PRODUCTION SYSTEMS ENGINEERING:

Optimality through Improvability

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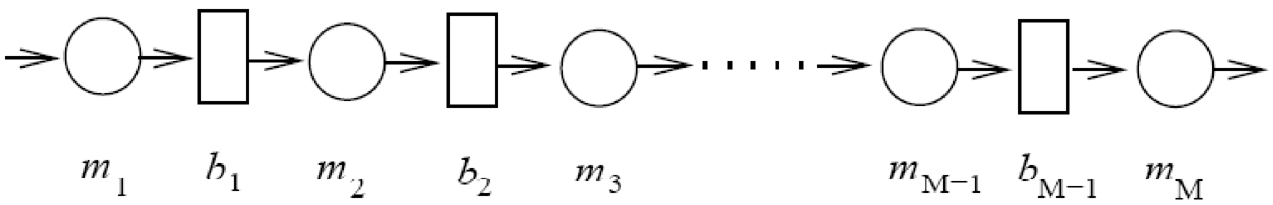
YEQT-IV (Young European Queueing Theorists Workshop)
"Optimal Control in Stochastic Systems“
Technical University of Eindhoven
November 25-27, 2010

PSE – AN OVERVIEW

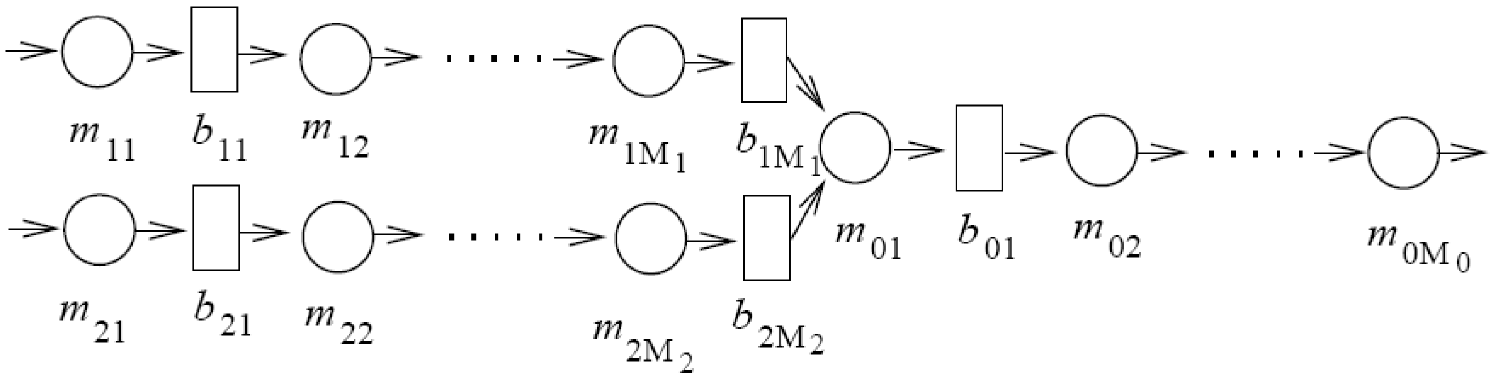
- ***Production Systems Engineering*** (PSE) is a emerging branch of Engineering intended to investigate fundamental laws that govern production systems and utilize them for the purposes of analysis, continuous improvement, and design.
- Every problem addressed in PSE has its origin on the factory floor.
- Practically every solution obtained in PSE has been implemented on the factory floor.
- The goals of this lecture:
  - provide a brief description of PSE;
  - discuss the issue of optimality vs. improvability.

