Performance Analysis of Production Lines with Continuous Material Flows and Finite Buffers

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ELA Doctorate Workshop

Technische Universiteit
Eindhoven
University of Technology

Where innovation starts

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  - Results
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Case study Heineken

- **Heineken Brewery ‘s-Hertogenbosch**

- **Packaging department**
- **Production line for retour bottles**
  - **Keyword: efficiency**

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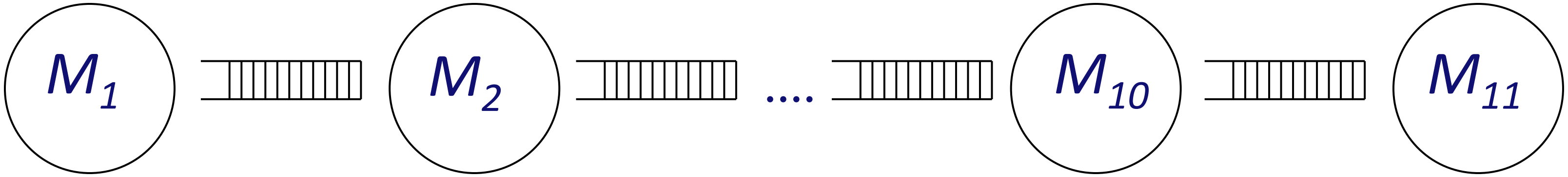
Case study Heineken

- **Production line 15A**



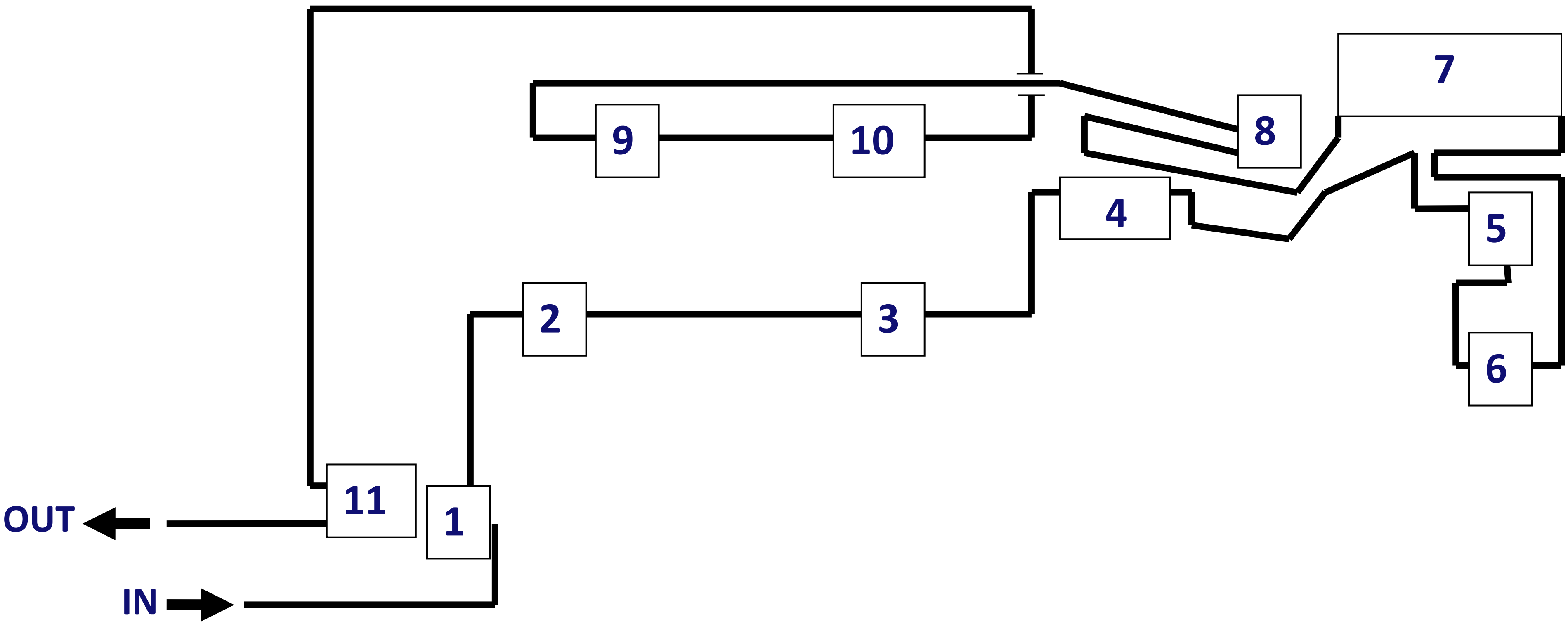
- **Eleven machines**

Depalletizer	**Pasteurizer**
Logo detection	**Labeller**
Depacker	**Packer**
Washing machine	**Crate packer**
Empty bottle inspector	**Palletizer**
Filler	

