Operating policies: routing

- Find the best sequence in which products are to be retrieved from storage.



Routing demonstration at: roodbergen.com/warehouse

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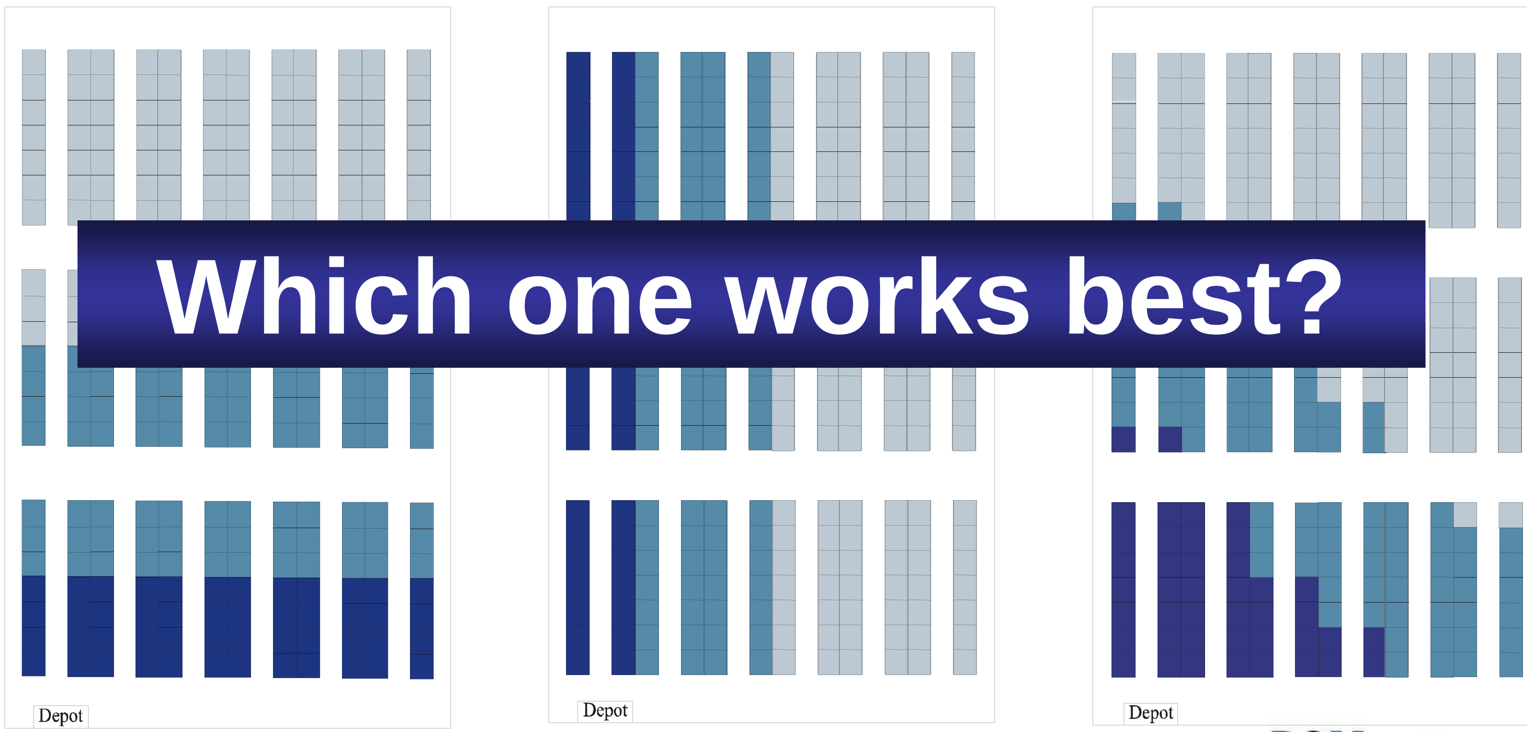
Operating policies: storage assignment

