Energy Consumption in Coded Queues for Wireless Information Exchange

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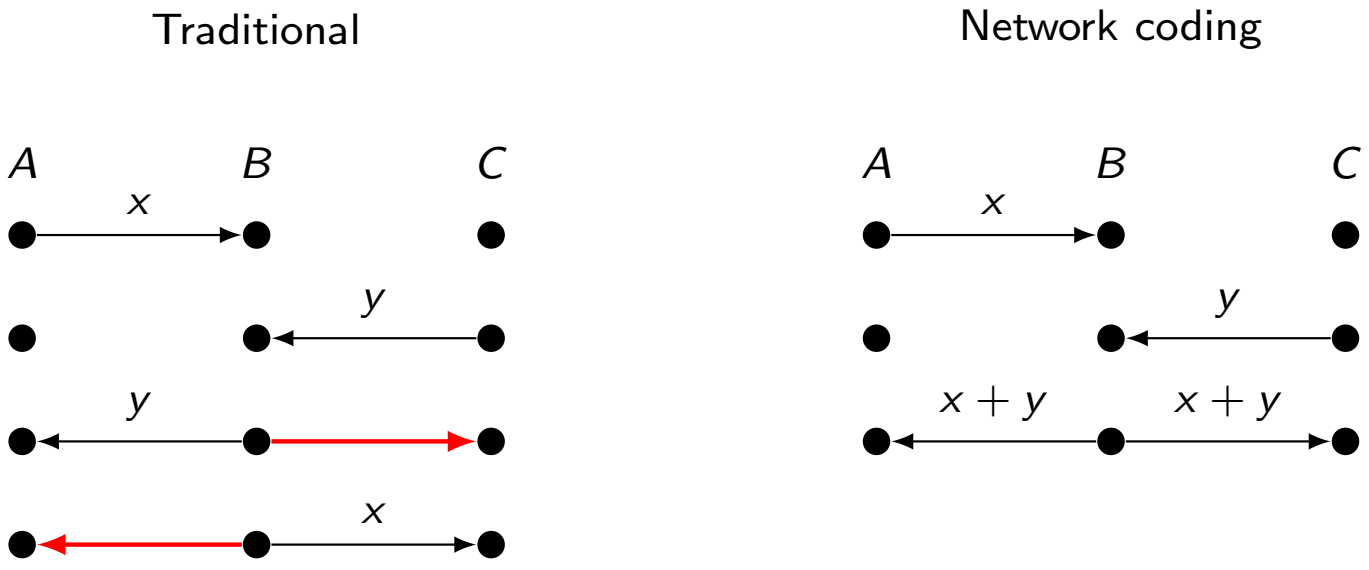
September 29 - October 1, 2010

Plan

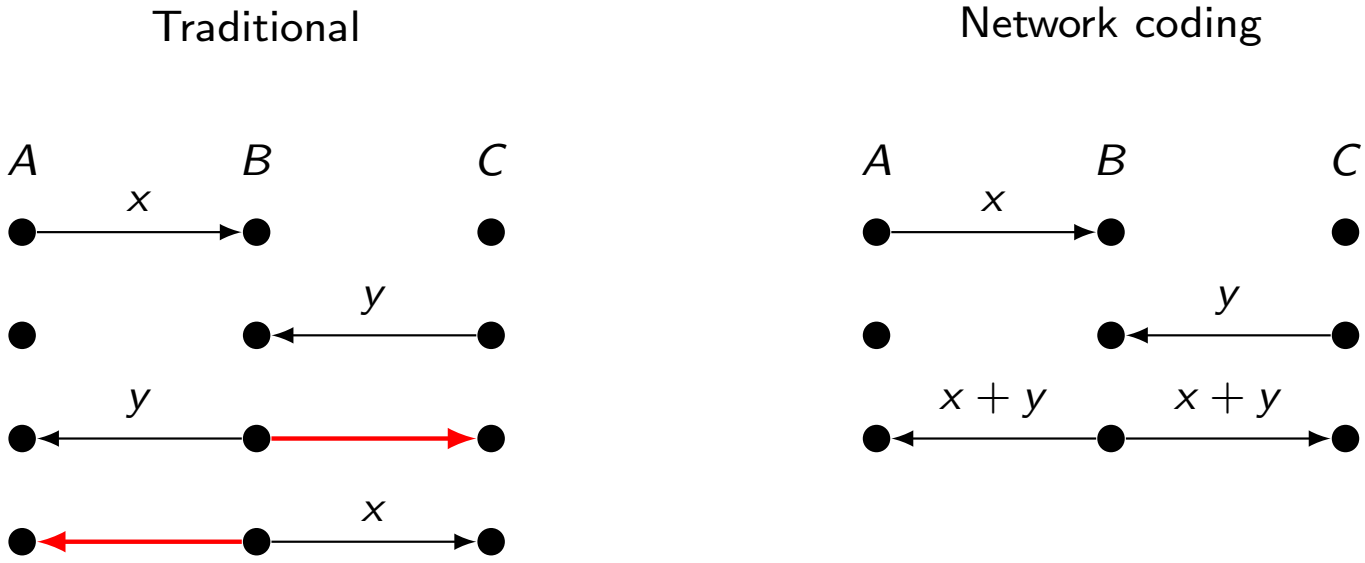
- network coding
- model
- energy consumption
- bounding queueing models
- bounds on energy consumption

Information Exchange



- Exploiting broadcast allows for more efficient use of resources.
- Reduce energy consumption by decreasing # transmissions.
- In this example: $4 \rightarrow 3$, Benefit $= \frac{4}{3}$.
- Energy savings of 50% at B by coding.

