Supply Disruptions in Multi-Echelon Inventory Systems

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Where innovation starts

Supply Uncertainty

- **Stochastic Leadtimes:** Delivery time is random
- **Yield Uncertainty:** Supply quantity is random
- **Disruptions:** Supply processes are interrupted
  - **Reasons:** natural disasters, labor strikes, terrorist attacks, etc.

Supply Disruptions

- **Example:** Hurricane Mitch (November, 1998)
  - 10% of the worldwide crop was destroyed
  - **Dole:** Lost 70% of its capacity in the region, struggled to find alternative sources of supply and suffered revenue declines
  - **Chiquita:** Maintained a steady supply by purchasing and increasing its production in other regions, reported revenue increase in 1998

Supply Disruptions

- **Mitigation Strategies (Tomlin (2006)):**
  - Passive acceptance
  - Inventory mitigation
  - Sourcing mitigation
  - Contingent rerouting
  - Demand management
  - Mixed strategies
- **Snyder and Shen (2006):** demand uncertainty models do not apply to supply disruption problems
- **Literature Review:** Snyder et al. (2010), Atan and Snyder (2010)

Multi-Echelon Supply Chains

Literature:

- Distribution Systems: Schmitt et al. (2008b)
- Assembly Systems: DeCroix (2010)

Contribution:

- Show the optimality of base-stock policies for serial systems subject to supply disruptions (Atan, Rong and Snyder (2010))
- Solve for the base-stock levels of OWMR systems subject to supply disruptions (Atan and Snyder (2010))

Single-stage inventory system

- Discrete-time, infinite-horizon discounted cost problem
- Demands across periods are iid
- h: unit holding cost
- b: unit backorder cost
- α: discount factor

Disruptions:

- Order may not arrive at any period with probability $1 - p$
- If order does not arrive, it disappears

The sequence of events:

1. Observe the current on-hand inventory level, $x(t)$
2. Decide on the order quantity, $q(t)$
3. Delivery (if any) arrives. On hand inventory now equals

$$\begin{cases} x(t) + q(t), & w.p.\ \ p \\ x(t), & w.p.\ \ 1 - p \end{cases}$$

4. Demand $D(t)$ is realized
5. Holding and shortage costs are incurred

Single-stage inventory system

- System dynamics:

$$x(t+1) = x(t) + Uq(t) - d(t)$$

  where U has Bernoulli distribution with success probability p

- The total expected cost for any ordering policy $\pi \in \Pi$:

$$V(x|\pi) = E_D\left[\sum_{t=0}^{T} E_U \alpha^{T-t}\left(h(x(t) + Uq(t) - d(t))^+ + b(x(t) + Uq(t) - d(t))^-\right)\right]$$

- **Plan:**

  1. Solve the problem assuming finite-horizon with length T
  2. Extend the results to infinite-horizon

Finite Horizon

- $\bar{C}_n(x,q)$: the single period expected cost.
- $C_n^*(x)$: the optimal cost for n periods remaining in the horizon:

$$C_n^*(x) \;=\; \min_{q\geq 0}\left\{\bar{C}_n(x,q) + \alpha E_U E_D(C_{n-1}^*(x+Uq-d))\right\}$$

- Equivalently with a single argument:

$$\begin{aligned} J_n(x,q) &= \bar{C}_n(x,q) + \alpha E_U E_D(C_{n-1}^*(x+Uq-d)) \\ J_n(x,q) &= E_U[\bar{J}_n(x+Uq)] \end{aligned}$$

where $\bar{J}_n(y)$ is

$$\begin{aligned} \bar{J}_n(y) &= \bar{C}_n^+(y) + \alpha E_D[C_{n-1}^*(y-d)] \\ \bar{C}_n^+(y) &= E_D[h(y-d_n)^+ + b(y-d_n)^-] \end{aligned}$$

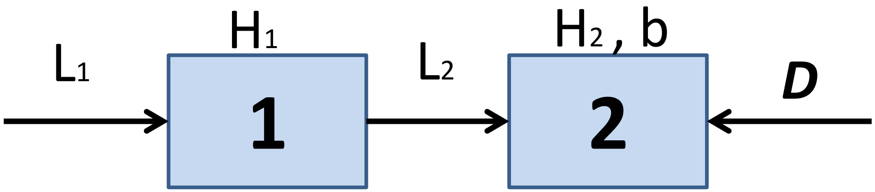
- $\bar{J}_n(y)$ is convex in $y \Rightarrow$ Base-stock policy is optimal.

