## TU/e

## Soft loss systems

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## TU/e

Alternative title

## Intensive Care Queues

## TU/e



## TU/e

## Costs of an IC bed

- If you stay overnight in a hospital, you'll probably pay at least $\$ 500$. Plus another $\$ 500$ if you got there in an ambulance
- An ICU bed costs you about $\$ 3500$ a night
- A kidney transplant operation only costs 4 K


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- In NL about 2000 IC beds
- IC units are different
- Within a radius of 7 km 4 hospitals


## TU/e

## Linda Green

## TU/e

- Waiting for an IC bed is difficult
- Accepting IC as a loss system is doubtful
- What to do?


## TU/e

## Aim

- Get some insight into what happens
- Get a feeling for what might be useful models


## TU/e

## Outline

- Playing with a number of simple models
- Looking at numbers
- Hoping to get some insight


## TU/e

## Simple Models

IC beds


## Model 1: Pure loss

- Poisson arrivals (in all models)
- Parameters
- Beds
- Load
- Performance
- Blocked patients


## TU/e

## Loss formula

$$
B(\rho, s)=\frac{\rho^{s} / s!}{\sum_{l=0}^{s} \rho^{l} / l!}
$$

## TU/e

## How much load?

- Given a loss probability
- Given the number of beds


## TU/e

## Loss probability 0.1

| s | Total off load | Acc load / bed | Beds / patient |
| ---: | :---: | :---: | :---: |
| 4 | 2.046 | 0.460 | 2.172 |
| 6 | 3.758 | 0.564 | 1.774 |
| 8 | 5.597 | 0.630 | 1.588 |
| 10 | 7.510 | 0.676 | 1.479 |
| 12 | 9.474 | 0.711 | 1.407 |
| 14 | 11.474 | 0.738 | 1.356 |
| 16 | 13.500 | 0.759 | 1.317 |
| 18 | 15.546 | 0.777 | 1.287 |
| 20 | 17.613 | 0.793 | 1.262 |

## TU/e

## Loss probability 0.05

| s | Total off load | Acc load / bed | Beds / patient |
| ---: | :---: | :---: | :---: |
| 4 | 1.524 | 0.362 | 2.764 |
| 6 | 2.960 | 0.469 | 2.133 |
| 8 | 4.543 | 0.540 | 1.854 |
| 10 | 6.216 | 0.590 | 1.694 |
| 12 | 7.950 | 0.629 | 1.589 |
| 14 | 9.730 | 0.660 | 1.515 |
| 16 | 11.545 | 0.685 | 1.459 |
| 18 | 13.386 | 0.706 | 1.415 |
| 20 | 15.248 | 0.724 | 1.381 |

## TU/e

## Model 2: temporary overflow

- If all beds are occupied, the patient is taken care of elsewhere (wrong bed)
- As soon as a bed becomes available, the patient is moved to the right bed


## TU/e

## In wrong bed; off load $0.7,0.8$ and 0.9

| s | 0.7 | 0.8 | 0.9 |
| ---: | :---: | :---: | :---: |
| 4 | 0.253 | 0.394 | 0.571 |
| 6 | 0.242 | 0.420 | 0.661 |
| 8 | 0.222 | 0.428 | 0.725 |
| 10 | 0.201 | 0.426 | 0.773 |
| 12 | 0.181 | 0.419 | 0.811 |
| 14 | 0.161 | 0.408 | 0.840 |
| 16 | 0.143 | 0.395 | 0.864 |
| 18 | 0.127 | 0.382 | 0.883 |
| 20 | 0.113 | 0.367 | 0.898 |

## TU/e

## Model 3; priority

- One reserved bed for emergency patients
- Two types 50/50
- Exponential stay times, mean 3 days
- Offered load 0.8 per bed