Case study Heineken

Case study Heineken

- **Production line 15A – lay-out**

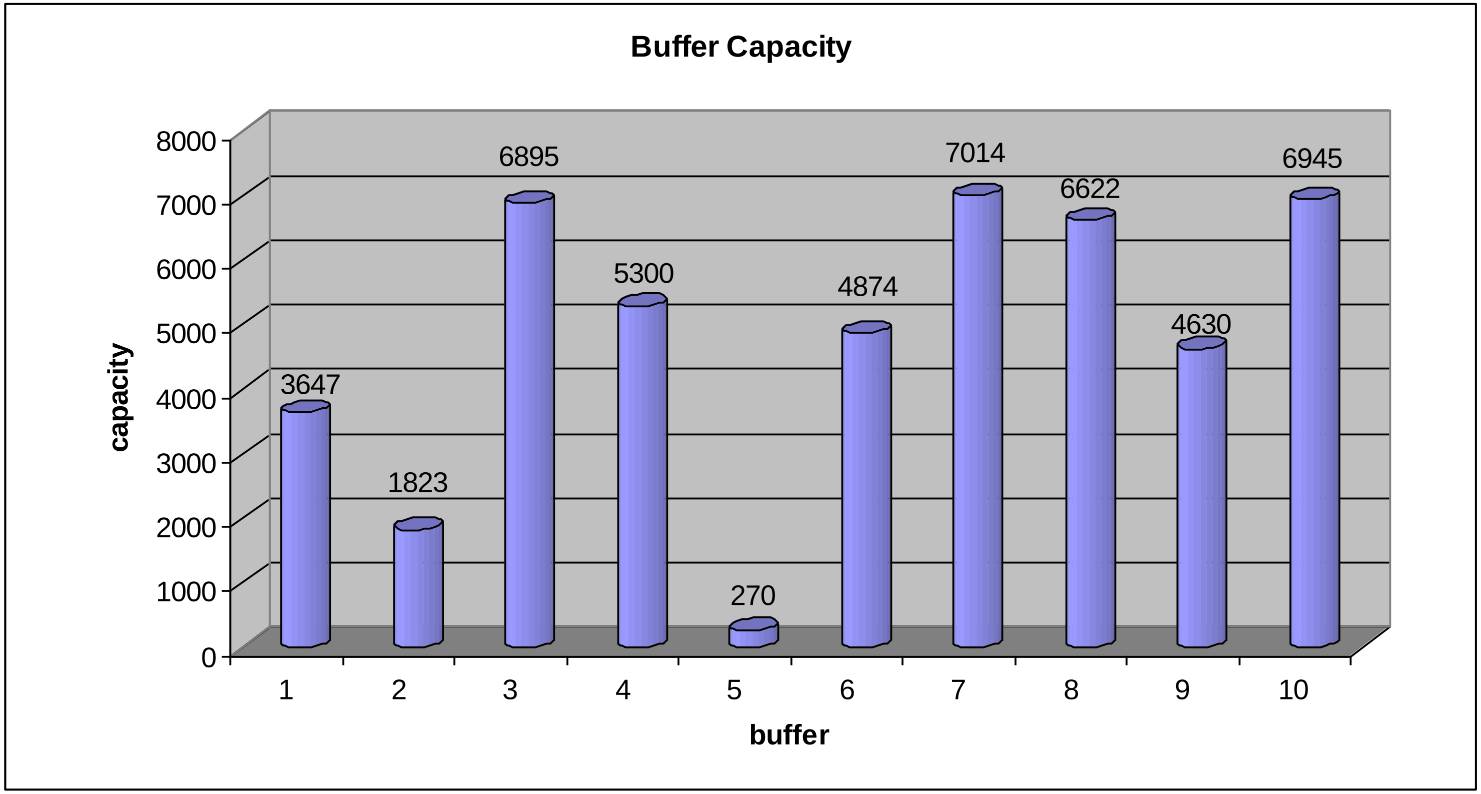


Case study Heineken

- Input variables
  - Buffer sizes (e.g. 5000 bottles)
  - Machine speeds (e.g. 39000 bottles/hour)
- Up and down behavior
  - Each machine breaks down now and then, because of e.g. failures, cleaning, change over, ...
  - We aggregate the breakdown (up) times into one single distribution
  - Same for the repair (down) times
  - Assumption: operational dependent failures
- Machines adjust their speed if necessary
- Assumption: flow of bottles behaves like a fluid

