

The dynamics of a low-order model for the Atlantic Multidecadal Oscillation

Alef Sterk
University of Groningen

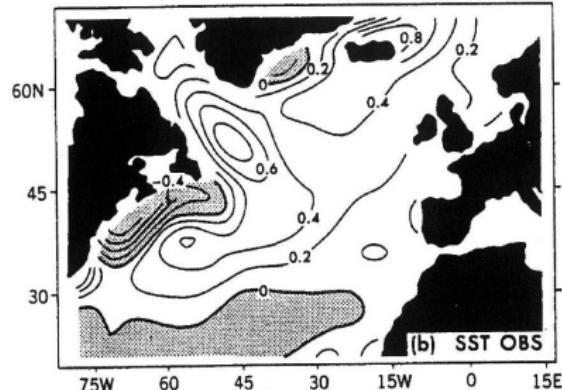
NDNS+ Workshop Eindhoven, April 15, 2010

Joint work with Henk Broer (Groningen), Henk Dijkstra (Utrecht),
Carles Simó (Barcelona) & Renato Vitolo (Exeter)

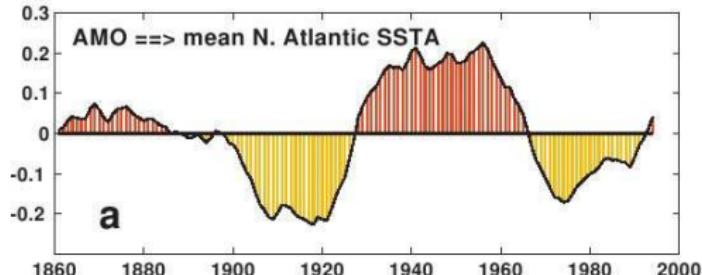
The Atlantic Multidecadal Oscillation (AMO)

Pattern of sea surface temperature variability in North Atlantic Ocean

SST (1950–1964) minus SST (1970–1984)

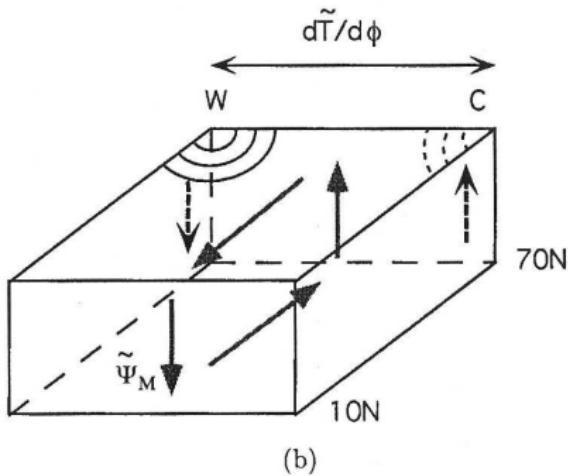
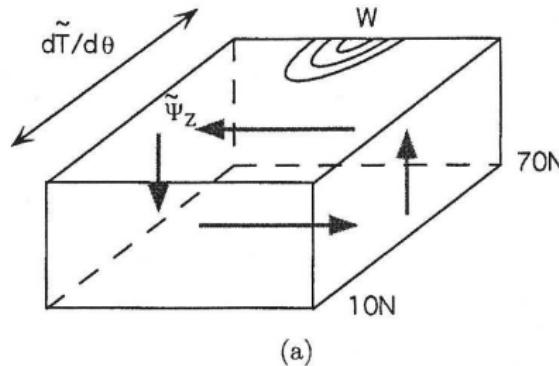


Average SST anomaly over entire North Atlantic basin



The AMO as an internal mode of the ocean

Te Raa & Dijkstra (2002) : interaction between temperature & transport anomalies



Model

Essential ingredients

Diagnostic equations (no time derivatives)

- ▶ Coriolis force ($-v, u$) balances (p_x, p_y)
- ▶ p_z linearly related with T
- ▶ mass conservation: $\operatorname{div}(u, v, w) = 0$