What if stochastic arrivals?

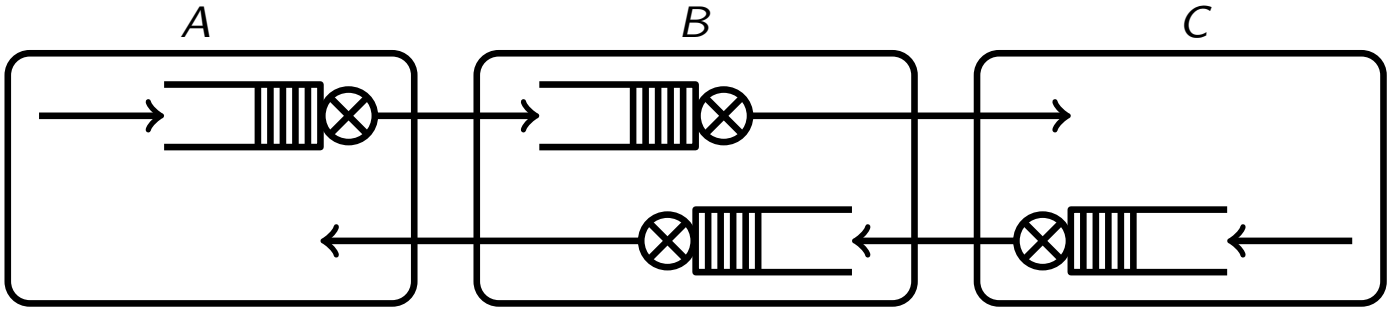


- Assumption: Always packets to send

Question

Energy savings if we have stochastic arrivals?

Information Exchange with Stochastic Arrivals



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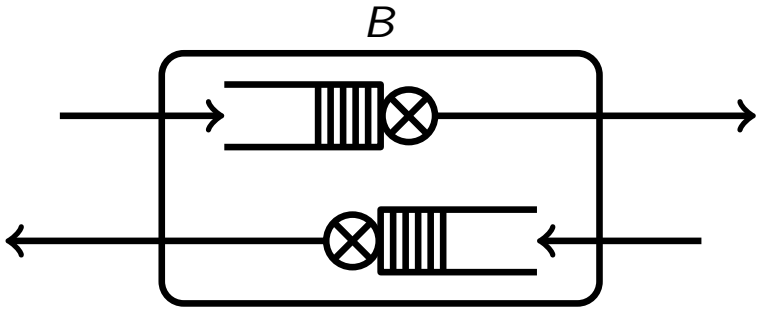
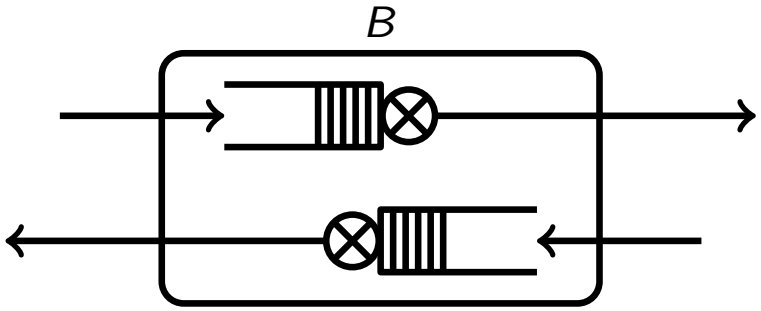


- Keep a queue for each connection
- Sending a coded packet corresponds to removing a packet from both queues
- Queues will sometimes be empty
- What to do if we have only one packet?

Wait for another packet
Delay $\Uparrow$
Energy $\Downarrow$

Transmit an uncoded packet
Delay $\Downarrow$
Energy $\Uparrow$

Information Exchange with Stochastic Arrivals



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Wait for another packet
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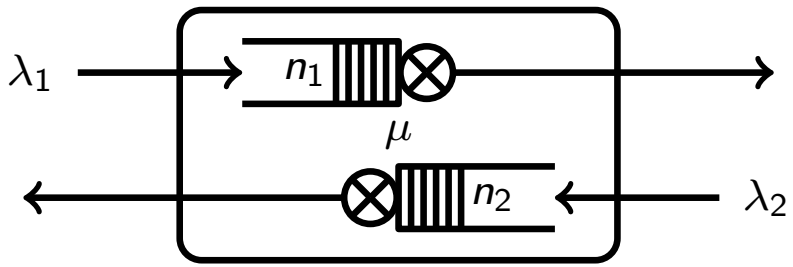
Energy $\Downarrow$