Infinite Horizon

- We show:
  1. The $\lim_{n\to\infty} C_n^*$ exists.
  2. The limit is $C^* = \lim_{n\to\infty} C_n^*$.
- **Theorem:** If $\{f_m(x)\}$ is a sequence of convex functions that converges pointwise to a function $f(x)$ for each x, then f is also convex.
- **Result**: Base-stock policy is optimal.

Two-Echelon Serial Systems

- Consider the classical Clark-Scarf model



- Introduce supply disruptions at the supply processes
  - ϕ^i= the state of the supply process of location i
  - $\phi^i = 0 \Rightarrow$ the link is not disrupted and order attempts are successful
  - $\phi^i = j \Rightarrow$ the link is disrupted for j^{th} consecutive period

Definitions

- w_1=local inventory at the warehouse
- w_2=local inventory at the retailer
- x_1=echelon inventory at the warehouse
- x_2=echelon inventory at the retailer
- y= size of the shipment from the outside supplier to the warehouse
- z=size of the shipment from the warehouse to the retailer
- $\tilde{y} = (y^1, \ldots, y^{L_1})$, $\tilde{z} = (z^1, \ldots, z^{L_2})$

Inventory Balance Equations:

$$\begin{aligned} x_{1(n-1)} &= x_{1n} + y_n^{L_1} - d_n \\ x_{2(n-1)} &= x_{2n} + z_n - d_n \\ \tilde{y}_{n-1} &= (y_n, y_n^1, \ldots, y^{L_1-1}) \end{aligned}$$

One Period Expected Cost

- System wide inventory at the end of a period:

$$
\begin{aligned}
& y^{L_1} + w_1 + \sum_{k=1}^{L_2-1} z^k + [w_2 + z^{L_2} - d]^+ \\
= \; & x_1 + y^{L_1} + \left\{ -(w_2 + z^{L_2}) + [w_2 + z^{L_2} - d]^+ \right\}
\end{aligned}
$$

- **One Period Expected Inventory Cost at Echelon 1**

$$
H_1 E\left[x_1 + y^{L_1} + \left\{ -\alpha^{L_2}(x_2 + z - d^{L_2}) + \alpha^{L_2}[x_2 + z - d^{L_2+1}]^+ \right\}\right]
$$

- **One Period Expected Cost at Echelon 2**

$$
\alpha^{L_2} \left\{ H_2 E[x_2 + z - d^{L_2+1}]^+ + bE[d^{L_2+1} - x_2 - z]^+ \right\}
$$

DP Algorithm

- Define

$$\begin{aligned} C^1(a) &= H_1 E[a] \\ C^2(a) &= -\alpha^{L_2}\left\{E[a-d^{L_2}] + (H_1+H_2)E[a-d^{L_2+1}]^+ + bE[d^{L_2+1}-a]^+\right\} \end{aligned}$$

- Overall Problem

$$\begin{aligned} G_n(\tilde{y}, x_1, x_2, \phi_n^1, \phi_n^2) = \min_{y,z} \Big\{ & C^1(x_1 + y^{L_1}) + C^2(x_2+z) \\ & + \alpha \sum_{j_1, j_2} P_{j_1|\phi_n^1, j_2|\phi_n^2} E[G_{n-1}(\tilde{y}_{n-1}, x_1 + y^{L_1} - d, x_2 + z - d, \phi_{n-1}^1, \phi_{n-1}^2)|\phi_n^1, \phi_n^2] \Big\} \end{aligned}$$

- **Claim:**

$$G_n(\tilde{y}, x_1, x_2, \phi_n^1, \phi_n^2) = G_n^1(\tilde{y}, x_1, \phi_n^1, \phi_n^2) + G_n^2(x_2, \phi_n^2)$$

- Proof by induction.