- Assign incoming loads to storage locations.
- For example: ABC storage



A items B items C items

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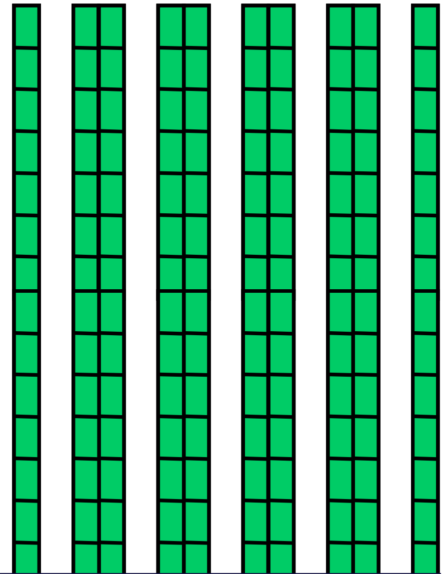
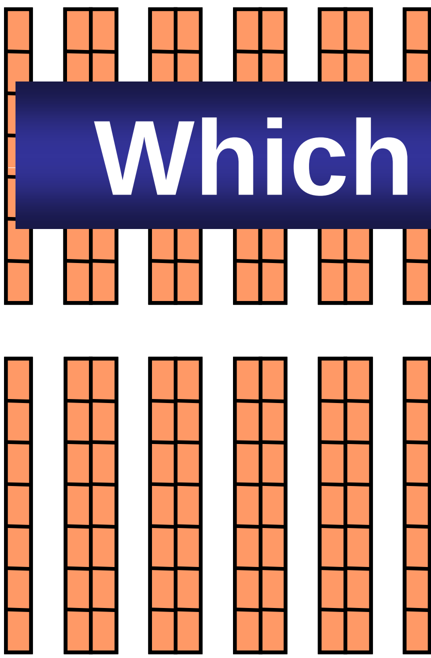
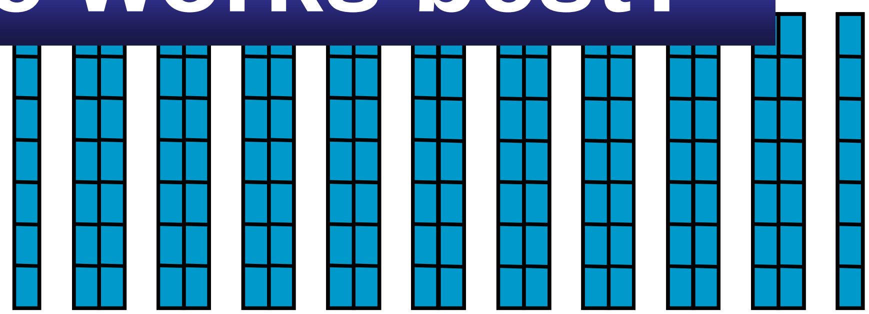
Layout

- number of aisles
- aisle length
- number of blocks





Which one works best?



Goal

Considering that

- *a myriad of layout parameters and operating policies determine the eventual efficiency of the operation.*

The goal is to

- find ways to obtain an overall optimization of the order picking process, including operating policies and layout parameters.

- *Three approaches follow ...*

The base optimization problem

$$\boxed{\begin{array}{ll} \min T_m(n,k,y) & \\ & S = n \cdot y \\ s.t. & n \geq 2 \\ & k \geq 1 \end{array}}$$

- There are typically less than a thousand feasible layouts.
- Main difficulty: quickly estimate travel time *T*

Options for goal function evaluations

- Closed-form expressions
  - Not available for all possible layout configuration and operating policy combinations,
  - Mathematically quite complex,
  - Easier to integrate in other applications.
- Simulation
  - Completely configurable,
  - Significant effort to develop / maintain / integrate.
- Or a combination of the two

Closed-form expressions

Approach

- Capture behavior of routing methods in formulas to find average travel time per route.

- Routing: S-shape
- Storage assignment: Random

- Results are also quite acceptable for some other routing methods (combined, largest gap).