Tradeoff

Transmit an uncoded packet
Delay $\Downarrow$
Energy $\Uparrow$

Model

- Continuous-time Markov Chain
- Two queues, N_i packets in queue i
- Poisson arrivals at rate λ_i
- Fixed size packets
- Exponential service time, rate μ
- γ_i: Probability of transmitting uncoded packet from queue i



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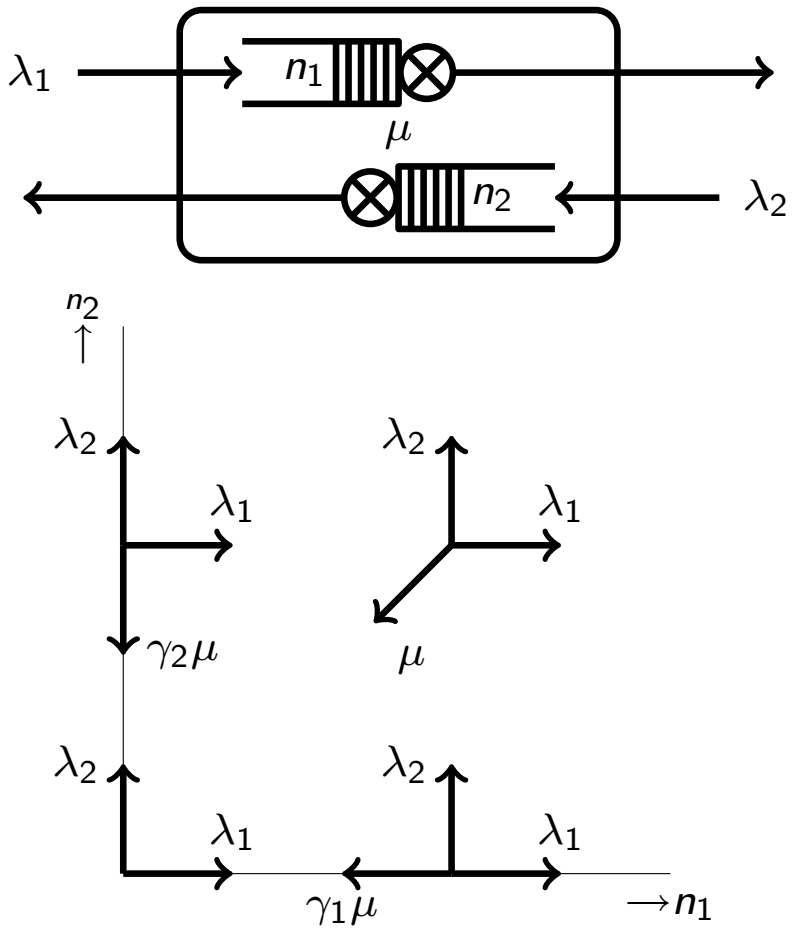

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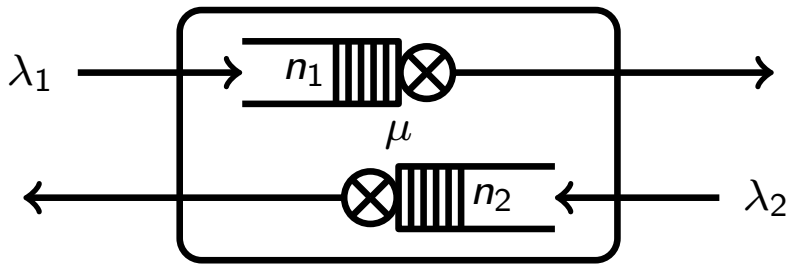
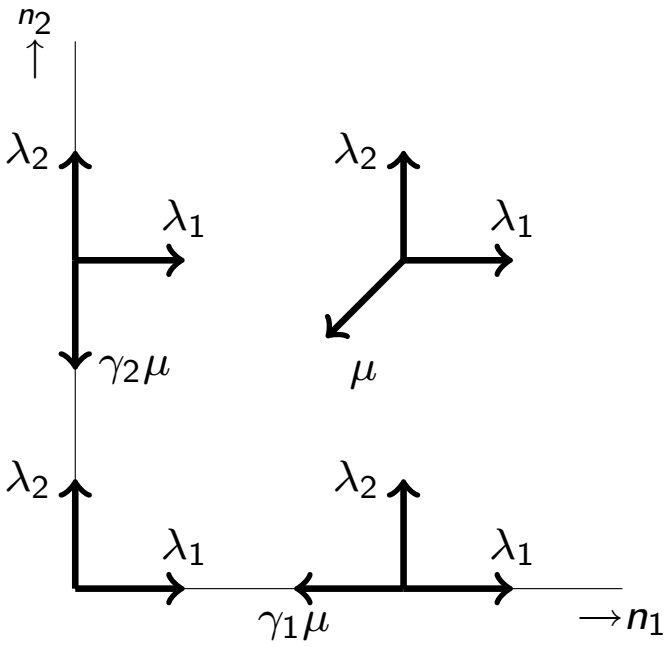