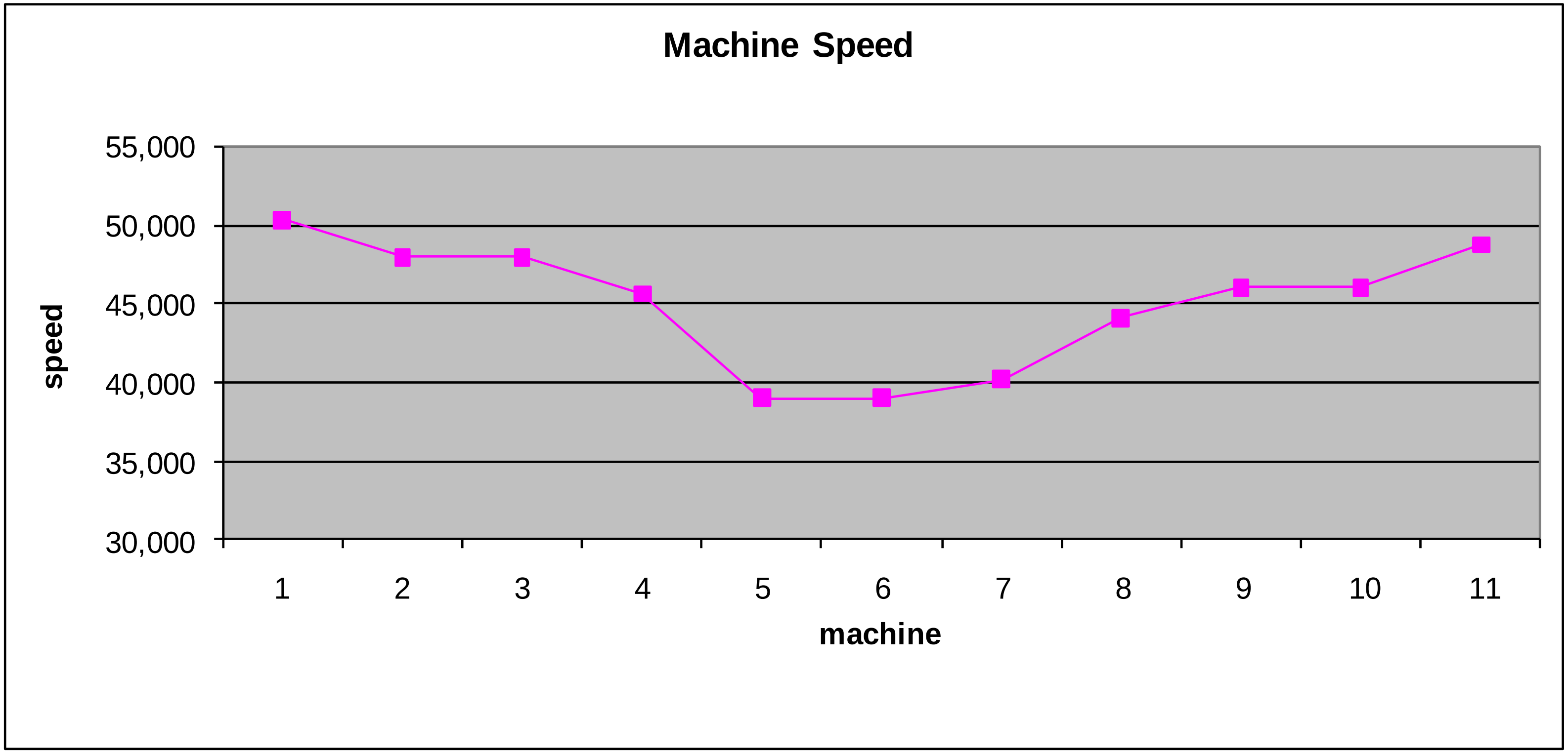
Case study Heineken

- **Buffer sizes**



Case study Heineken

- **Machine speeds**



Case study Heineken

nr	Machine	start date	start time	end date	end time	duration	cause	effect
15A	Etiketteerder	5-3-2008	01:55:37	5-3-2008	01:56:41	64	Leegloop	Ongeplande stilstand
15A	Etiketteerder	5-3-2008	01:56:41	5-3-2008	01:57:36	55	Productie	Goed product
15A	Etiketteerder	5-3-2008	01:57:36	5-3-2008	01:58:05	29	Interne Storing	Ongeplande stilstand
15A	Etiketteerder	5-3-2008	01:58:05	5-3-2008	01:59:00	55	Productie	Goed product
15A	Etiketteerder	5-3-2008	01:59:00	5-3-2008	01:59:18	18	Interne Storing	Ongeplande stilstand
15A	Etiketteerder	5-3-2008	01:59:18	5-3-2008	02:01:44	146	Productie	Goed product
15A	Etiketteerder	5-3-2008	02:04:54	5-3-2008	02:05:03	9	Volloop	Ongeplande stilstand
15A	Etiketteerder	5-3-2008	02:05:03	5-3-2008	02:20:11	908	Productie	Goed product
15A	Etiketteerder	5-3-2008	02:20:12	5-3-2008	02:23:30	198	Interne Storing	Ongeplande stilstand
15A	Etiketteerder	5-3-2008	02:23:30	5-3-2008	02:25:20	110	Productie	Goed product
15A	Etiketteerder	5-3-2008	02:25:21	5-3-2008	02:26:50	89	Interne Storing	Ongeplande stilstand
15A	Etiketteerder	5-3-2008	02:26:50	5-3-2008	02:27:45	55	Productie	Goed product
15A	Etiketteerder	5-3-2008	02:27:45	5-3-2008	02:28:23	38	Interne Storing	Ongeplande stilstand
15A	Etiketteerder	5-3-2008	02:28:23	5-3-2008	02:43:28	905	Productie	Goed product

Case study Heineken

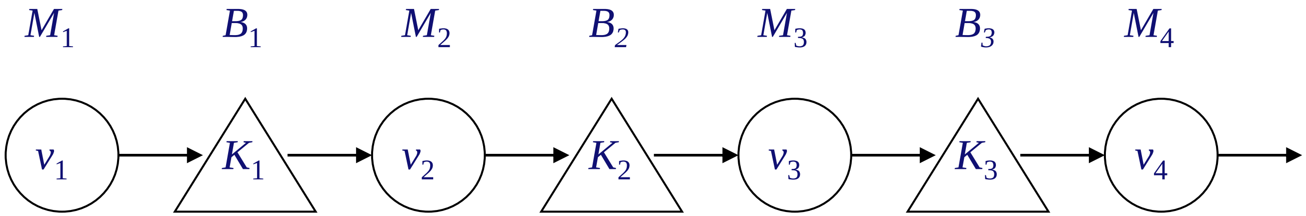
- Main objective: throughput
- Other output variables
  - Leadtimes
  - Buffer usage
  - Actual machine speeds
  - ...
- Use simulation to estimate output variables
  - Difference simulated<->actual throughput is less than 1%
- Question: can we do with less buffer space?