The goal function

$$nk\left[1-\left(\frac{nk-1}{nk}\right)^m\right]\cdot\left(\frac{y}{k}+w_c\right)+\frac{2ym}{m+nk}-y+w_a\sum_{i=1}^{n}\left((n-i)\cdot\left[\left(\frac{i}{n}\right)^m-\left(\frac{i-1}{n}\right)^m\right]\right).$$

$$+\left(\frac{y}{k}+w_c\right)\sum_{i=1}^{n}\sum_{j=1}^{k}\left[\left(\frac{i}{n}\right)^m-\left(\frac{i-1}{n}\right)^m\right]\cdot\left[\left(\frac{j}{k}\right)^m-\left(\frac{j-1}{k}\right)^m\right]\left[(j-1)\left(\frac{ij-1}{ij}\right)^m+(j-1)\left(\frac{ij-i+1}{ij}\right)^m\right]$$

$$+w_a\sum_{i=2}^{n}\sum_{j=1}^{k}A_{ij}\left[\sum_{g=1}^{i-1}\left\{(i-g)\sum_{u=1}^{m}B\left(u,m,\frac{i-1}{ij}\right)\left[\left(\frac{i-g}{i-1}\right)^u-\left(\frac{i-g-1}{i-1}\right)^u\right]\right\}\right],$$

$$+w_a\sum_{i=3}^{n}\sum_{j=1}^{k}A_{ij}\left[(j-1)\sum_{u=1}^{m-1}\left\{B\left(u,m,\frac{i-1}{ij}\right)\cdot\sum_{\ell=1}^{i-1}[\ell\cdot(i-1-\ell)\cdot Q(\ell)]\right\}\right]$$

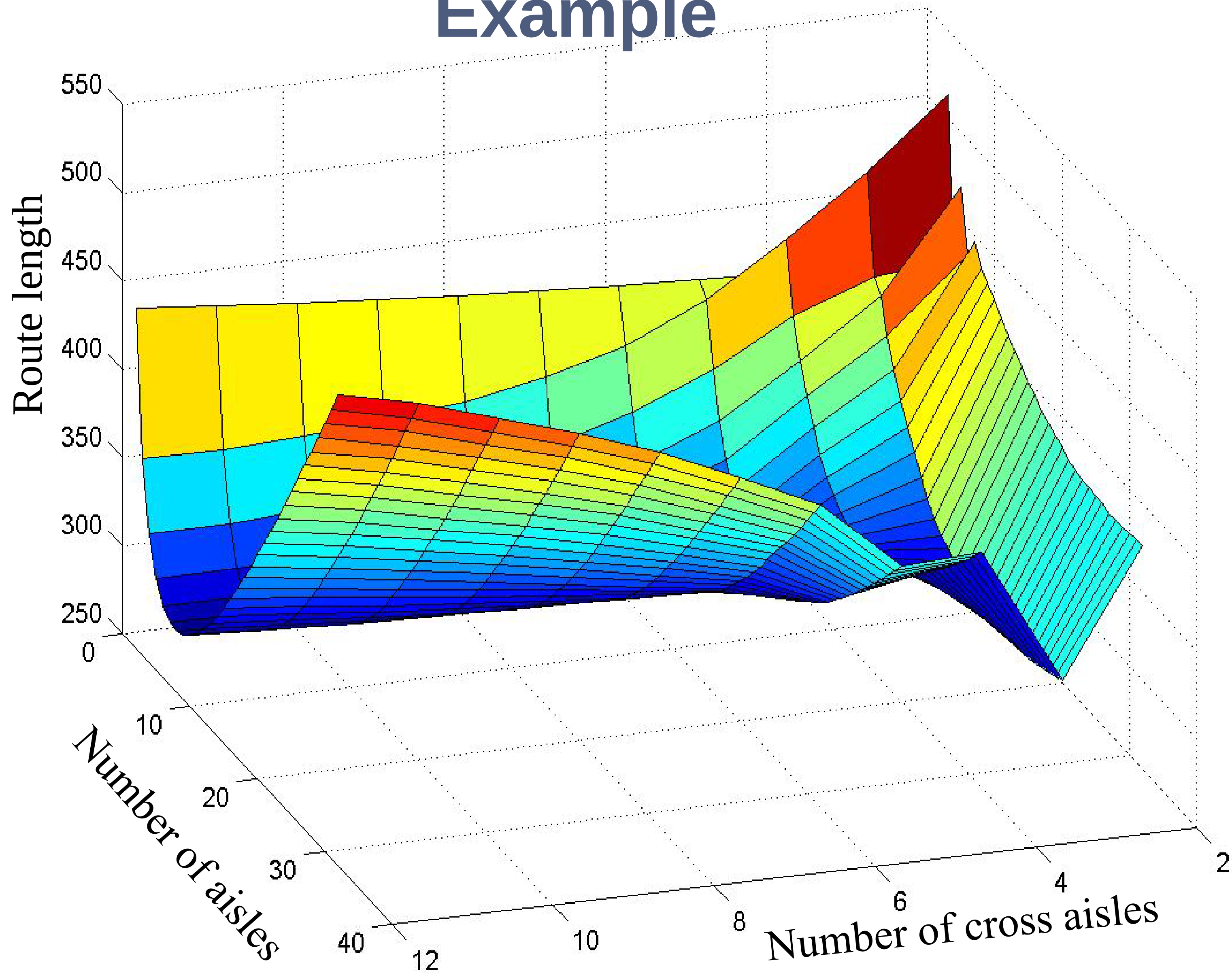
$$+\sum_{i=2}^{n}\sum_{j=1}^{k}A_{ij}\left[(j-1)\cdot\frac{1}{3}\cdot\left((i-2)\cdot w_a-E_{ij}^{(6)}\right)\cdot\left(1-\left(\frac{ij-i+1}{ij}\right)^m\right)\right]$$