Energy consumption: $\mathcal{E} = \mathbb{E}\left[\mathbf{e}(N_1, N_2)\right]$,

$$\mathbf{e}(n_1, n_2) = \gamma_1\mu\mathbf{1}_{\{n_1>0,n_2=0\}} + \gamma_2\mu\mathbf{1}_{\{n_1=0,n_2>0\}} + \mu\mathbf{1}_{\{n_1>0,n_2>0\}}$$

Literature

Sagduyu & Ephremides [ISIT '06]

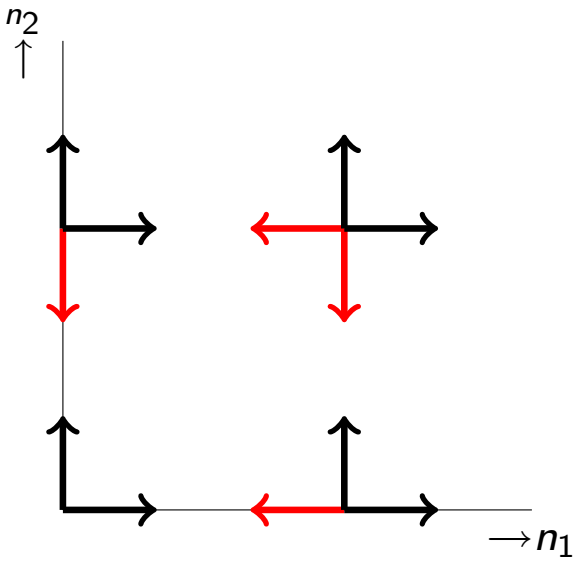
- Energy consumption, processing costs
- Scheduling between multiple sessions
- Discrete-time model

Lun et al. [Phys Comm '08]

- Random linear coding for lossy packet networks
- Propagation of innovative packets through network
- Jackson network model for single session

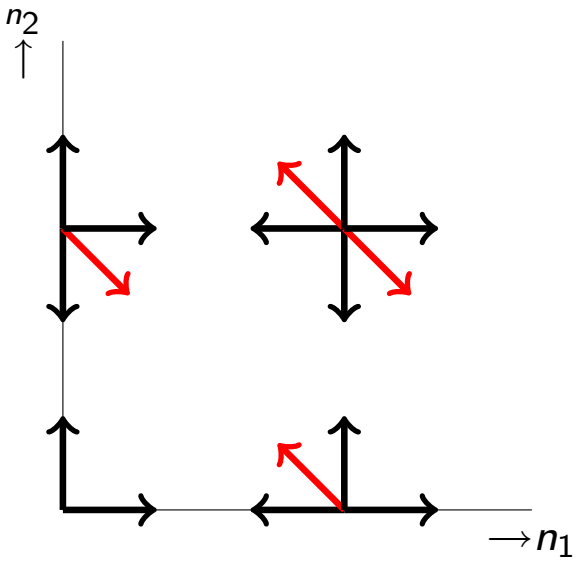
Networks of Queues

- Independent M/M/1 queues. After receiving service
  - leave network



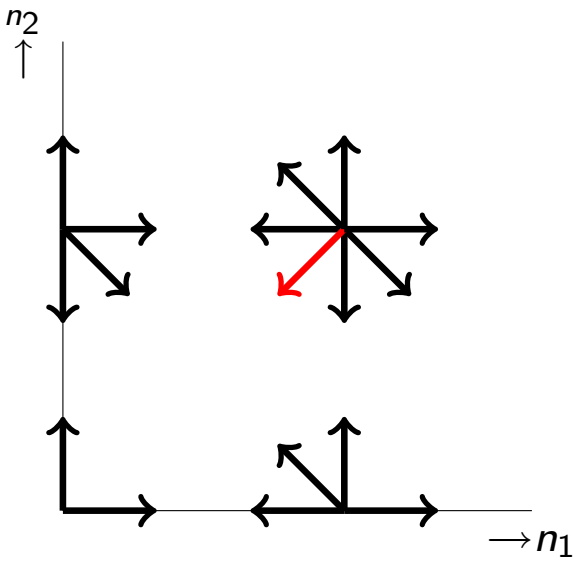
Networks of Queues

- Independent M/M/1 queues. After receiving service
  - leave network
- Jackson. After receiving service
  - leave network, or
  - enter another queue



Networks of Queues

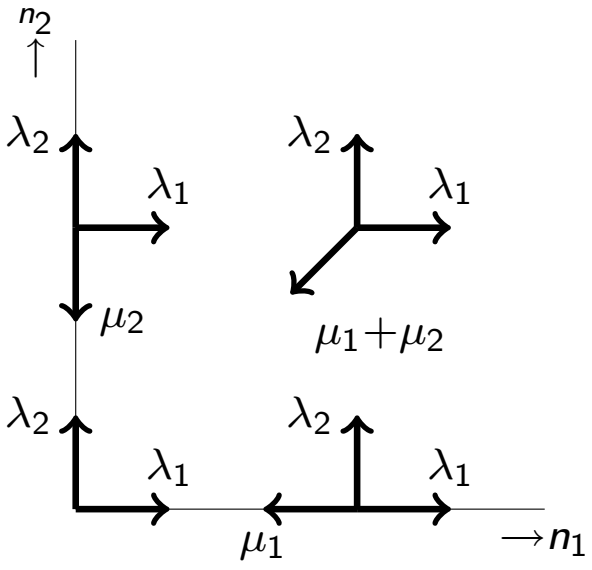
- Independent M/M/1 queues. After receiving service
  - leave network
- Jackson. After receiving service
  - leave network, or
  - enter another queue
- Gelenbe. After receiving service
  - leave network, or
  - enter another queue, or
  - remove a customer from another queue



