Neuroscience Campus Amsterdam

Uncovering deterministic and stochastic dynamics from macroscopic neuronal recordings

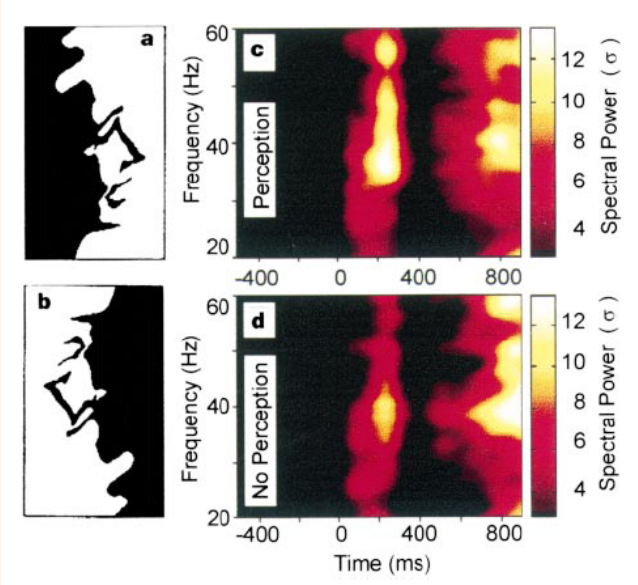
Rikkert Hindriks

VU medisch centrum

vrije Universiteit amsterdam

Background

- There exists an intimate link between psychological processes and electrical brain activity
- One of the major goals of cognitive neuroscience is to characterize this link
- Most of the studies however, use only simple statistical properties of the recorded brain signals
- Not much is known about their **dynamics** and its relation to cognition
- We therefore advocate **explicit dynamical modeling** of neuronal activity
- A recent development within this context is the use of **stochastic differential equations**



Rodriguez et. al., Nature, 1999

Stochastic differential equations

General form: $dx_t = b_\theta(x_t)dt + \sigma_\theta(x_t)dW_t$ with
drift function $b_\theta : R^n \to R^n$,
diffusion function $\sigma_\theta : R^n \to Mat_n(R)$,
driving n-dimensional Brownian motion W_t , and
parameter vector $\theta \in R^d$.

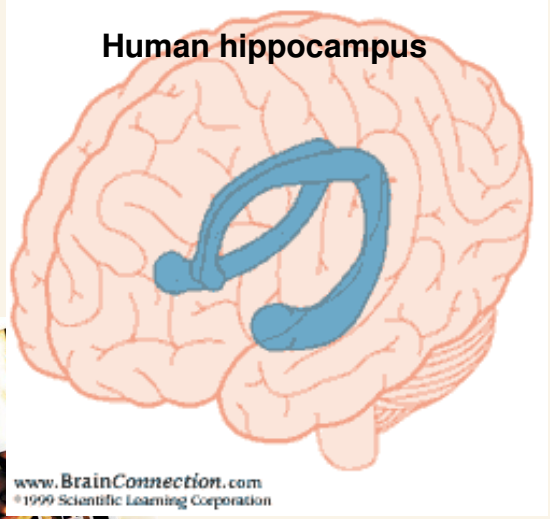
- The drift function models the deterministic dynamics of the system
- The diffusion function models the stochastic fluctuations affecting the system
- **Problem statement:** Estimate θ from discretely-sampled observations $X_0, X_\Delta, ..., X_{n\Delta}$.

Mouse hippocampal activity *in vitro*

- The hippocampus is a cortical structure known to play a fundamental role in the formation of memories

- *In vitro* activity in mouse hippocampus slices has characteristics comparable with activity in intact hippocampus

- *In vitro* recordings are used to investigate the genetics underlying hippocampal activity