## TU/e

## One reserved bed

| s | B 1 | $\mathrm{~B} 1 / \mathrm{yr}$ | B 2 | $\mathrm{~B} 2 / \mathrm{yr}$ |
| ---: | :---: | ---: | ---: | ---: |
| 4 | 0.129 | 25.1 | 0.451 | 87.7 |
| 6 | 0.097 | 28.3 | 0.339 | 99.1 |
| 8 | 0.078 | 30.3 | 0.272 | 106.0 |
| 10 | 0.065 | 31.5 | 0.227 | 110.3 |
| 12 | 0.055 | 32.2 | 0.193 | 112.9 |
| 14 | 0.048 | 32.6 | 0.168 | 114.1 |
| 16 | 0.042 | 32.7 | 0.147 | 114.5 |
| 18 | 0.037 | 32.6 | 0.130 | 114.1 |
| 20 | 0.033 | 32.3 | 0.116 | 113.1 |

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Model 4: two phases, push ahead


## TU/e

## Details

- Phase 1: exponential, mean 2 days (more critical)
- Phase 2: exponential, mean 1 day (almost ok)
- If full and new arrival: a phase 2 patient leaves
- Offered load 0.8 per bed


## TU/e

## Two phases; leaving early

| s | P earlier | P full |
| ---: | :---: | :---: |
| 4 | 0.2170 | 0.1094 |
| 6 | 0.2022 | 0.0636 |
| 8 | 0.1854 | 0.0394 |
| 10 | 0.1690 | 0.0253 |
| 12 | 0.1535 | 0.0166 |
| 14 | 0.1392 | 0.0111 |
| 16 | 0.1263 | 0.0075 |
| 18 | 0.1146 | 0.0051 |
| 20 | 0.1040 | 0.0035 |

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Model 5; variant of model 4

## TU/e

## More variation; $\mathrm{c}^{2}=13 / 9$



## TU/e

## Details

- Phase 1: exponential, mean 4 days
- Phase 2: exponential, mean 1 day
- Average time 3 days
- If full and a new phase 1 arrives: a phase 2 leaves


## TU/e

## Dismissed faster if full

| s | P Faster | B 1 | $\mathrm{~B} 1 / \mathrm{yr}$ | B 2 | $\mathrm{~B} 2 / \mathrm{yr}$ |
| ---: | :---: | :---: | ---: | :---: | ---: |
| 4 | 0.121 | 0.109 | 21.3 | 0.301 | 58.6 |
| 6 | 0.111 | 0.064 | 18.6 | 0.251 | 73.3 |
| 8 | 0.101 | 0.039 | 15.3 | 0.215 | 83.9 |
| 10 | 0.091 | 0.025 | 12.3 | 0.188 | 91.4 |
| 12 | 0.082 | 0.017 | 9.7 | 0.165 | 96.6 |
| 14 | 0.074 | 0.011 | 7.6 | 0.147 | 99.9 |
| 16 | 0.066 | 0.007 | 5.8 | 0.131 | 101.8 |
| 18 | 0.060 | 0.005 | 4.5 | 0.117 | 102.6 |
| 20 | 0.054 | 0.003 | 3.4 | 0.105 | 102.4 |

## TU/e

Summary; load 0.8 and 10 beds

- 1: Loss 0.122


## TU/e

## Summary; load 0.8 and 10 beds

- 1: Loss 0.122
- 2: To wrong bed 0.28 , in wrong bed 0.43


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- 1: Loss 0.122
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- 4: Loss 0.025, Early 0.17


## TU/e

## Summary; load 0.8 and 10 beds

- 1: Loss 0.122
- 2: To wrong bed 0.28 , in wrong bed 0.43
- 3: Loss 1 0.065, Loss 20.23
- 4: Loss 0.025, Early 0.17
- 5: Loss 1 0.025, Loss 2 0.19, Early 0.09


## TU/e

## Conclusions

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- Many relevant models


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- Many relevant models
- No clear answers


## TU/e

## Conclusions

- Many relevant models
- No clear answers
- Small IC units are very expensive and inefficient