$$+w_a\cdot\sum_{i=1}^{n}\sum_{j=2}^{k}p_jA_{ij}\left[\sum_{g=1}^{i-1}\sum_{u=1}^{m-1}\left(B\left(u,m,\frac{i-1}{ij}\right)\cdot(n-g)\cdot\left[\left(\frac{g}{i-1}\right)^u-\left(\frac{g-1}{i-1}\right)^u\right]\right)\right],$$

$$+w_a\cdot\sum_{i=1}^{n}\sum_{j=2}^{k}(1-p_j)A_{ij}\left[\sum_{g=1}^{i-1}\sum_{u=1}^{m-1}\left(B\left(u,m,\frac{i-1}{ij}\right)\cdot(n-g)\cdot\left[\left(\frac{i-g}{i-1}\right)^u-\left(\frac{i-g-1}{i-1}\right)^u\right]\right)\right]$$

$$+w_a\sum_{i=1}^{n}\sum_{j=1}^{k}A_{ij}\left(n-\frac{i}{2}\right)\left(\frac{ij-i+1}{ij}\right)^m+w_a\sum_{i=1}^{n}\sum_{g=1}^{n}\left(A_{i1}\cdot(g-1)\cdot\left[\left(\frac{g}{n}\right)^m-\left(\frac{g-1}{n}\right)^m\right]\right).$$

Example


Route length
550
500
450
400
350
300
250
0
10
20
30
40
Number of aisles
12
10
8
6
4
2
Number of cross aisles

Combining simulation and closed-form expressions

Approach (1)

- The basis is one simple closed-form expression for travel time in a given layout:

$$T = \frac{y}{t_a} \cdot A(n,k,m) + \frac{w_a}{t_c} \cdot C(n,k,m) + \frac{w_c}{t_c} \cdot E(n,k,m)$$

- which calls three “functions” A, C and E.