Electrode grid over hippocampal slice

Mathematical model

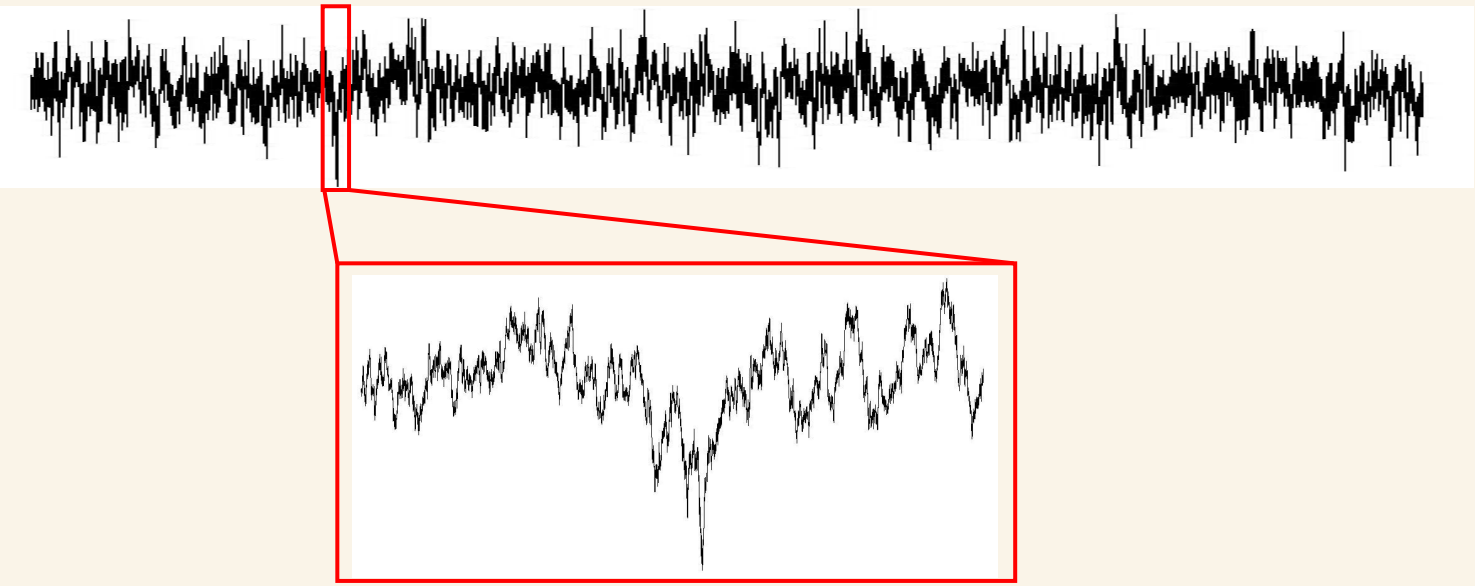
One-dimensional system with parabolic diffusion

$$dx_t = -\theta_1 x_t dt + \sqrt{\theta_2 + \theta_3 x_t + \theta_4 x_t^2}\, dW_t$$

thus $\theta = (\theta_1, \theta_2, \theta_3, \theta_4)$,

drift function $b_\theta(x) = -\theta_1 x$, and

diffusion function $\sigma_\theta(x) = \sqrt{\theta_2 + \theta_3 x + \theta_4 x^2}$.



Statistical inference

Numerically solve the estimating equation $G_n(\theta) = 0$ where

$$G_n(\theta) = \sum_{i=1}^{n} w(X_{(i-1)\Delta};\theta) \begin{pmatrix} X_{i\Delta} - m_1(X_{(i-1)\Delta};\theta) \\ (X_{i\Delta} - m_1(X_{(i-1)\Delta});\theta)^2 - m_2(X_{(i-1)\Delta};\theta) \end{pmatrix}$$

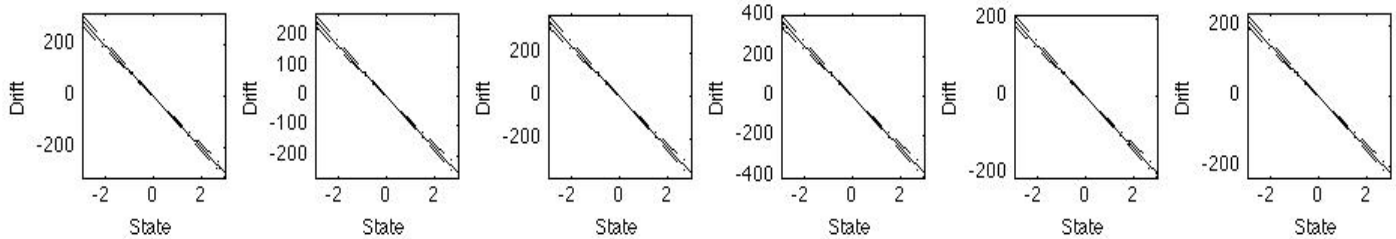
where $m_1(x;\theta) = \int y p_\Delta(y \mid x;\theta) dy$