Case study Heineken

- Results
  - If you decrease the right buffers, total buffer space can be reduced by almost 25% without any loss in throughput
  - Increasing certain other buffers can increase throughput
  - Making the filler and labeller more reliable will give the highest gain in throughput

- Step towards optimization
  - Need for more efficient estimates: analytical model

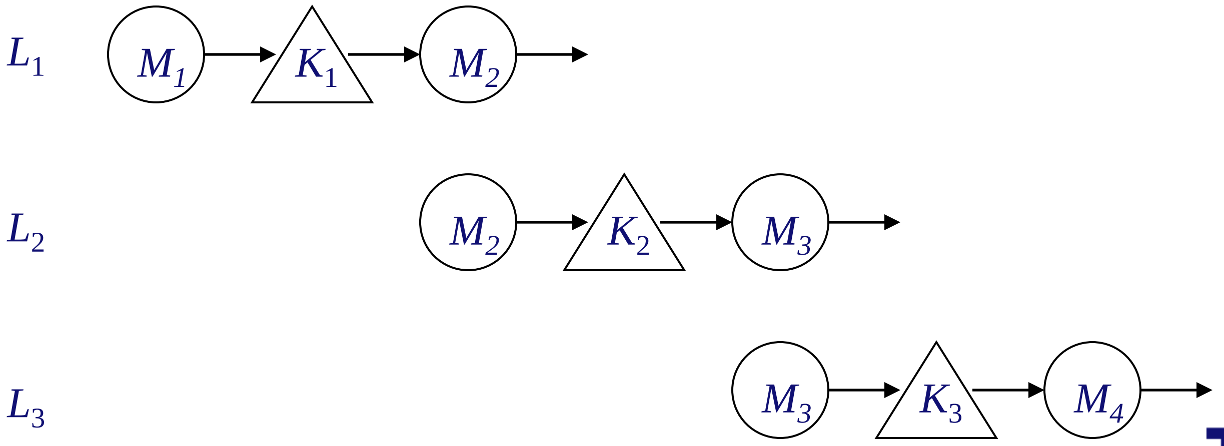
Production lines with exponential up- and downtimes



- **Production line with servers M_i in tandem and buffers B_i of size K_i in between**
- **Flow through the machines is continuous (fluid flow)**
- **For each machine M_i, the maximum speed v_i is known**
- **Machine is 'up' for a random time with mean E[U_i], after which it is 'down' for a random time with mean E[V_i]**
  - **Assumption: up- and downtimes are exponentially distributed**
  - **Later: generally distributed up- and downtimes**
- **Operational dependent failures**
- **Goal: estimate throughput and sojourn time**

Production lines with exponential up- and downtimes

- **Exact analyses exist for two machines with a buffer in between**
- **Decompose the production line into subsystems**
- **Obtain the throughput and sojourn time of the production line by using iteration**



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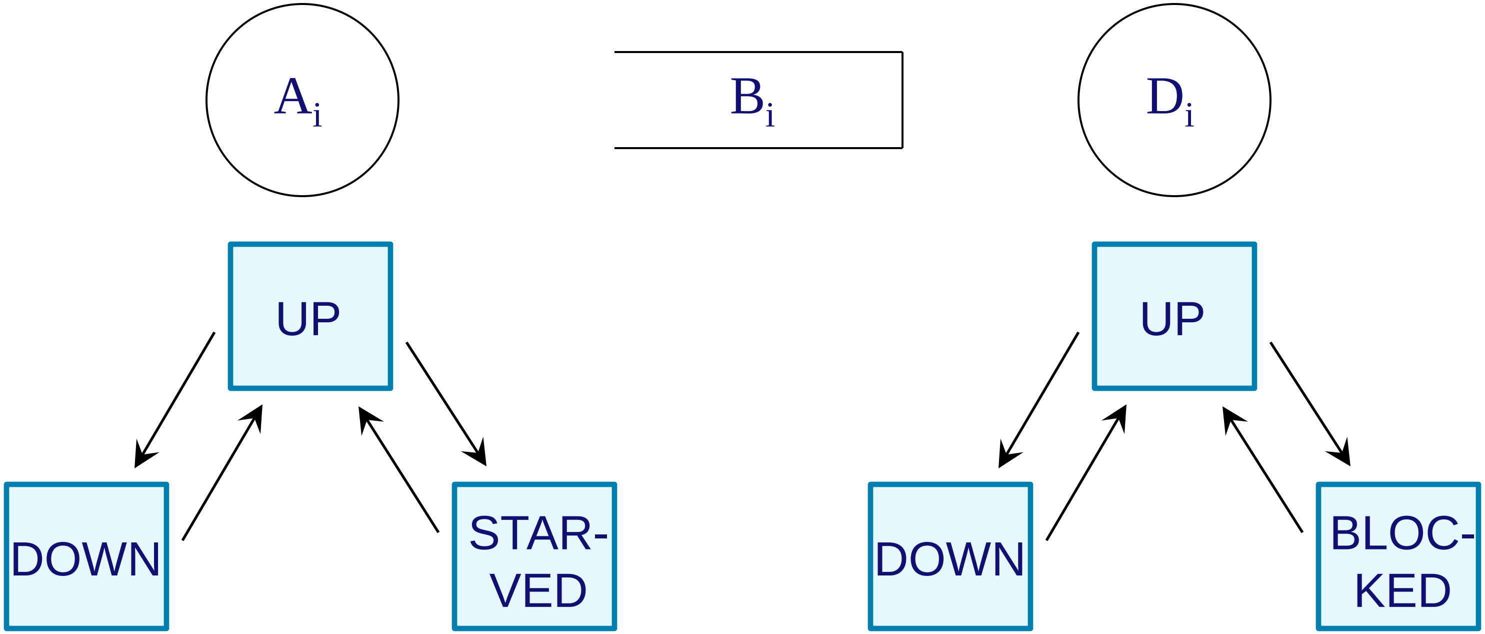
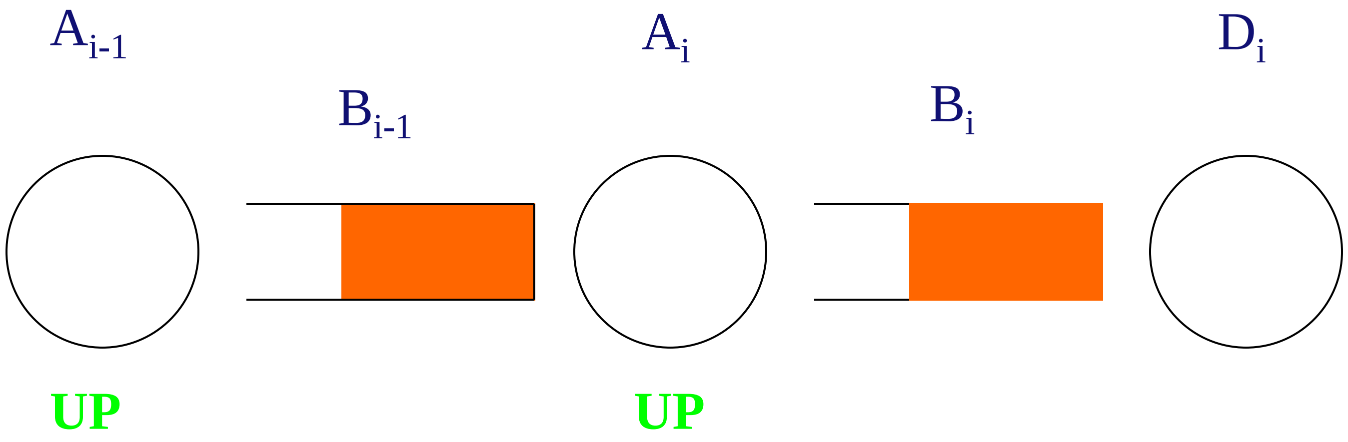
Production lines with exponential up- and downtimes