Approach (2)

- Function *A*(*n*,*k*,*m*)
  - The expected travel distance within aisles
- Function *C*(*n*,*k*,*m*)
  - The expected travel distance within cross aisles
- Function E(*n*,*k*,*m*)
  - The number of times an aisle is entered

- For a layout with
  - *n* aisles,
  - *k* blocks,
  - *m* picks per route

- Normalized aisle length
- Normalized cross aisle length

Approach (3)

- The functions A, C, E would typically be difficult to obtain as closed-form expressions.
- We generated A, C, E through simulation and stored the result for all values of n, k, m in a spreadsheet.
- Calculation of A, C, E is an one-time effort!
  It never needs to be repeated.

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Approach (4)

- An estimate *T* for travel time for any layout can be calculated by means of a combination of
  - Database lookup, and
  - A fairly simple formula.

- Implemented in Microsoft Excel.

- A layout optimization cycle takes less than a second.

Resulting spreadsheet

Microsoft Excel - layout optimization table v4 (max 100 picks).xls

Warehouse layout optimization spreadsheet

For more information, refer to the paper:
Roodbergen, K.J. and Vis, I.F.A. (2009)
A spreadsheet-based approach to warehouse layout

total aisle length (m)	400
center distance between aisles (m)	2.5
width of cross aisles (m)	1
average speed within aisles (m/s)	0.6
average speed outside aisles (m/s)	0.6
additional time to change aisles (s)	0
Routing method	Combined+

best options	# aisles	# cross aisles	routelength
1	12	5	546.10
2	10	7	546.15
3	13	5	546.74
4	8	9	546.77
5	11	7	546.90
6	9	7	547.14
7	11	5	547.17
8	9	9	547.61
9	14	5	547.80
10	8	7	549.22
11	10	5	549.30
12	15	5	549.70
13	7	9	549.86
14	12	7	550.04
15	8	10	550.81

order profile	
lines per order	frequency
1	0.2
2	0.2
3	0.2
4	0.2
5	0.2
6	
7	
8	
9	
10	
11	
12	
13	
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31	

front / ranking / Sa / Sc / Se / La / Lc / Le / Ca / Pa / Pc / Pe / Aa / Ac / Ae

Ready

Results

- Travel time estimation for
  - Many routing heuristics
  - Random storage
  - Layouts with any number of aisles and blocks

- Test: our hybrid method versus true simulation
  - Travel time estimates differ by
    - less than 1% on average
    - 3% at most.
  - Top 5 layouts
    - complete match in 60% of the cases;
    - match of 4 or more in 97% of the cases.

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Simulation

Case study

- New facility to be build for Cito Benelux
  - Large diversity of products
  - Pick routes will visit <u>three</u> different areas (pallets, shelves, flow racks).
  - On average fairly small orders (about 5 picks/route), but individual lists may have more than 100 picks.
  - Demand is significantly skewed