Temperature advection in rectangular basin ($6000 \times 6000 \times 4$ km)

$$T_t + uT_x + vT_y + wT_z = \text{Diffusion} + \text{Forcing}$$

$$\text{Forcing} = Bi \left[\underbrace{15 + \frac{\Delta}{2} \cos\left(\frac{\pi y}{L_y}\right)}_{\text{atmosphere temp.}} - T \right]$$

Model

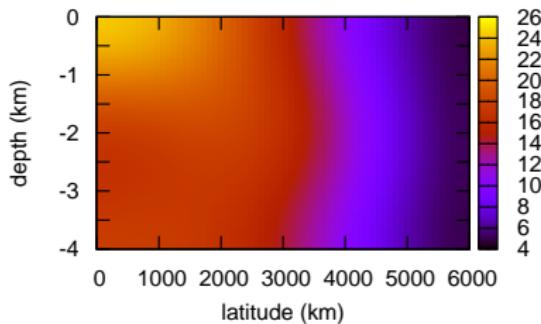
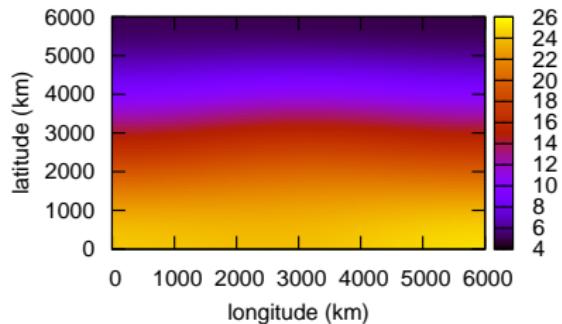
Galerkin projection

- ▶ expand T in Fourier modes:

$$T(x, y, z, t) = \sum \hat{T}_{m,n,k}(t) \cos\left(\frac{m\pi x}{L_x}\right) \cos\left(\frac{n\pi y}{L_y}\right) \cos\left(\frac{k\pi z}{L_z}\right)$$

- ▶ truncate: $0 \leq m, n, k \leq 2 \implies 27$ ODEs for $\hat{T}_{m,n,k}$
- ▶ (p_x, p_y, p_z) and (u, v, w) are eliminated

Stable steady state for $\Delta = 20^\circ\text{C}$



Velocity field: upwelling in the south & downwelling in the north

Modify the forcing term

Changes stability of the steady state...

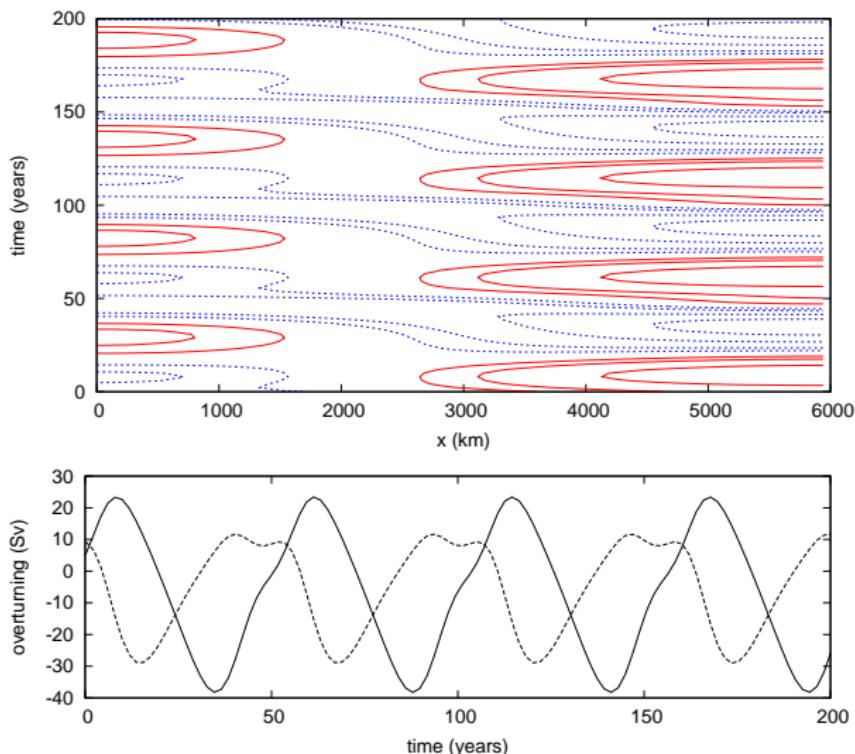
$$\text{Forcing} = Bi \left(\underbrace{15 + \frac{\Delta}{2} \cos\left(\frac{\pi y}{L}\right)}_{\text{atmosphere temp.}} - (1 - \gamma)T - \gamma T_{eq} \right)$$

What happens if γ increases from 0 to 1?