- **Three-state Markov chains for server behaviors**



- **No transitions possible between down and starved/blocked**
- **To determine**
  - **speed in upstate: $v_A(i)$, $v_D(i)$**
  - **transition rates**

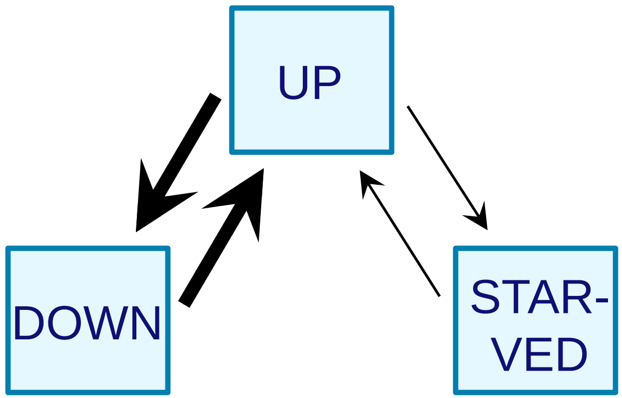
Production lines with exponential up- and downtimes



- **Arrival machine can take two possible speeds in state 'up':**
  - **Original speed of arrival machine (v_i) when the buffer in front is not empty**
  - **Speed of previous arrival machine ($v_A(i-1)$) when the buffer is empty, A_{i-1} is up and $v_A(i-1) < v_i$**

Production lines with exponential up- and downtimes

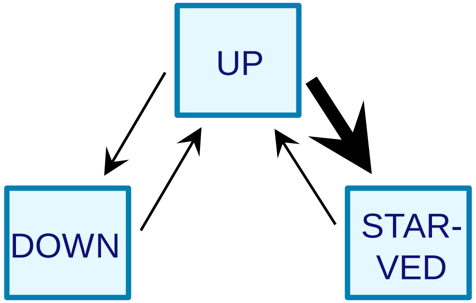
- **Transition rates (arrival machine)**