Optimization

- The problem for the retailer:

$$G_n^2(x_2, \phi_n^2) = \min_z \left\{ C^2(x_2 + z) + \alpha \sum_{j_2} P_{j_2|\phi_n^2} E[G_{n-1}^2(x_2 + z - d|\phi_n^2] \right\}$$

- **Result:** State dependent base-stock policy is optimal for the retailer.
- The problem for the warehouse:

$$G_n^1(\tilde{y}, x_1, \phi_n^1, \phi_n^2) = \min_y \{ C^1(x_1 + y^{L_1}) + P_n(x_1 + y)$$
$$+ \alpha \sum_{j_1, j_2} P_{j_1|\phi_n^1, j_2|\phi_n^2} E[G_{n-1}^1(\tilde{y}_{n-1}, x_1 + y^{L_1} - d, \phi_{n-1}^2)|\phi_n^1, \phi_n^2] \Big\}$$

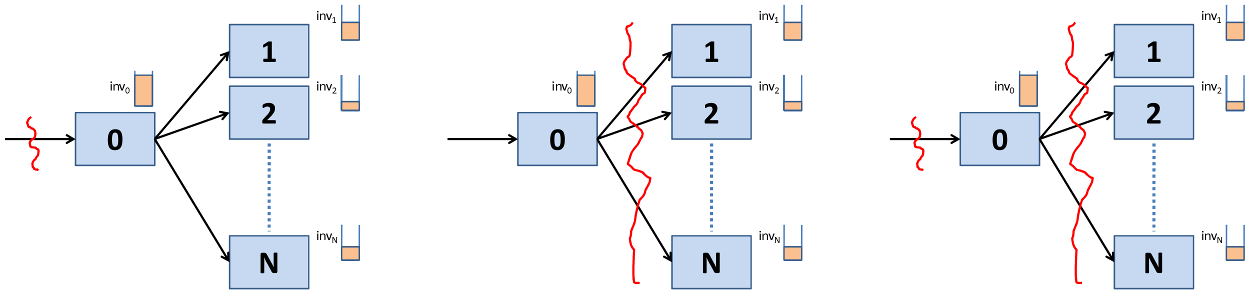
- **Result:** State dependent base-stock policy is optimal for the warehouse.

State-dependent Policies

	Warehouse Base-Stock	Retailer Base-Stock
Disruptions at the warehouse	X	-
Disruptions at the retailer	X	X

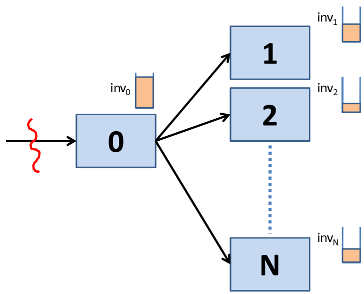
OWMR Systems

1. Disruptions at the warehouse
2. Disruptions at the retailers
3. Disruptions at both the warehouse and the retailers



- Disruptions are modeled using a discrete-time Markov Chain
- State variable: # of consecutive disrupted periods

Warehouse-Assumptions



- During the disruption:
  - the warehouse cannot receive units from the outside supplier
  - the warehouse satisfies the demands of the retailers as long as it has enough inventory
- At the end of the disruption:
  - the inventory levels of both the warehouse and the retailers are increased to their base-stock levels

Warehouse-Solutions

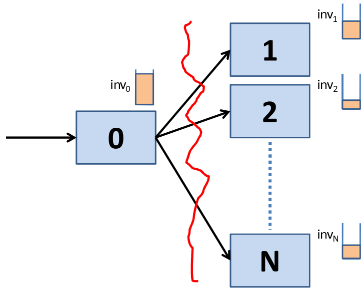
Identical Retailers: Exact Solution

- Closed-form expressions
- Location with smaller holding cost holds the extra inventory