- ▶ $\Delta = 20$: Hopf bifurcation at $\gamma \approx 0.95 \implies$ AMO!
- ▶ $\Delta > 20$: period doubling bifurcations & Hénon-like SA

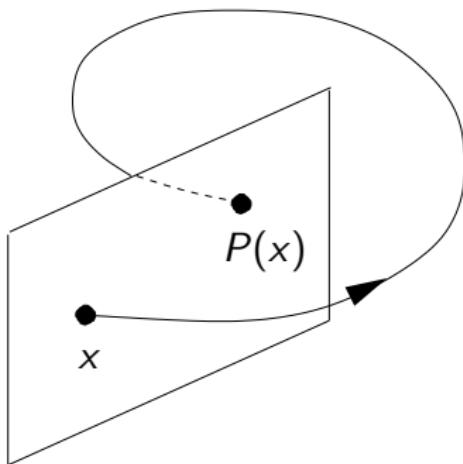
Periodic orbit has physical characteristics of the AMO

- 1) Westward travelling temperature anomalies
- 2) phase difference between anomalous E-W & S-N transports



The Poincaré return map

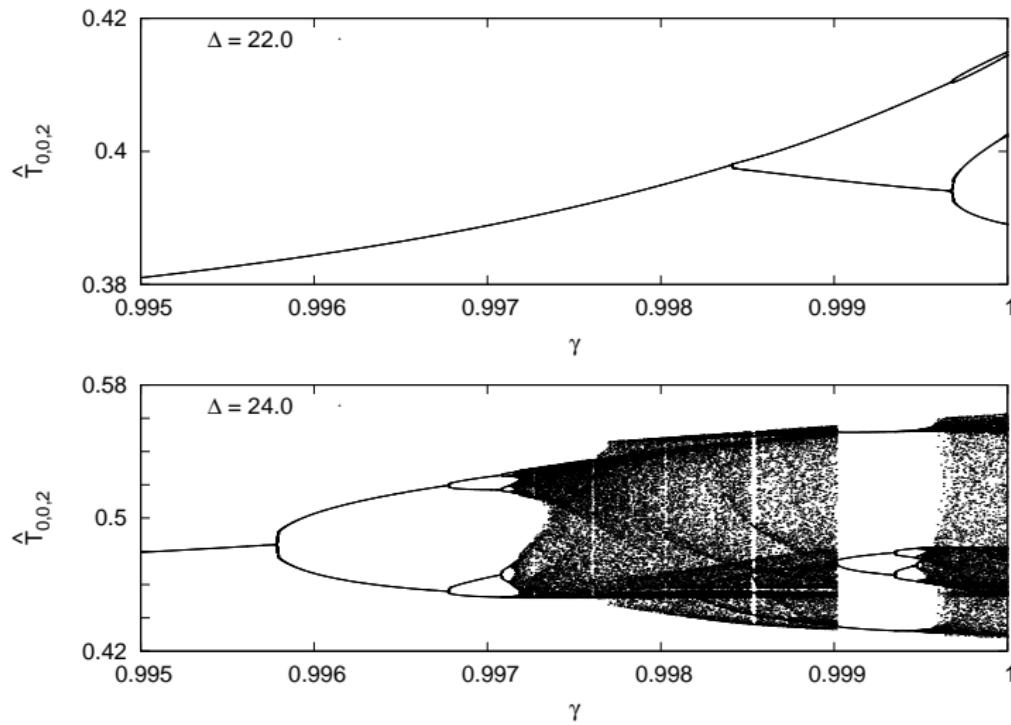
Intersections of integral curves with a hyperplane