Systems addressed in PSE

- Open serial lines

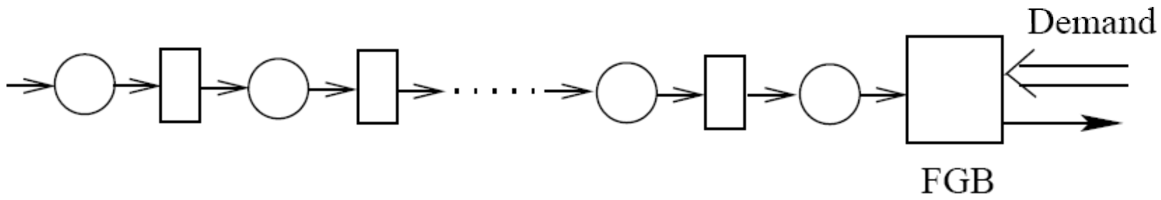


- Assembly systems

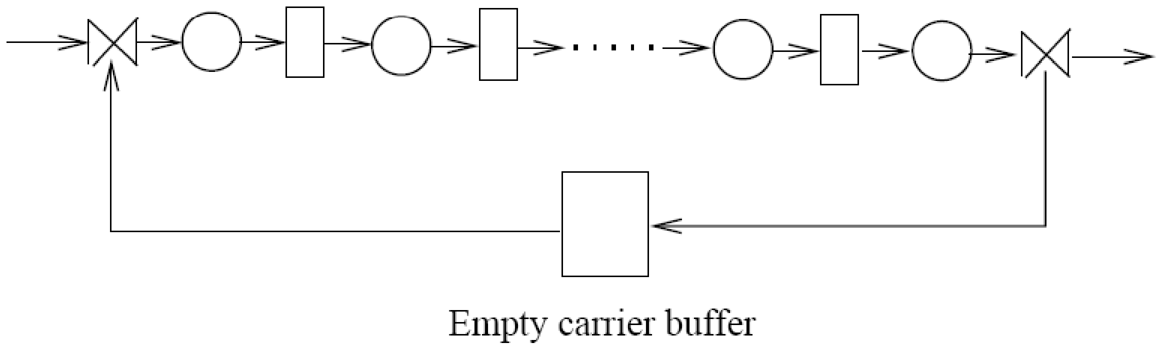


Systems addressed (cont)

- Serial lines with finished goods buffers (FGB)

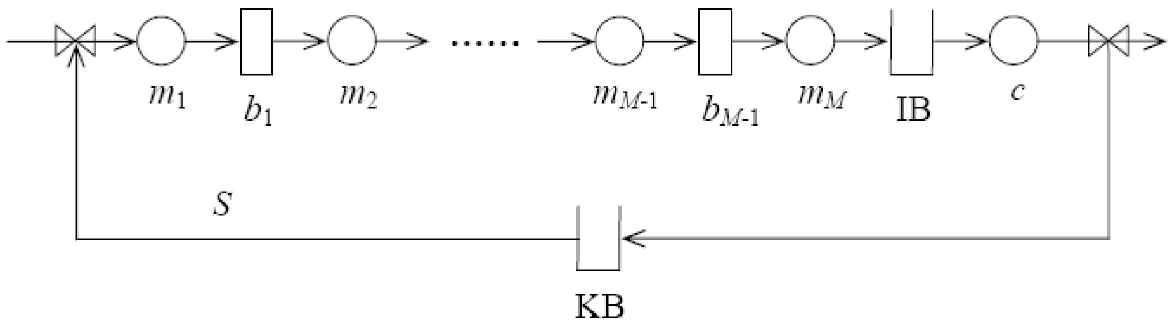


- Closed (palletized) serial lines



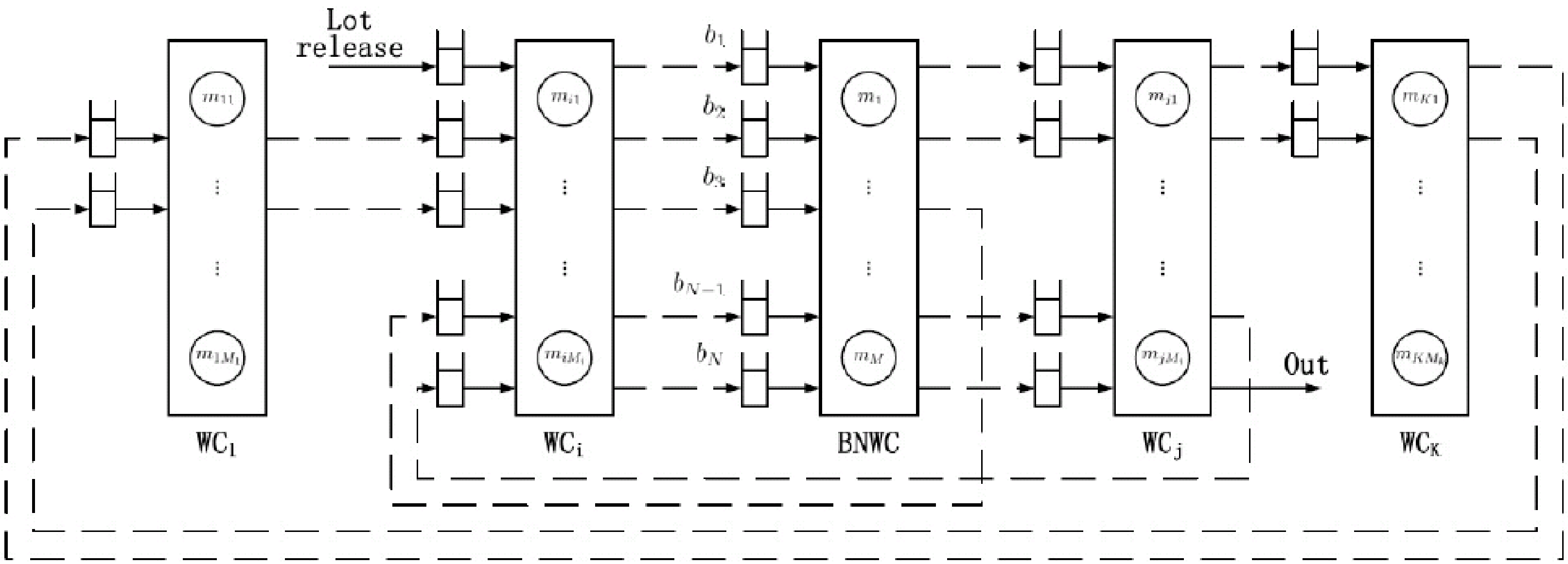
Systems addressed (cont)