Queueing network with negative and positive customers

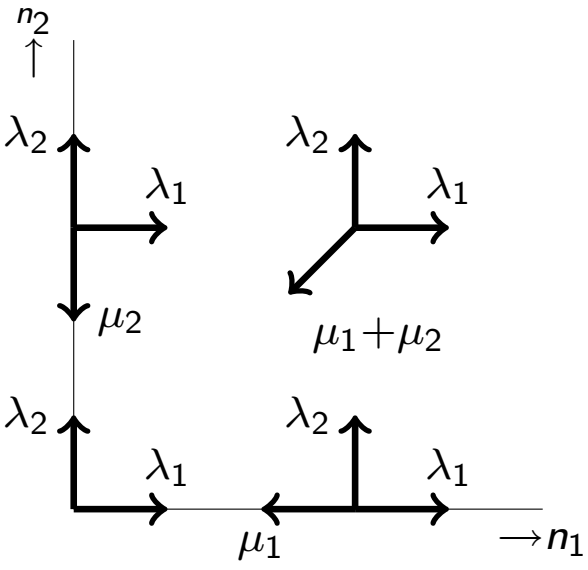
Generalizations: Remove customers from multiple queues, string transitions, etc.

Queueing Network with Negative and Positive Customers

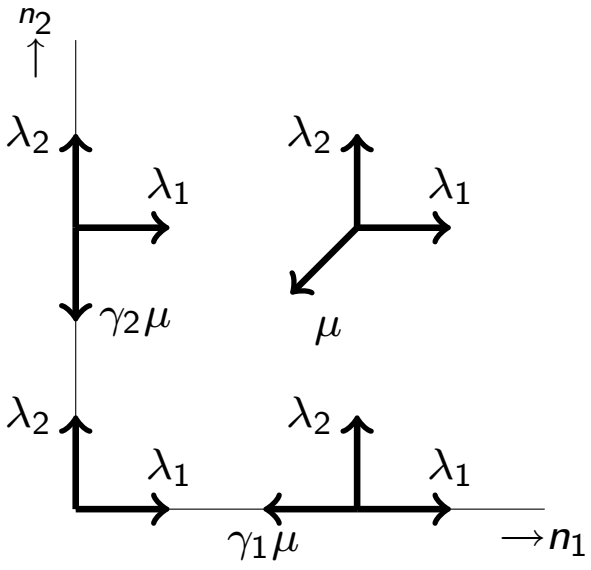


- Service at rate μ_i at queue i, $\mu_1 + \mu_2 = \mu$
- When a packet is leaving a queue, it takes along a packet from the other queue (negative customer)

Queueing Network with Negative and Positive Customers



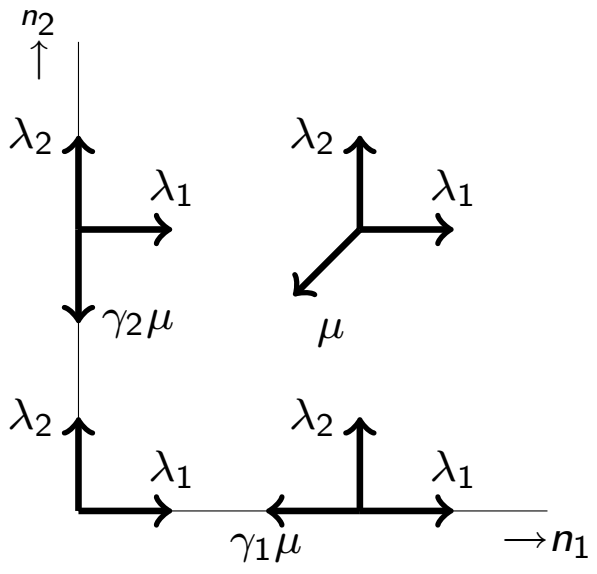
Information Exchange



- Service at rate μ_i at queue i, $\mu_1 + \mu_2 = \mu$
- When a packet is leaving a queue, it takes along a packet from the other queue (negative customer)
- Share resources, even if one queue is empty
- Require $\gamma_1 + \gamma_2 = 1$ for product-form