and $m_2(x;\theta) = \int (y - m_1(x;\theta))^2 p_\Delta(y \mid x;\theta) dy$

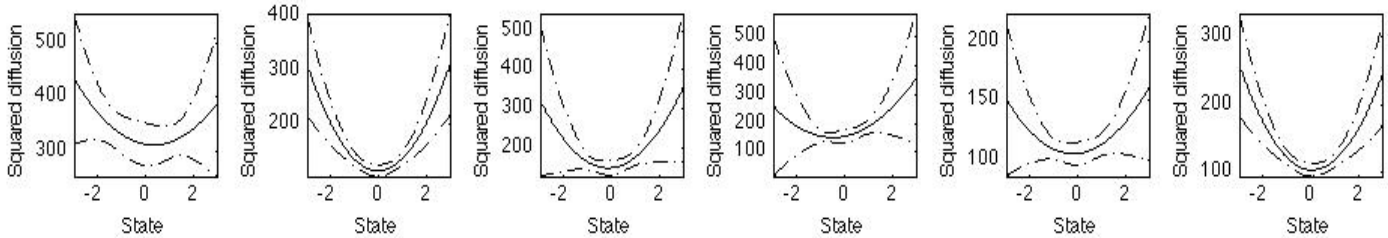
are the first and second conditional centered moments (Bibby & Sorensen, 1998).