- Production lines with feedback-controlled release (e.g., kanban)



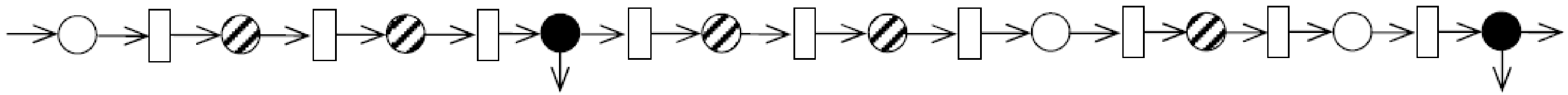
1.1 Structural Models (cont)

- Re-entrant lines

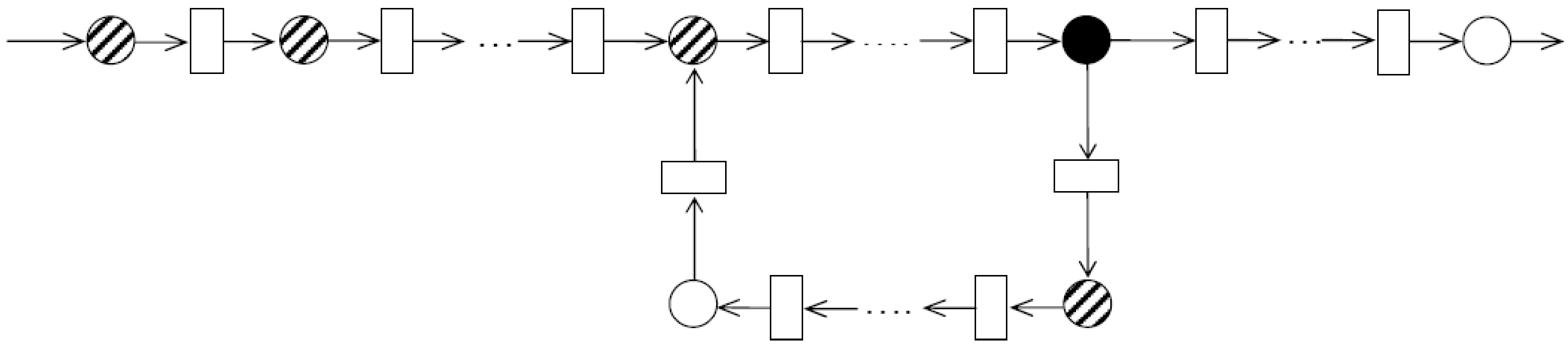


Systems addressed (cont)

- Serial lines with non-perfect quality and inspection machines



- Serial lines with rework



- **Practically every production line in large volume manufacturing can be reduced to one of the above (with a sufficient level of fidelity)**