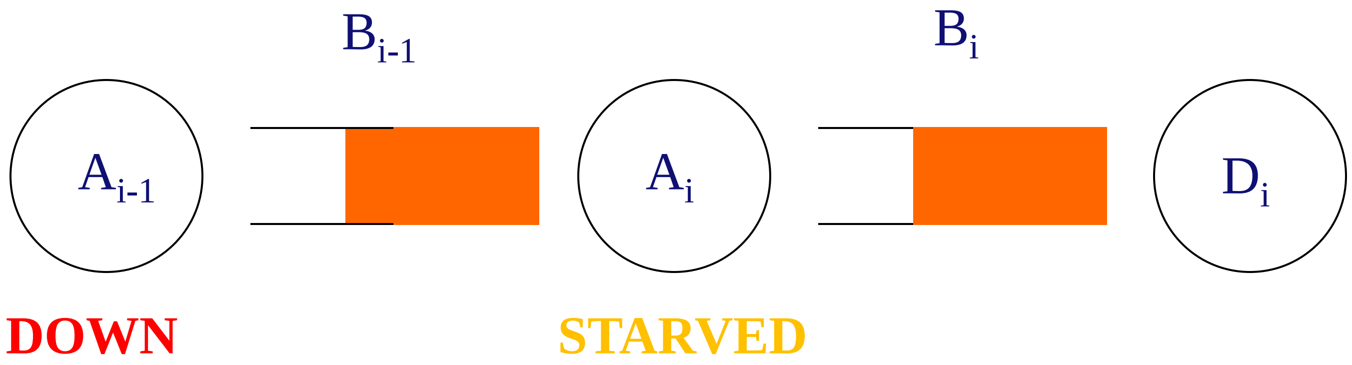


- **Up to down: Actual breakdown with rate $1/E(U_i)$**
- **Down to up: Repair time with rate $1/E(V_i)$**

Production lines with exponential up- and downtimes

- **Transition rates (arrival machine)**

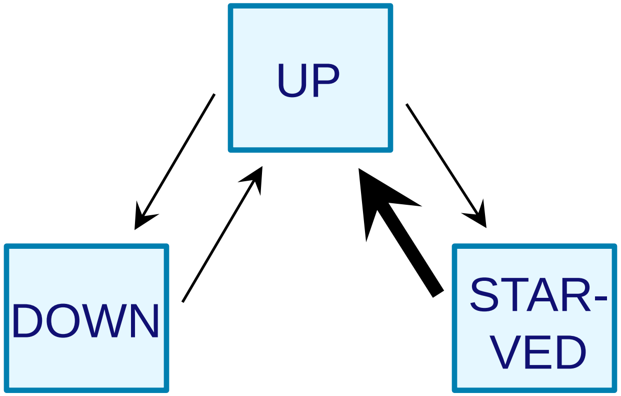




- **Arrival machine can go from up to starved when:**
  - **Buffer B_{i-1} is empty and arrival machine A_{i-1} goes down**
  - **Arrival machine A_{i-1} is already down and B_{i-1} goes from non-empty to empty**

Production lines with exponential up- and downtimes

- **Transition rates (arrival machine)**



- **Arrival machine can be starved for two reasons:**
  - **Previous arrival machine is down**
  - **Previous arrival machine is starved**
- **Take a weighted average over the expected times down->up and down->starved of A_{i-1}**

Production lines with exponential up- and downtimes

- **Iteration:**
  - **Step 0: Initialize Markov chains and average lower speeds for all subsystems**
  - **Step 1: For subsystems $i=1,\dots,N-1$;**
    - **Update parameters for the arrival machine**
    - **Determine the buffer content distribution using matrix analytic techniques**
  - **Step 2: For subsystems $i=1,\dots,N-1$;**
    - **Update parameters for the departure machine**
  - **Step 3: Repeat step 1 and 2 until the throughputs have converged**

Production lines with exponential up- and downtimes

- Test set of 1728 cases
- Parameters:
  - Number of machines: 4, 8, 12, 16
  - Mean uptimes: 5, 10, 20
    - Imbalance by dividing values for even machines by two
  - Mean downtimes: 0.5, 1, 2
    - Imbalance by dividing values for even machines by two
  - Machine speeds:
    - 10,10,10,10,…
    - 10,15,10,15,…
    - 15,…,10,…,15 (V-shape)
  - Buffer sizes: 1, 10, 25, 50

Production lines with exponential up- and downtimes

- **Results; number of machines in production line**

	% difference in throughput		**% difference in mean total buffer content**	
	Two-state	Three-state	Two-state	Three-state
4	0,50%	0,44%	0,63%	0,60%
8	1,39%	1,24%	0,55%	0,50%
16	2,13%	1,93%	0,57%	0,52%
24	2,74%	2,52%	0,63%	0,58%

Production lines with exponential up- and downtimes

- **Results; mean uptimes**

	% difference in throughput		% difference in mean total buffer content	
	Two-state	Three-state	Two-state	Three-state
5,5,5,5,…	2,22%	2,01%	0,72%	0,66%
5,2.5,5,2.5,…	2,32%	2,08%	0,72%	0,67%
10,10,10,10,…	1,59%	1,45%	0,60%	0,55%
10,5,10,5,…	1,78%	1,62%	0,58%	0,54%
20,20,20,20,…	1,01%	0,93%	0,50%	0,46%
20,40,20,40,…	1,21%	1,11%	0,47%	0,43%

Production lines with exponential up- and downtimes

- **Results; mean downtimes**

	% difference in throughput		% difference in mean total buffer content	
	Two-state	Three-state	Two-state	Three-state
0.5,0.5,0.5,0.5,…	0,88%	0,88%	0,28%	0,28%
0.5,0.25,0.5,0.25,…	0,95%	0,83%	0,27%	0,26%
1,1,1,1,…	1,54%	1,54%	0,31%	0,31%
1,0.5,1,0.5,…	1,81%	1,53%	0,42%	0,36%
2,2,2,2,…	2,08%	2,08%	0,79%	0,79%
2,4,2,4,…	2,87%	2,34%	1,51%	1,31%

Production lines with exponential up- and downtimes

- **Results for increasing imbalance in mean downtimes**
  - **Six-machine line, uptimes of 1, speeds of 10, buffers of 10**

	% difference in throughput	
	Two-state	Three-state
1,1,1,…	1,42%	1,42%
1.33,1,0.67,…	2,06%	1,78%
2,1,0.5,…	3,71%	2,51%
4,1,0.25,…	7,96%	3,64%
10,1,0.01,…	12,09%	4,21%