- ▶ $P : \mathbb{R}^{27} \rightarrow \mathbb{R}^{27}$ maps each intersection to the next
- ▶ fixed points correspond with periodic orbits

Attractors of the Poincaré map as a function of γ

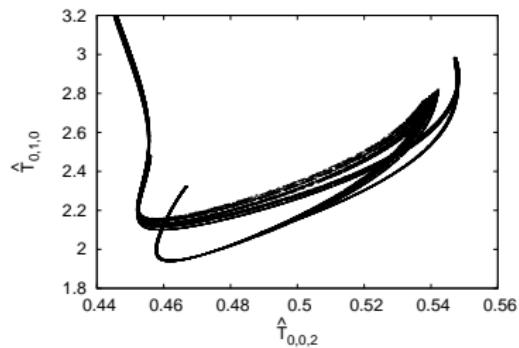
Two period doublings for $\Delta = 22^\circ\text{C}$, a cascade for $\Delta = 24^\circ\text{C}$



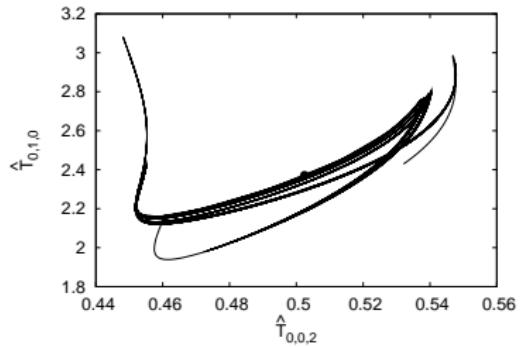
Hénon-like strange attractors after PD cascade