Queueing Network with Negative and Positive Customers

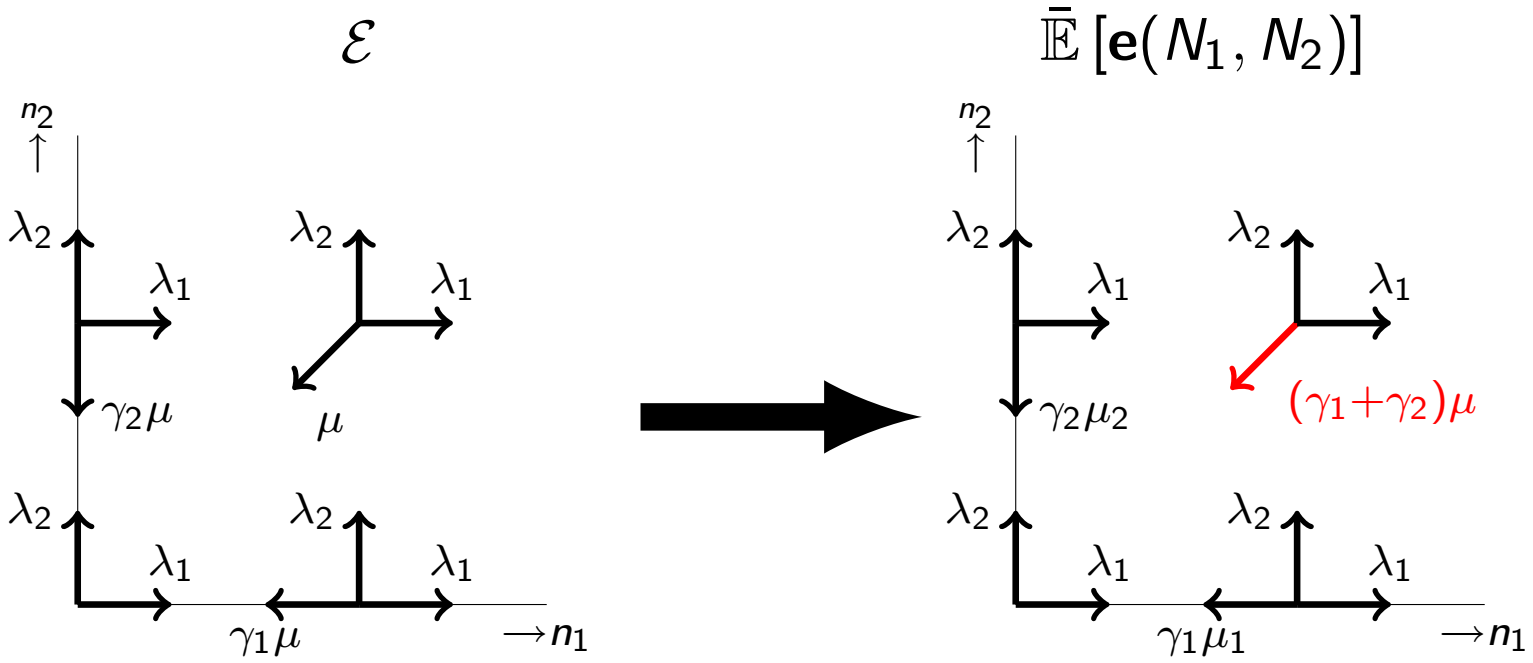
- $\gamma_1 + \gamma_2 = 1$: Product form stationary distribution (Gelenbe)
- $\pi(n_1, n_2) = (1 - q_1)q_1^{n_1}(1 - q_2)q_2^{n_2}$
  $q_1 = \lambda_1/(\mu_1 + q_2\mu_2)$
  $q_2 = \lambda_2/(\mu_2 + q_1\mu_1)$
- Distribution can be computed, energy consumption known in closed form
- Otherwise: stationary distribution unknown, not a product form



Energy consumption in case $\gamma_1 + \gamma_2 \neq 1$:

- Perturbation of process to get product form, such that
- $\bar{\mathbb{E}}\left[\mathbf{e}(N_1, N_2)\right]$ perturbed system gives upper/lower bound on $\mathcal{E}$