Production lines with exponential up- and downtimes

- **Results; machine speeds**

	% difference in throughput		% difference in mean total buffer content	
	Two-state	Three-state	Two-state	Three-state
10,10,10,10,…	1,52%	1,37%	0,24%	0,24%
10,15,10,15,…	2,88%	2,68%	1,46%	1,34%
15,…,10,…,15	0,66%	0,55%	0,09%	0,07%

Production lines with exponential up- and downtimes

- **Results; buffer sizes**

	% difference in throughput		% difference in mean total buffer content	
	Two-state	Three-state	Two-state	Three-state
1,1,1,1,…	0,86%	0,78%	0,18%	0,19%
10,10,10,10,…	2,42%	2,16%	0,32%	0,31%
25,25,25,25,…	2,02%	1,83%	0,54%	0,47%
50,50,50,50,…	1,46%	1,36%	1,35%	1,23%

Production lines with exponential up- and downtimes

- Heineken case
  - Production line with 11 machines
  - Imbalance in up- and downtimes
  - Speeds are ordered in a V-shape
- Comparison with simulation
- Results
  - Two state approach: 1,0% error in throughput
  - Three-state approach: 0,8% error in throughput

- Difference with actual throughput: 18,4%

Production lines with exponential up- and downtimes

- **Heineken test case: coefficients of variation**

Machine	Coeff of variaton	
	Uptimes	Downtimes
1	1.24	4.04
2	1.48	6.17
3	1.67	3.19
4	1.38	5.89
5	1.76	6.04
6	1.64	4.35
7	2.51	1.82
8	1.45	4.37
9	1.70	4.14
10	2.02	1.82
11	1.52	5.04

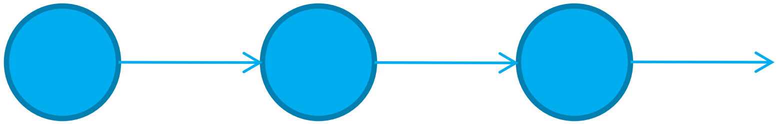
- **Conclusion: we need to include the variance of up- and downtimes**

Production lines with generally distributed up- and downtimes

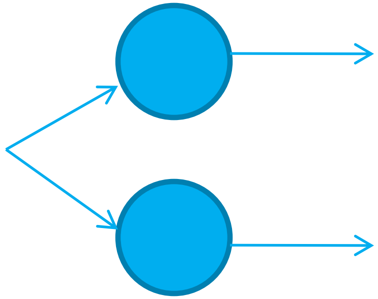
- **Next step: up- and downtimes can be non-exponential**
- **Machine is 'up' for a random time with first and second moment E[U_i] and Var[U_i], after which it is 'down' for a random time with first and second moment E[V_i] and Var[V_i]**
- **Use phase-type distributions**

Production lines with generally distributed up- and downtimes

- Exponential distribution: memory-less property
- Phase-type distributions: consist of more "phases" of the exponential distribution
- For each random variable X with mean E(X) and variance Var(X), we can fit a phase-type distribution
- Examples
  - Erlang distribution