Non-identical Retailers: Heuristic

- $C(s_0, \bar{s}_r)$ is convex in each s_r, $r \in \{1, 2, ..N\}$
- $s_r^*(s_0)$ has closed-form expression
- Approximated $C(s_0, \bar{s}_r^*(s_0))$ to make it convex in s_0
- $\bar{\epsilon}$=0.420%, σ_ϵ= 1.982%
- In 92.71% of the 1080 randomly generated instances $\epsilon = 0$

Retailers



- retailers cannot receive the items send by the warehouse
- warehouse keeps track of the demands of the retailers

Exact Solution (Identical and Non-identical Retailers):

s_0^*	s_r^*
$\sum_{r=1}^{N} d_r$	$d_r\left(\left(F^r\right)^{-1}\left(\frac{b_r}{b_r+b_r}\right)+1\right)$

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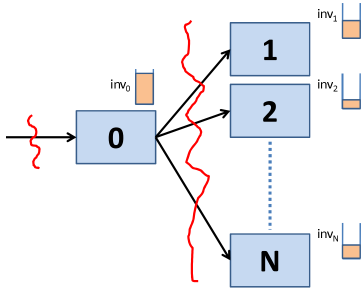
Both Locations



THEOREM:
For fixed s_0, the cost function $C(s_0, \mathbf{s_r})$ is convex in $\mathbf{s_r}$.

Both Locations

Identical Retailers: Heuristic

- For $h_0 \geq h_r$, $s_0^* = Nd_r$
- For $h_r \geq h_0$: approximate closed-form expressions
- Performance:
  - 0% error in numerical experiments
  - Conjecture: Heuristic always generates the exact solution

Both Locations

Non-identical Retailers: Heuristic

- Decompose to serial systems
- Use the heuristic proposed for identical-retailer case to solve for serial systems
- Aggregate using backorder matching
- 1080 randomly generated cases: $\bar{\epsilon}$=6.45%, σ_ϵ= 4.92%

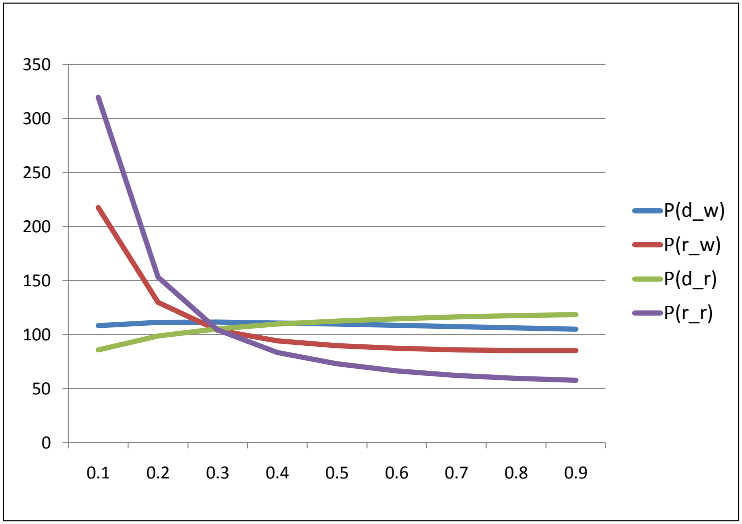
Summary of the Results

OWMR with Identical Retailers	
Warehouse	Exact Solution
Retailers	Exact Solution
Warehouse and Retailers	Heuristic with 0% mean cost error

OWMR with Non-Identical Retailers	
Warehouse	Heuristic with 0.42% mean cost error
Retailers	Exact Solution
Warehouse and Retailers	Heuristic with 6.45% mean cost error

Effects of System Parameters

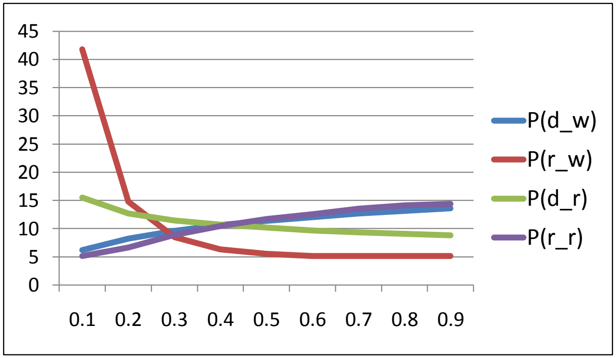
- Expected Cost vs. Disruption/Recovery Probabilities