Preliminary results

Estimated drift functions

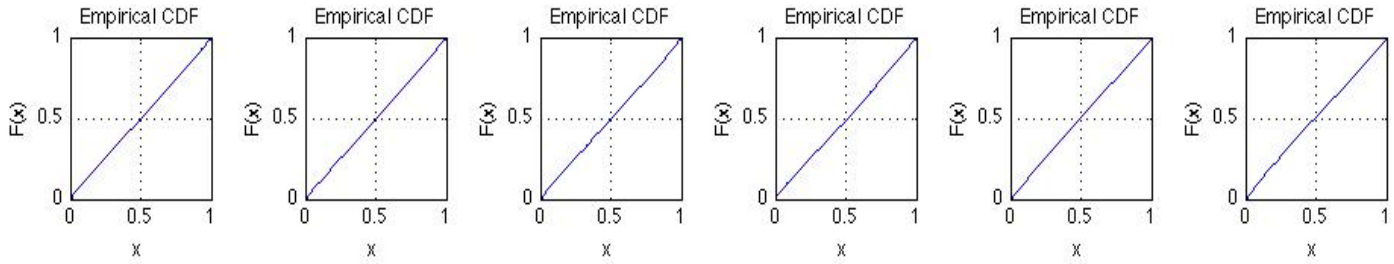


Estimated squared diffusion functions

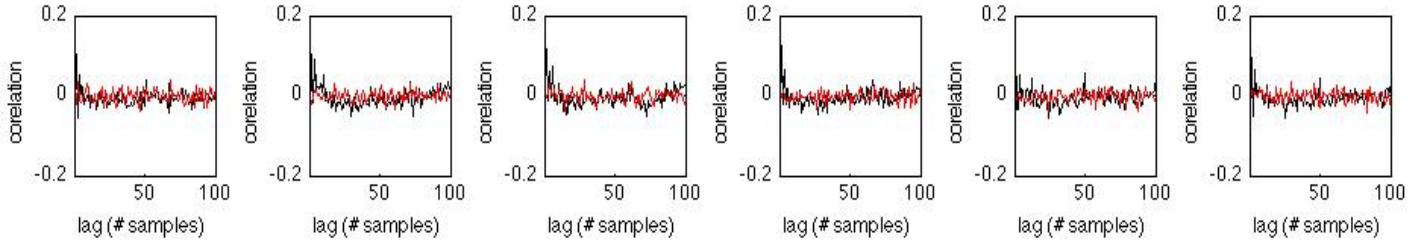


Preliminary results

Cumulative distribution of the residuals

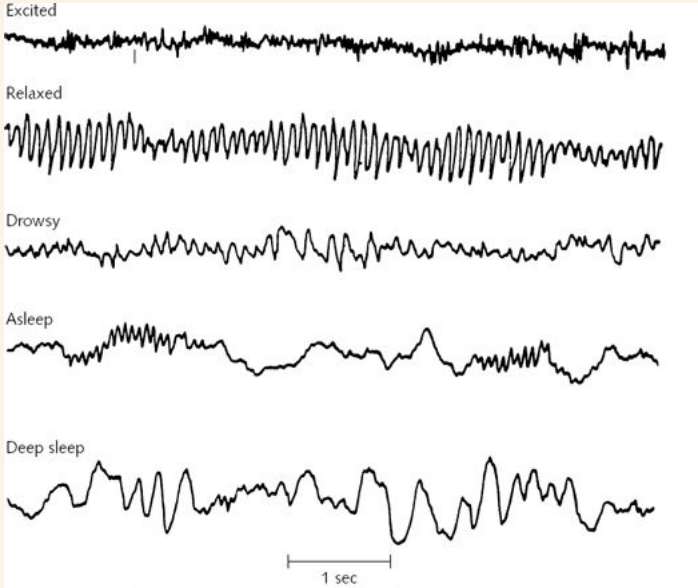


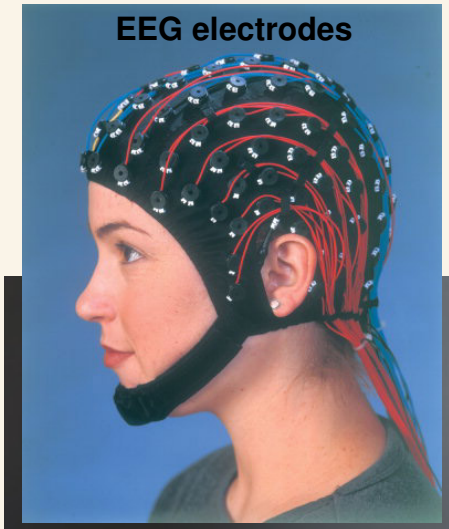
Autocorrelation functions of the residuals



Human brain rhythms

- Electrical activity from human brains is measured with EEG or MEG
- The measured signals reflect the summed activity of local cortical patches
- And is typically rhythmic





MEG scanner

Mathematical model

- Damped harmonic oscillator driven by additive white noise:

$$\ddot{x} + \gamma\dot{x} + \omega^2 x = \sigma\xi(t).$$

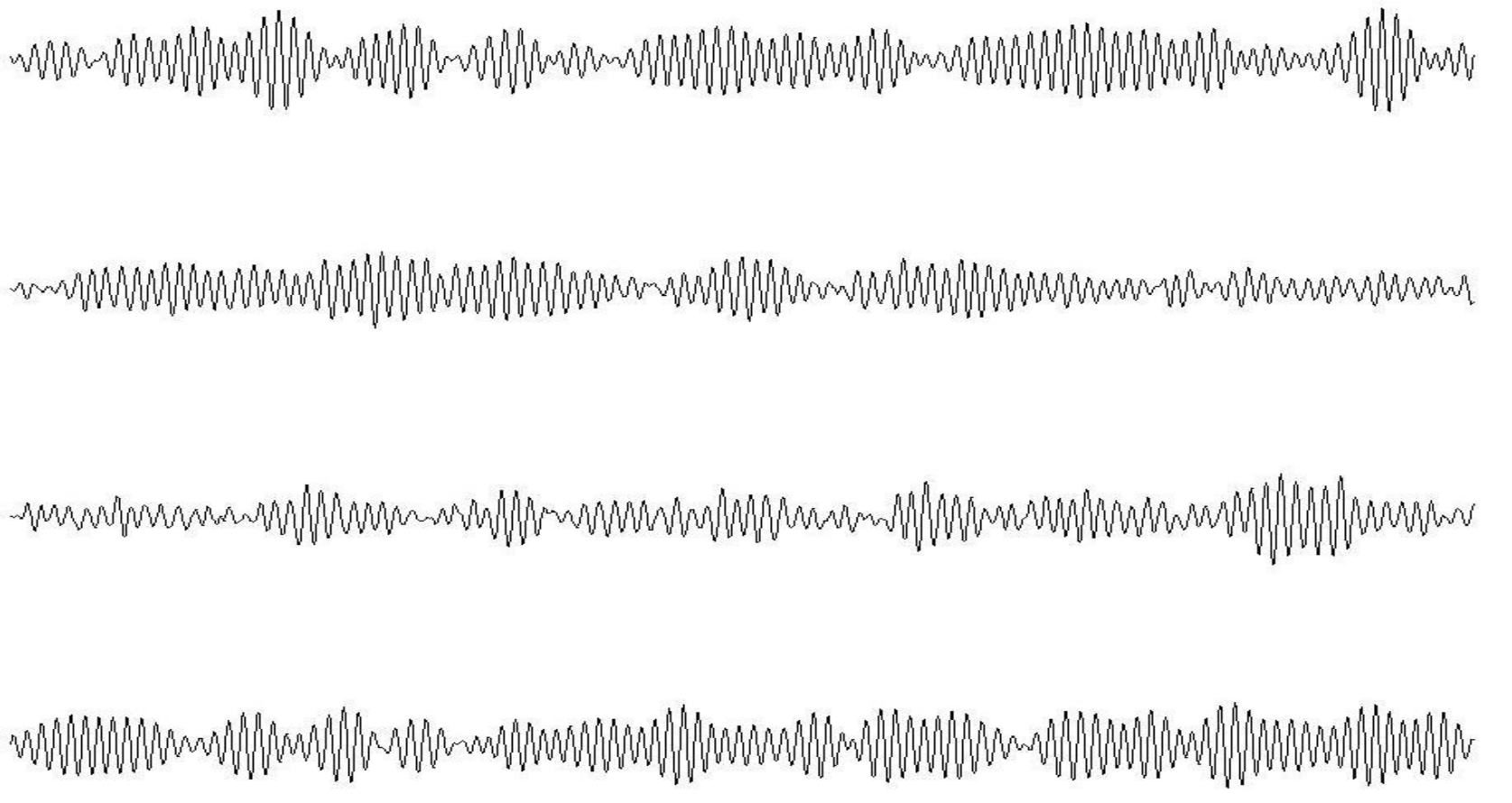
- Let $v = \dot{x}$ then $\begin{pmatrix} dx_t \\ dv_t \end{pmatrix} = -A\begin{pmatrix} x_t \\ v_t \end{pmatrix}dt + \sigma\begin{pmatrix} 0 \\ 1 \end{pmatrix}dW_t$ where $A = \begin{pmatrix} 0 & -1 \\ \omega^2 & \gamma \end{pmatrix}$