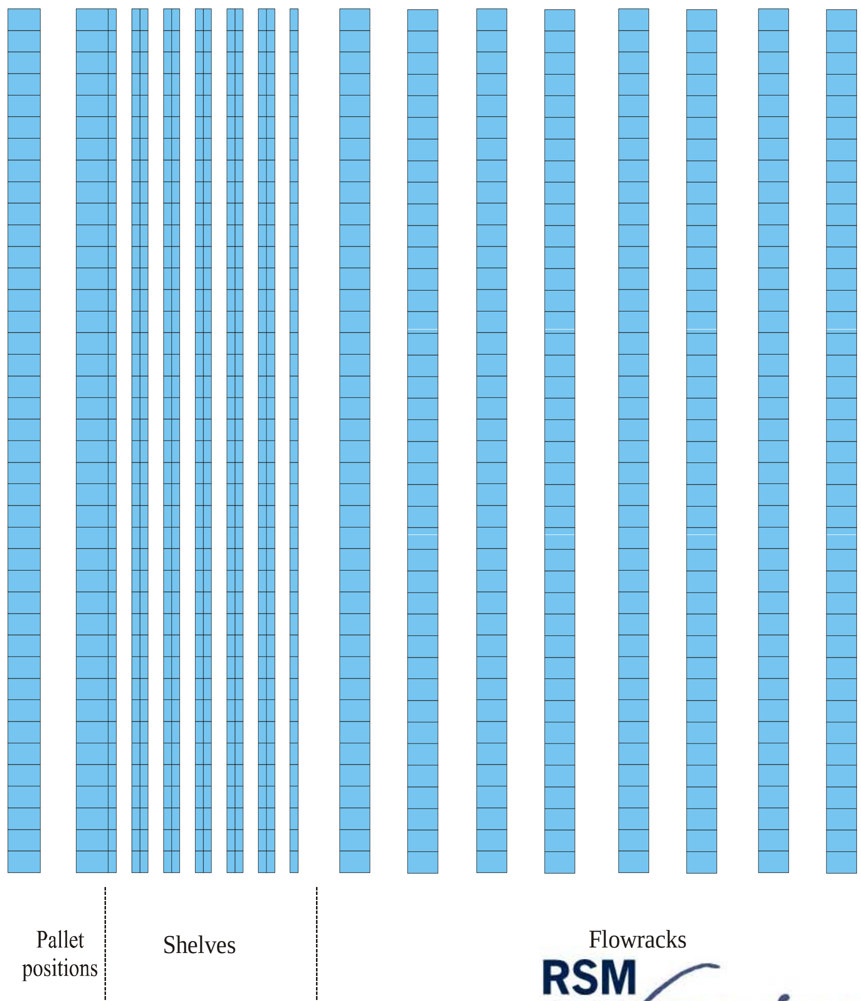
- Four scenario’s
  - rule of thumb: “twice as deep as wide”.
  - rule of thumb: “square-in-time”.
  - simultaneous optimization.
  - optimized per area.

Baseline option 1

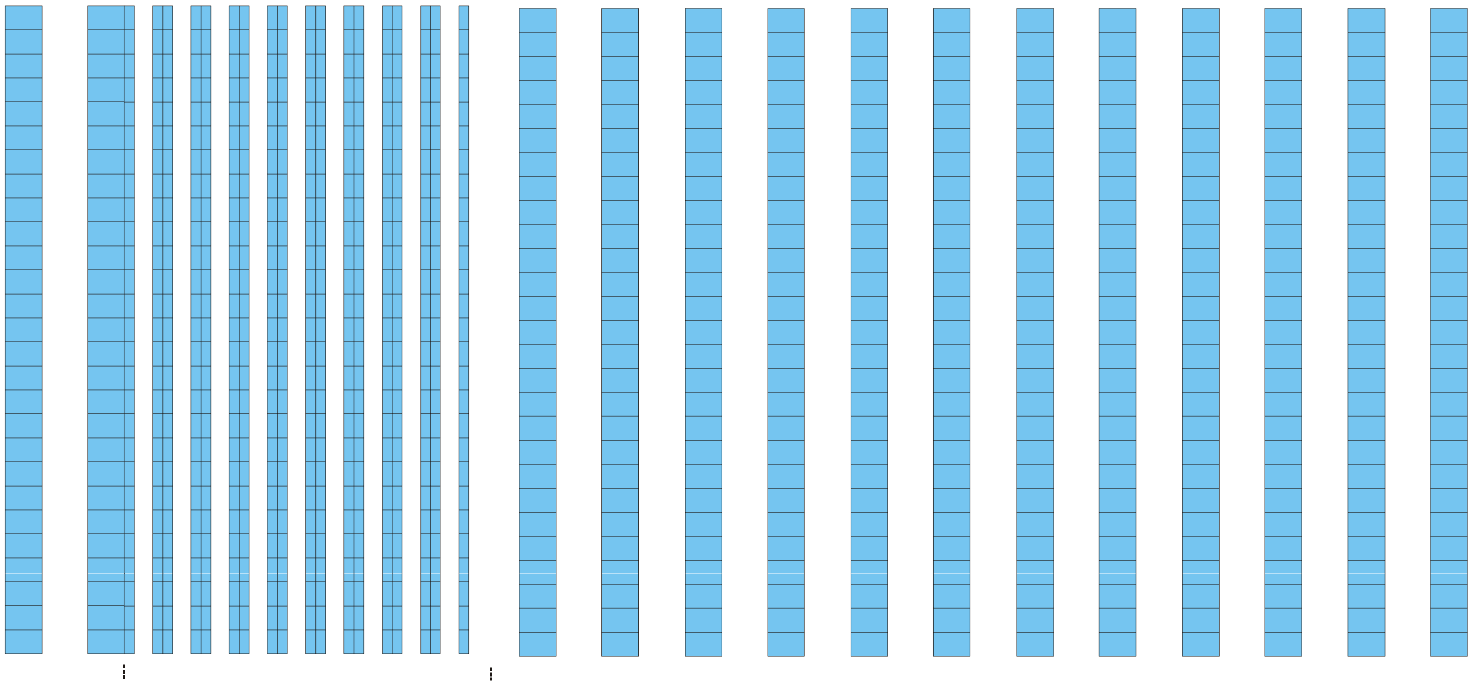
- Square-in-time layout
  - 1 pallet aisle
  - 6 aisles with shelves
  - 4 aisles with flow racks