Machine reliability models considered

- **The machines are described by the following reliability models:**
  - Bernoulli model: the status of the machine is an i.i.d. sequence of Bernoulli random variables;
  - Geometric model (constant breakdown and repair probabilities): the state of the machine is described by a Markov chain;
  - Exponential model (constant breakdown and repair rates): the state of the machine is described by a continuous time, discrete space Markov process;
  - Non-Markovian models (time-dependent breakdown and repair rates): Weibull, Rayleigh, gamma, log-normal pdf's;
  - General model: the state of the machine is described by a random process of an unknown nature.
- **In this formalization, a production system is a set of random processes interacting in a nonlinear manner. Their analysis using system-theoretic tools is the approach of PSE.**

Problems addressed in PSE

- **Mathematical modeling:** Provides methods for constructing mathematical models of production systems at hand with acceptable fidelity.
- **Performance analysis:** Offers analytical tools for calculating the steady state throughput, work-in-process, probabilities of machine blockages and starvations, the level of customer demand satisfaction, etc.
- **Constrained improvability:** Develops methods for re-allocating limited resources (such as buffer capacity or workforce or cycle time) so that the throughput is increased.

Problems addressed (cont)

- **Unconstrained improvability:** Provides methods for identifying bottleneck machines and bottleneck buffers, i.e., machines and buffers, which affect the production rate in the strongest manner.
- **Lean buffer design:** Offers analytical tools for calculating the smallest buffer capacity, which is necessary and sufficient to obtain the desired throughput of a production system.
- **Customer demand satisfaction:** Develops formulas for calculating the Due Time Performance, i.e., the probability to ship to the customer the desired number of parts during a fixed time interval.
- **Product quality:** Presents methods for analysis and improvement of production systems with non-perfect quality of parts produced.

Problems addressed (cont)

- **System-theoretic properties:** Investigates fundamental structural properties of production systems, such as reversibility, monotonicity, and the effects of up- and downtime
- **Transient analysis:** Studies temporal properties of reaching steady states in production systems.
- **Preventive maintenance:** For machines with maintenance-reliability coupling, determines PM's feasibility and, if so, optimal PM rate.
- **PSE Toolbox:** Provides a user-friendly set of C++ programs that implement the methods and algorithms developed in PSE.
- **Case studies:** Describes numerous applications of PSE in various production systems.

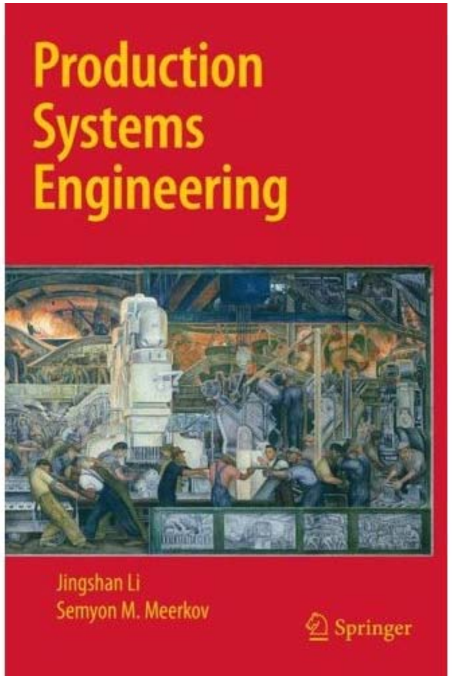
Solution paradigm

- *Theorems* – for problems that can be solved analytically.
- *Numerical Facts* – for problems that are investigated numerically.
- *Improvability Indicators* and *Continuous Improvement Procedures* – for practical implementation of the results obtained.

The textbook

- The main results of PSE are summarized in a textbook:

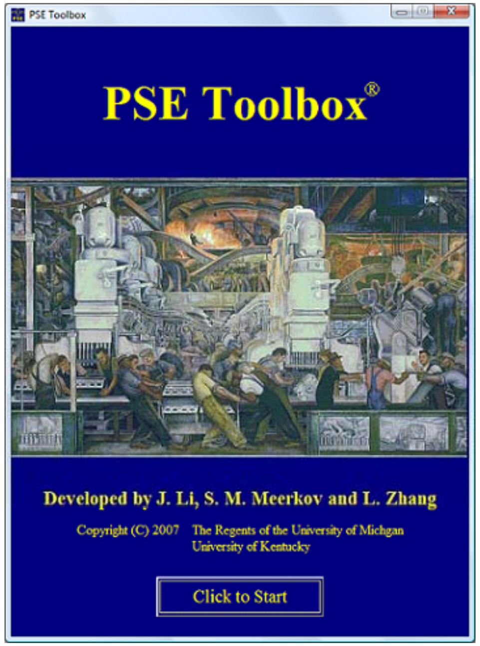
  J. Li and S.M. Meerkov, ***Production Systems Engineering***, Springer, 2009