SA = closure of the unstable manifold of an unstable fixed point

attractor

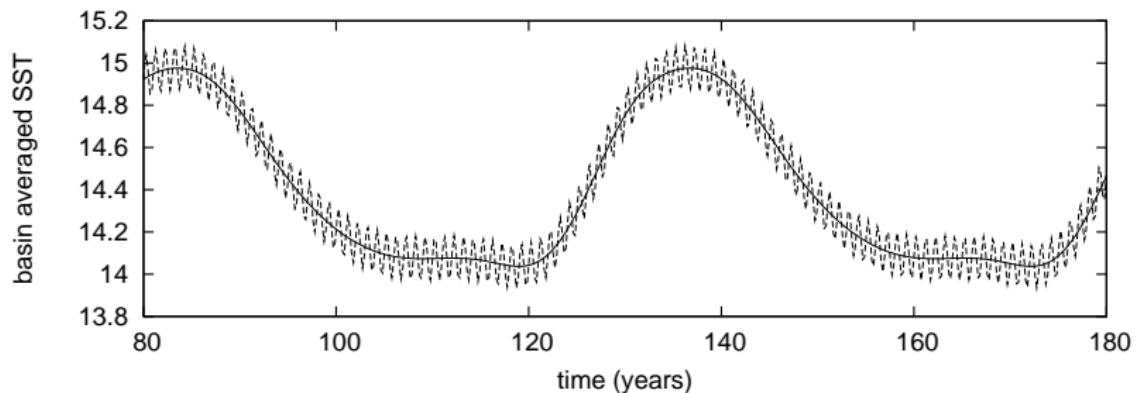


unstable manifold fixed point



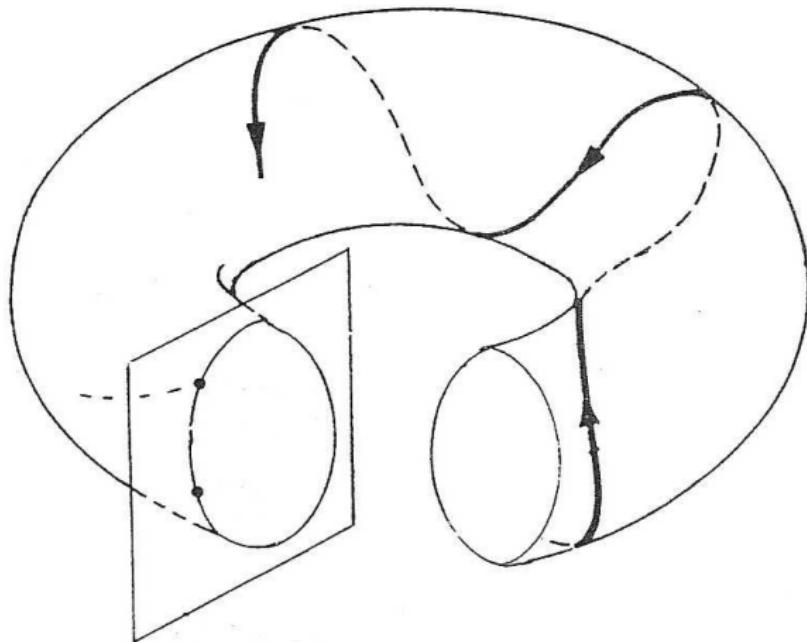
Next step: add annual variation to the forcing term

Two periods (AMO & forcing) : dynamics on a 2-torus



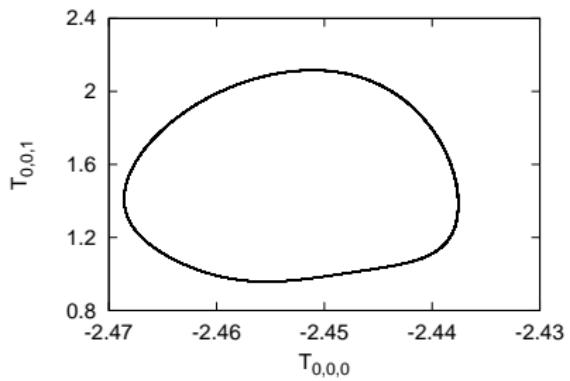
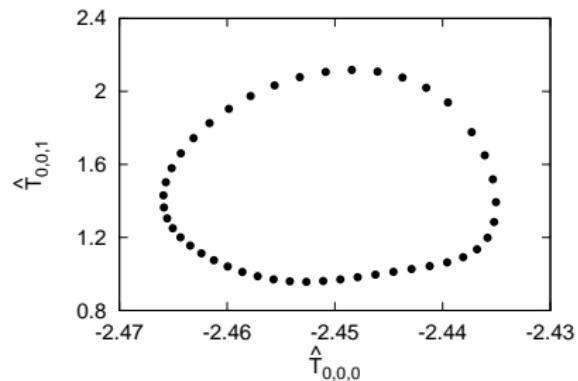
The Poincaré stroboscopic map

Snapshots of integral curves at multiples of forcing period



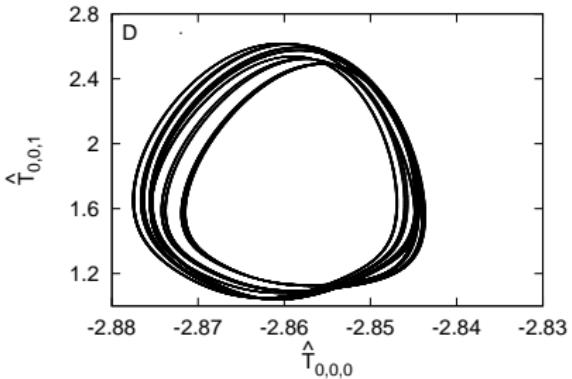
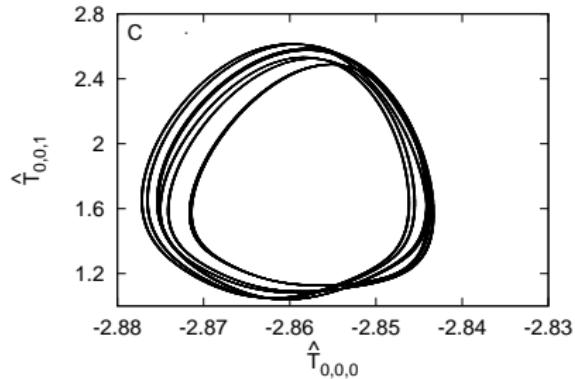
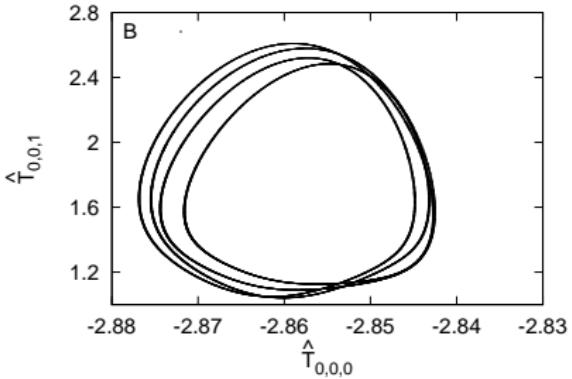
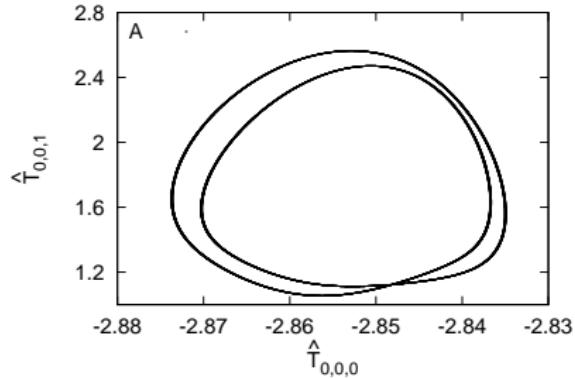
Dynamics of the stroboscopic map on invariant circles