  - Operating policies as available from the current WMS.

- Route length:
  172.2 meters



Baseline option 2



- Twice-as-wide-as-deep
  - 1 pallet aisle
  - 9 aisles with shelves
  - 6 aisles with flow racks
  - Operating policies as available from the current WMS.
- Route length: 159.7 m.

The challenge (1)

- What are the issues when trying to find the best alternative using simulation?

- Typical number of alternatives
  - We have 25 possible combinations of operating policies that can be used (5 routing methods, 5 storage methods).
  - And about 500 possible layout configurations.
  - Resulting in about 12,500 alternatives.

- How many replications do we need to get statistically valid conclusions?

The challenge (2)

- We cannot just compare means. Differences may not be significant.
- High variability in route length already for theoretical instances (with fixed pick list size).
- Variability increases when using actual pick list size distributions.
- Individual observations are not normally distributed in about 10% of the instances.
- If more alternatives must be compared, the required number of replications per alternative increases rapidly for a given significance level.

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Approach

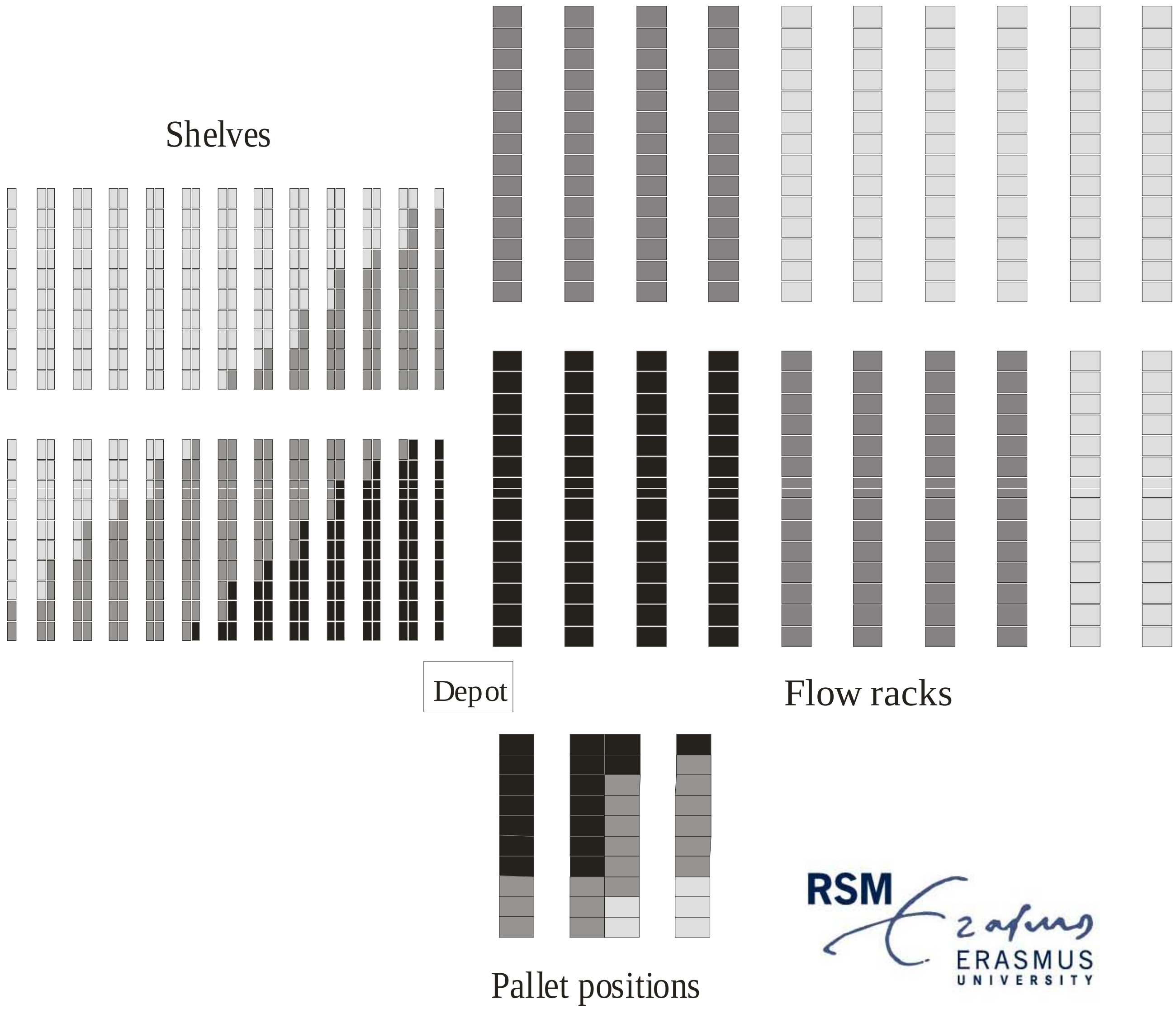
- Screening and selection procedure
  - Screening phase: Calculate for each alternative the average travel distances by simulating a small number of replications.
  - Retain only those alternatives that are most likely to turn out to be the best alternative.