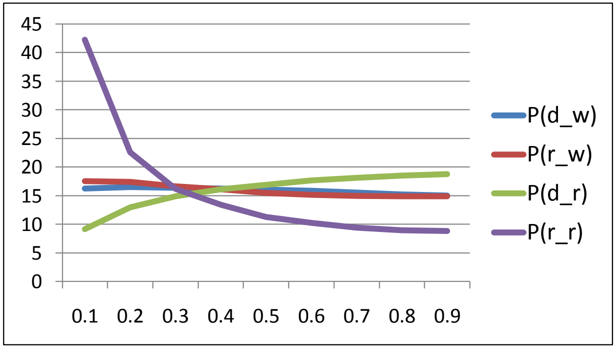


- Firms should focus more on reducing the duration of disruptions than on reducing their probability of occurrence

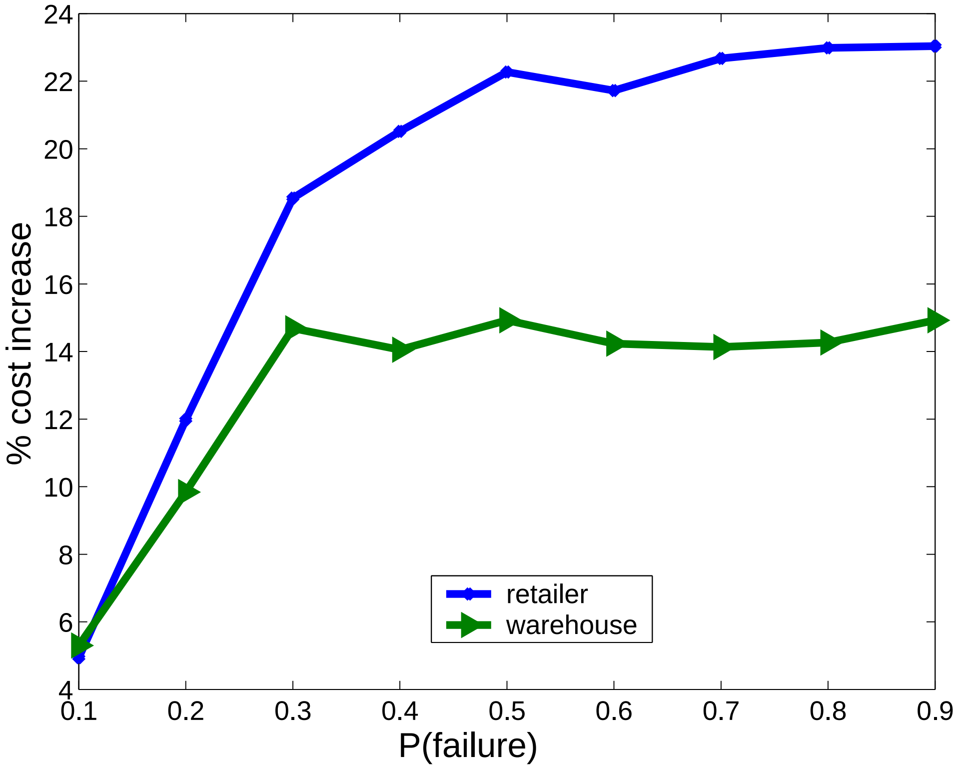
Effects of System Parameters

- Optimal warehouse/retailer base-stock levels vs. Disruption/Recovery Probabilities





- Ignoring supply disruptions close to customers is more costly than ignoring disruptions elsewhere



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Conclusions

- Serial Systems:
  - If disruptions are independent of any other historical event, stationary echelon order-up-to policies are optimal
  - For more general disruption processes, state-dependent order-up-to policies are optimal
  - The state dependent base-stock levels are monotonically increasing in the number of disrupted periods
- Distribution Systems:
  - Disruptions affect the optimal inventory decisions of all locations
  - Companies should concentrate more on reducing the duration of disruptions which happen close to the customers