The toolbox

- A demo of the ***PSE Toolbox*** is available at :

  `www.ProductionSystemsEngineering.com`



- **The remainder of this lecture addresses the issues of optimality vs. improvability of production systems**

Problems addressed in this talk

1. Constrained improvability
2. Unconstrained improvability
3. Leanness
4. Transients
5. PSE Toolbox
6. Summary

1. CONSTRAINED IMPROVABILITY
1.1 Improvability of Open Serial Lines

- Constrains:
  - *Buffer capacity (BC) constraint:* $\sum_{i=1}^{M-1} N_i = N^*$.
  - *Work force (WF) constraint:* $\prod_{i=1}^{M} p_i = p^*$.
  - *Cycle time (CT) constraint:* $\sum_{i=1}^{M} \tau_i = \tau^*$, where $\tau_i = 1/c_i$ is the cycle time of the i-th machine.
- Optimality question: How should the resources be allocated, so that the throughput is maximized?
- Improvability question: Can the resources be re-allocated, so that the throughput is increased?
- Clearly, the unimprovable system is optimal.
- Which approach is more conducive in applications?

1.1 Improvability of Open Serial Lines (cont)

Definition 5 *A serial line is:*

- BC-improvable *if there exists a sequence* $N'_1, \ldots, N'_{M-1}$ *such that* $\sum_{i=1}^{M-1} N'_i = N^*$ *and*

$$PR(p_1, \ldots, p_M, N'_1, \ldots, N'_{M-1}) > PR(p_1, \ldots, p_M, N_1, \ldots, N_{M-1}) \tag{20}$$

- WF-improvable *if there exists a sequence* $p'_1, \ldots, p'_M$ *such that* $\prod_{i=1}^{M} p'_i = p*$ *and*

$$PR(p'_1, \ldots, p'_M, N_1, \ldots, N_{M-1}) > PR(p_1, \ldots, p_M, N_1, \ldots, N_{M-1}) \tag{21}$$

- CT-improvable *if there exists a sequence* $\tau'_1, \ldots, \tau'_M$ *such that* $\sum_{i=1}^{M} \tau'_i = \tau^*$ *and*

$$TP(\tau'_1, \ldots, \tau'_M) > TP(\tau_1, \ldots, \tau_M), \tag{22}$$

where TP *is the throughput of the system.*

1.1 Improvability of Open Serial Lines (cont)

- Recursive aggregation procedure (Bernoulli lines):

$$
\begin{aligned}
p_i^b(s+1) &= p_i[1 - Q(p_{i+1}^b(s+1), p_i^f(s), N_i)], \quad i = 1, \dots, M-1, \\
p_i^f(s+1) &= p_i[1 - Q(p_{i-1}^f(s+1), p_i^b(s+1), N_{i-1})], \quad i = 2, \dots, M, \qquad (1) \\
& s = 0, 1, 2, \dots,
\end{aligned}
$$

with initial conditions

$$p_i^f(0) = p_i, \quad i = 1, \dots, M, \qquad (2)$$

and boundary conditions

$$p_1^f(s) = p_1, \quad p_M^b(s) = p_M, \quad s = 0, 1, 2, \dots, \qquad (3)$$

where

$$Q(x, y, N) = \begin{cases} \frac{(1-x)(1-\alpha)}{1-\frac{x}{y}\alpha^N}, & \text{if } x \neq y, \\ \frac{1-x}{N+1-x}, & \text{if } x = y, \end{cases} \qquad (4)$$

$$\alpha = \frac{x(1-y)}{y(1-x)}. \qquad (5)$$

1.1 Improvability of Open Serial Lines (cont)

Theorem 1 *Aggregation procedure (1)-(5) has the following properties:*

(i) The sequences, $p_2^f(s), \ldots, p_M^f(s)$ *and* $p_1^b(s), \ldots, p_{M-1}^b(s)$, $s = 1, 2, \ldots$, *are convergent, i.e., the following limits exist:*

$$p_i^b := \lim_{s\to\infty} p_i^b(s), \quad p_i^f := \lim_{s\to\infty} p_i^f(s). \tag{6}$$