  - Selection phase: Calculate for each of the remaining options the average travel distance by performing a sufficient number of replications.
  - Select the alternative with the lowest average total travel distance.
- Final choice is within a tolerance of δ from the best configuration with a confidence level of $1-\alpha$.

Simultaneous optimization

- For every area:
  - aisle-by-aisle routing
  - across-aisle storage
- Layout
  - *Length = 42.20 m.*
  - *Width = 39.90 m.*
  - 4 cross aisles
- *Average travel distance:118.80 m.*
- A saving of 31%

Optimized per area

- Routing:
  - combined
- Storage:
  - nearest-location
  - nearest-subaisle
- *Average travel distance 74.99 m.*
- A saving of 56%



Results

- Simulation
  - 11,500 scenario's
  - screening phase: 1.3 million replications
  - selection phase: 116 million replications
  - Total calculation time: about 3 days on 5 PCs.

- Many aspects of the solution for CITO can be considered atypical when compared to literature and/or practice, which proves the point of considering layout and operating policies together.

Conclusions

Conclusions

- The interactions are important.
  - Layout + routing + storage + zoning + batching = efficiency
- Closed-form expressions
  - powerful, fast, but limited in applicability
- Simulation
  - Maybe not be as straightforward as it seems.
  - High number of alternatives.
  - Route lengths are not always normally distributed: test and compensate.
  - Enormous amount of replications required for achieving statistical significance.

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