Bound 1



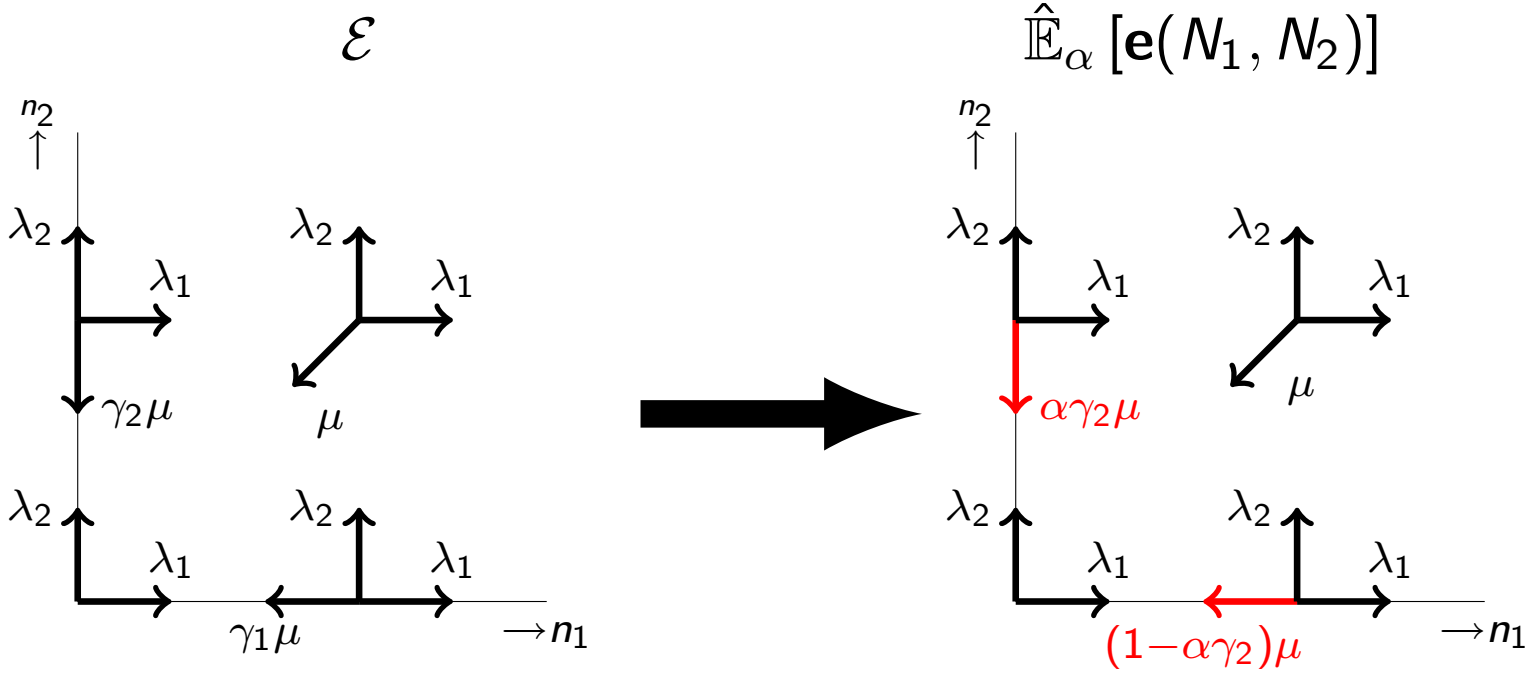
$\gamma_1 + \gamma_2 > 1$:

$$\mathcal{E} \geq \bar{\mathbb{E}}\left[\mathbf{e}(N_1, N_2)\right]$$

$\gamma_1 + \gamma_2 < 1$:

$$\mathcal{E} \leq \bar{\mathbb{E}}\left[\mathbf{e}(N_1, N_2)\right]$$

Bound 2



$\gamma_1 + \gamma_2 > 1$:

$$\mathcal{E} \leq \hat{\mathbb{E}}_\alpha\left[\mathbf{e}(N_1, N_2)\right]$$

$$\alpha = \frac{1-\gamma_1}{\gamma_2}$$

$\gamma_1 + \gamma_2 < 1$:

$$\mathcal{E} \geq \hat{\mathbb{E}}_\alpha\left[\mathbf{e}(N_1, N_2)\right]$$

$$\alpha = \frac{1}{2\gamma_2}$$

Bounds

$\gamma_1 + \gamma_2 > 1$:

$$\bar{\mathbb{E}}\left[\mathbf{e}(N_1, N_2)\right] \leq \mathcal{E} \leq \hat{\mathbb{E}}_\alpha\left[\mathbf{e}(N_1, N_2)\right] \qquad \alpha = \frac{1-\gamma_1}{\gamma_2}$$

$\gamma_1 + \gamma_2 < 1$:

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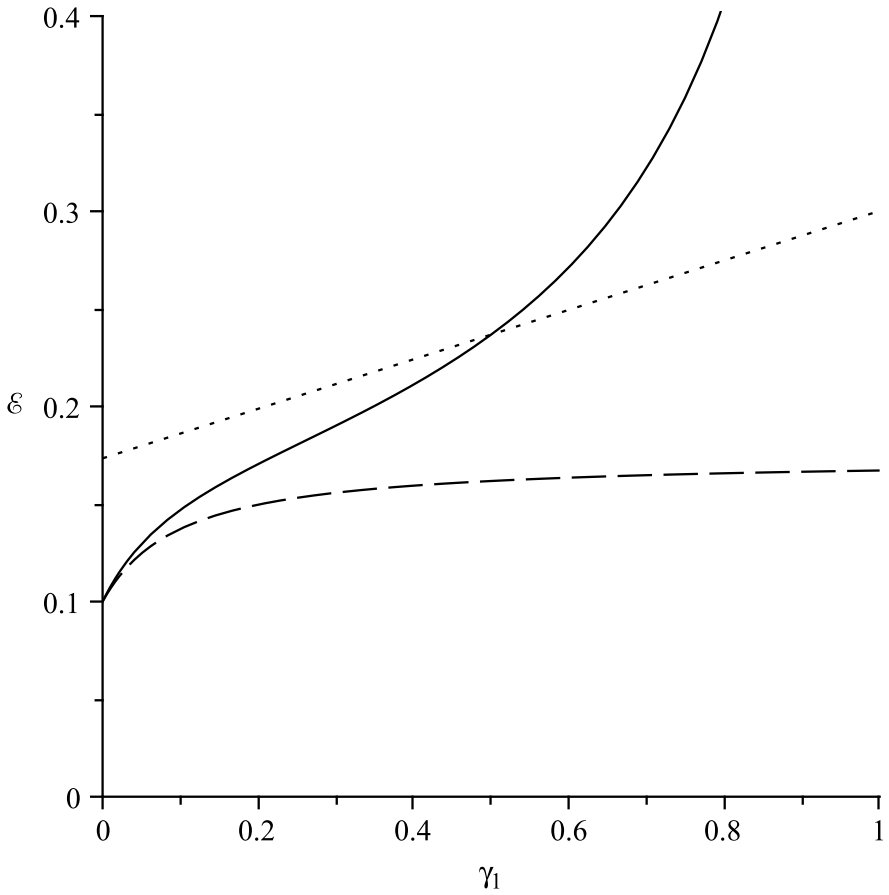
Proof: error bound technique - Van Dijk & Puterman [AAP '88]
uniformized system, $V^k(i)$ expected k step reward starting from i
Theorem: If for all $i \in S$ and $k \geq 0$