- Which has solution $\begin{pmatrix} x_t \\ v_t \end{pmatrix} = e^{-At}\begin{pmatrix} x_0 \\ v_0 \end{pmatrix} + \sigma\int_0^t e^{-A(t-s)}\begin{pmatrix} 0 \\ 1 \end{pmatrix}dW_s.$

- The covariance function of x can be explicitly calculated.

- The parameter vector $\theta = (\gamma, \omega, \sigma)$ is estimated by matching the theoretical and observed covariance functions

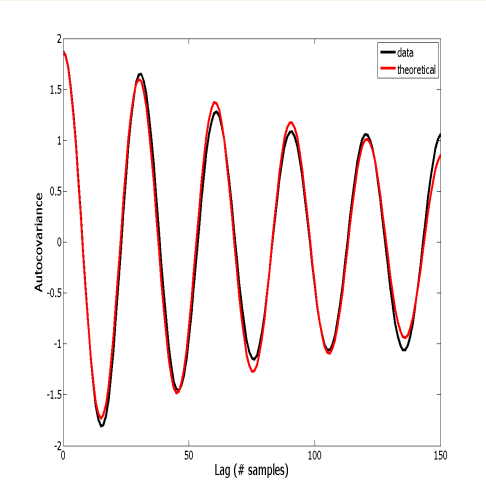
Preliminary results



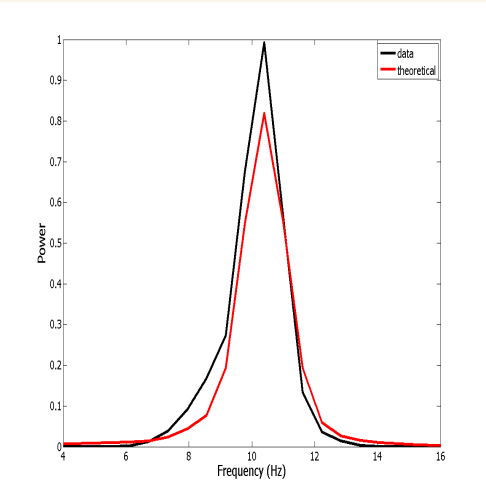
Preliminary results

observed
fitted

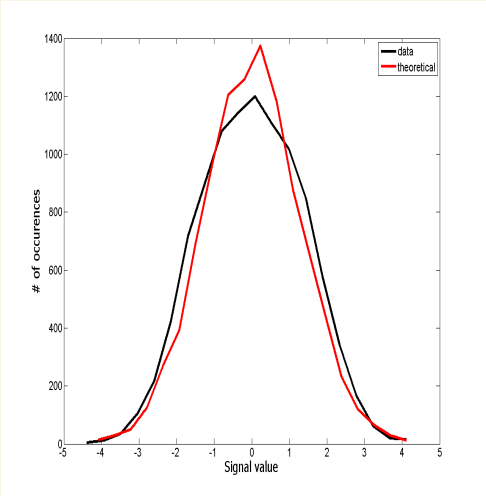
Autocovariance function



Power spectrum



Amplitude distribution



Conclusions

- The dynamics of diverse ongoing neuronal activity can be described by stochastic differential equations (mouse hippocampus activity *in vitro,* spontaneous human alpha rhythm)

- The statistical techniques to estimate the model parameters heavily depend on the type of model (estimation equations, generalized moments estimation)

- The neuroscientific relevance of these models has to be established by applying them to different experimental paradigms (genetic mouse lines, cognitive conditions)