Periodic or quasi-periodic



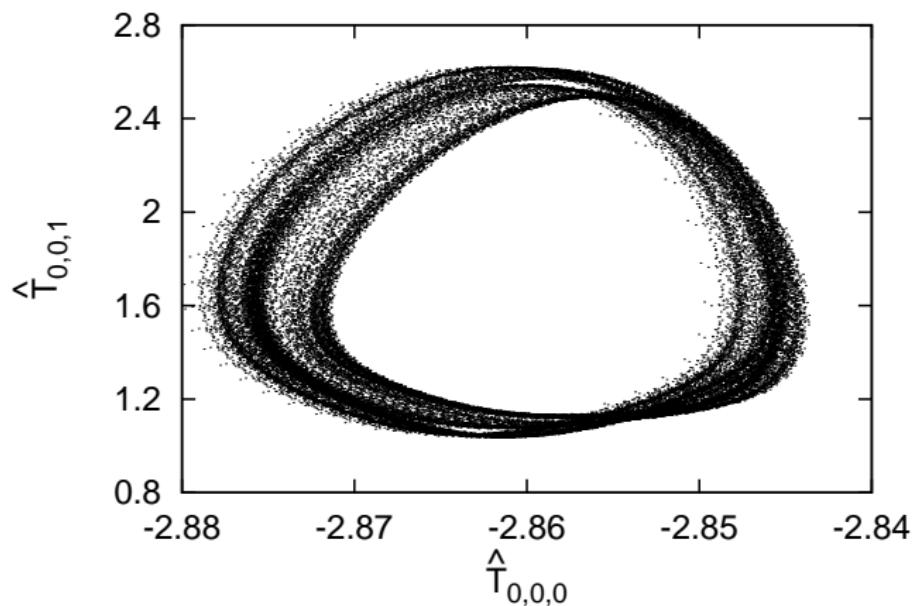
Doublings of invariant circles ($\Delta = 24^\circ\text{C}$, increasing γ)

Inherited from the period doublings of periodic orbits



A quasi-periodic Hénon-like strange attractor

SA = closure of unstable manifold of invariant circle?



References

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Appendix: model equations

Relation velocity & temperature gradients

$$-\nu + p_x = E_H(u_{xx} + u_{yy}) + E_V u_{zz}$$

$$u + p_y = E_H(v_{xx} + v_{yy}) + E_V v_{zz}$$

$$p_z = Ra T$$

Mass conservation

$$u_x + v_y + w_z = 0$$

Temperature advection

$$T_t + u T_x + v T_y + w T_z = P_H(T_{xx} + T_{yy}) + P_V T_{zz} + B(T_S - T)G(z)$$