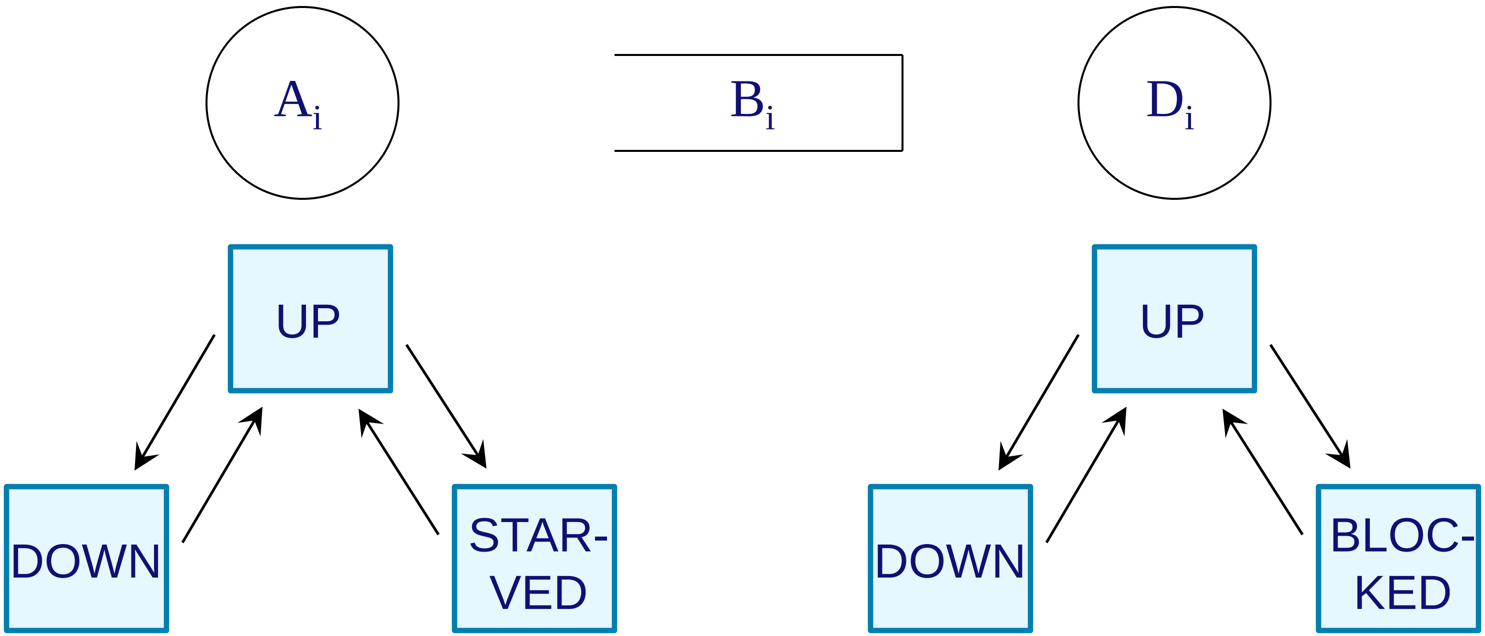


  - Hyper-exponential distribution



Production lines with generally distributed up- and downtimes

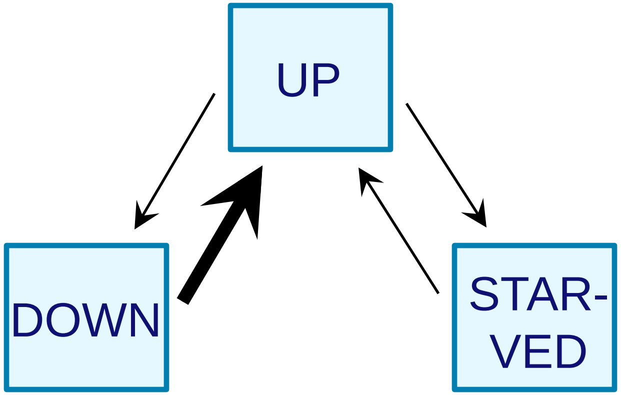
- **Use previously defined three-state approach**



- **Each state consists of a number of phases**

Production lines with generally distributed up- and downtimes

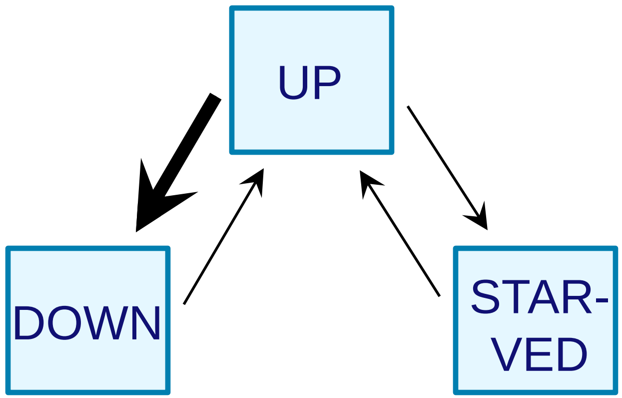
- **Transition rates (arrival machine)**



- **Down to up: fit phase-type distribution on mean repair time $E(V_i)$ and variance $Var(V_i)$**

Production lines with generally distributed up- and downtimes

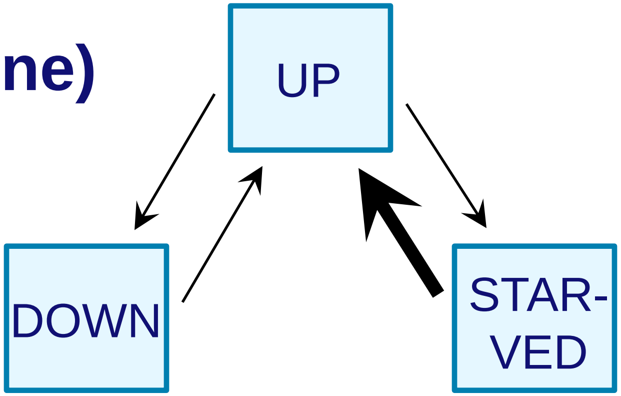
- **Transition rates (arrival machine)**



- **Up to down: fit phase-type distribution on mean breakdown time $E(U_i)$ and variance $Var(U_i)$**

Production lines with generally distributed up- and downtimes

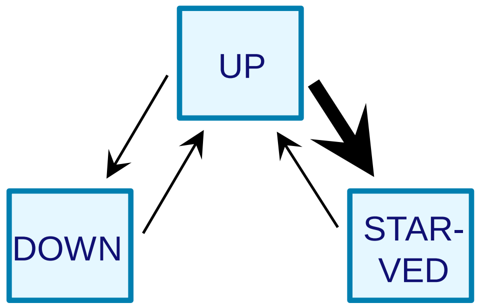
- **Transition rates (arrival machine)**



- **Arrival machine can be starved for two reasons:**
  - **Previous arrival machine is down**
  - **Previous arrival machine is starved**
- **Obtain mean and second moment by taking a weighted average over mean and second moment of transitions down->up and down->starved of A_{i-1}**
- **Fit a phase-type distribution**

Production lines with generally distributed up- and downtimes

- **Transition rates (arrival machine)**



- **Second moment / variance of this transition is hard to determine**
- **Start with exponential distribution**
- **Find other estimations for this variance**

Production lines with generally distributed up- and downtimes

- Heineken test case
  - Difference when using approximation for exponential up- and downtimes: 18,4%
  - Difference when using first approximation for general up- and downtimes: 1,4%

- Conclusions
  - model for general up- and downtimes works significantly better for the Heineken case
  - Test on a larger test set
  - Find estimations for the variance of the transitions up->starved and up->blocked