$$\sum_{j \in S} (\bar{q}(i,j) - q(i,j))[V^k(j) - V^k(i)] \geq 0$$

then $\bar{\mathbb{E}}\left[\mathbf{e}(N_1, N_2)\right] \geq \mathcal{E}$

Numerical Results

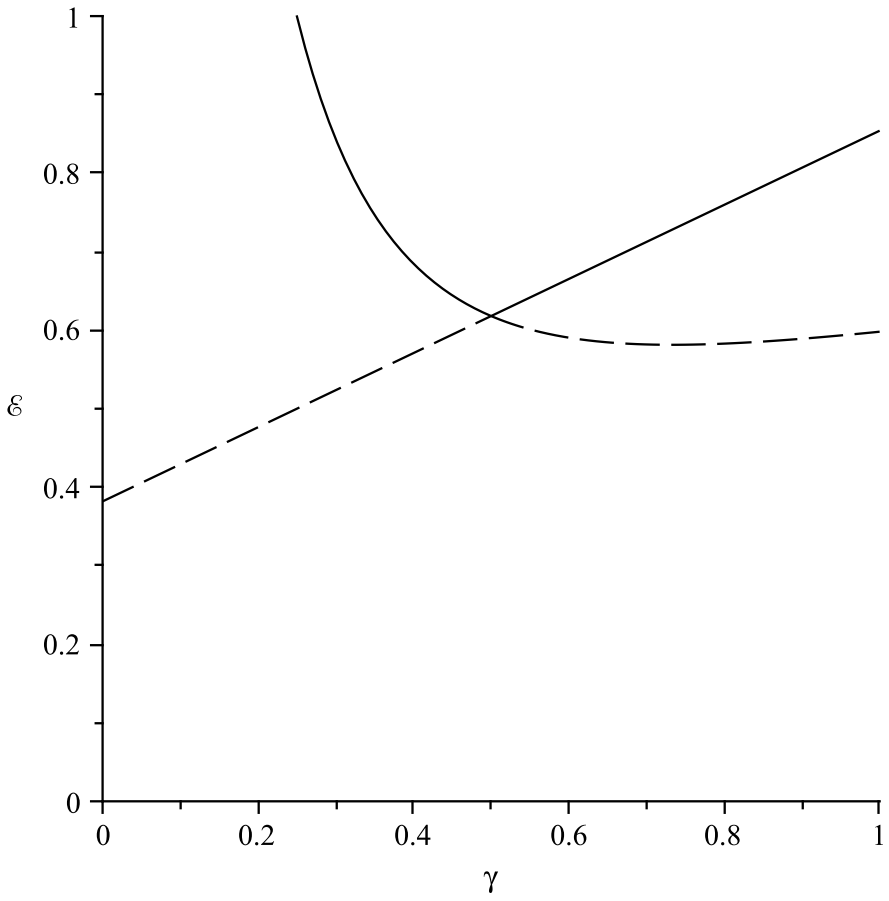
- Asymmetrical system at low load
- $\mu = 1$, $\lambda_1 = .09$, $\lambda_2 = .1$
- $\gamma_2 = 1$, so always transmit uncoded packet from queue 2
- γ_1 as a parameter



- Energy consumption of an uncoded system: 0.19
- Obtain maximum energy savings for $\gamma_1 \to 0$

Numerical Results

- Symmetrical system
- $\mu = 1$, $\lambda_1 = \lambda_2 = 0.5$
- $\gamma_1 = \gamma_2 = \gamma$ as a parameter



- Exact results at $\gamma = 0.5$
- Bounds not useful for all parameters

Conclusions

- Relation between network coded systems and queuing networks with negative and positive customers
- Obtain product-form for specific parameters
- Use Markov reward techniques to obtain bounds on energy consumption

Ongoing work:

- Generalize to other networks
- Obtain bounds on other performance measures, *i.e.*, delay and queue sizes