(ii) These limits are unique solutions of the steady state equations corresponding to (1), i.e., of

$$\begin{aligned} p_i^f &= p_i[1 - Q(p_{i-1}^f, p_i^b, N_{i-1})], \qquad 2 \le i \le M, \\ p_i^b &= p_i[1 - Q(p_{i+1}^b, p_i^f, N_i)], \qquad 1 \le i \le M-1, \\ & p_1^f = p_1, \qquad p_M^b = p_M. \end{aligned} \tag{7}$$

(iii) In addition, these limits satisfy the relationships:

$$\begin{aligned} p_M^f &= p_1^b \\ &= p_{i+1}^b[1 - Q(p_i^f, p_{i+1}^b, N_i)] \\ &= p_i^f[1 - Q(p_{i+1}^b, p_i^f, N_i)], \qquad i = 1, \ldots, M-1. \end{aligned} \tag{8}$$

1.1 Improvability of Open Serial Lines (cont)

Using the steady state of this recursive procedure, the performance measures of a Bernoulli line are estimated as follows:

$$\begin{aligned} \widehat{PR} &= p_i^f[1 - Q(p_{i+1}^b, p_i^f, N_i)] \\ &= p_{i+1}^b[1 - Q(p_i^f, p_{i+1}^b, N_i)], \end{aligned} \tag{9}$$

$$\widehat{WIP}_i = \begin{cases} \frac{p_i^f}{p_{i+1}^b - p_i^f \alpha^{N_i}(p_i^f, p_{i+1}^b)} \left[\frac{1-\alpha^{N_i}(p_i^f, p_{i+1}^b)}{1-\alpha(p_i^f, p_{i+1}^b)} \right. \\ \qquad \left. - N_i \alpha^{N_i}(p_i^f, p_{i+1}^b) \right], & \text{if } p_i^f \neq p_{i+1}^b, \\ \frac{N_i(N_i+1)}{2(N_i+1-p_i^f)}, & \text{if } p_i^f = p_{i+1}^b, \end{cases} \tag{10}$$

$$i = 1, \ldots, M-1,$$

$$\widehat{WIP} = \sum_{i=1}^{M-1} \widehat{WIP}_i, \tag{11}$$

$$\widehat{BL}_i = p_i Q(p_{i+1}^b, p_i^f, N_i), \quad i = 1, \ldots, M-1, \tag{12}$$

$$\widehat{ST}_{i+1} = p_i^b Q(p_{i-1}^f, p_i^b, N_{i-1}), \quad i = 2, \ldots, M. \tag{13}$$

- The accuracy of $\widehat{PR}$ is within 1%; lower for other performance measures.

1.1 Improvability of Open Serial Lines (cont)

Theorem 5 *A Bernoulli line is unimprovable with respect to WF if and only if*

$$p_i^f = p_{i+1}^b, \quad i = 1, \ldots, M-1. \tag{23}$$

- Implementation:
  - Direct (optimality): Optimal WF allocation:

$$\begin{aligned} p_1^* &= \Big(\frac{N_1+1}{N_1+\widehat{PR}^*}\Big)\widehat{PR}^*, \\ p_i^* &= \Big(\frac{N_{i-1}+1}{N_{i-1}+\widehat{PR}^*}\Big)\Big(\frac{N_i+1}{N_i+\widehat{PR}^*}\Big)\widehat{PR}^*, i = 2, \ldots, M-1 \\ p_M^* &= \Big(\frac{N_{M-1}+1}{N_{M-1}+\widehat{PR}^*}\Big)\widehat{PR}^*. \end{aligned}$$

where $\widehat{PR}^*$ s the limit of the following recursive procedure:

$$x(n+1) = (p^*)^{\frac{1}{M}} \prod_{i=1}^{M-1} \Big(\frac{N_i + x(n)}{N_i+1}\Big)^{\frac{2}{M}}, \quad x(0) \in (0,1)$$

  - Disadvantage: All systems parameters must be known..

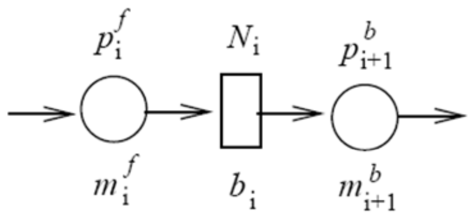
1.1 Improvability of Open Serial Lines (cont)

- Implementation (cont):
  - Indirect (improvability): In a WF unimprovable system,

$$\widehat{WIP}_i = \frac{N_i(N_i+1)}{2(N_i+1-p_i^f)}, \qquad i = 1, \ldots, M-1$$

  - In other words,

$$\frac{N_i}{2} < \widehat{WIP}_i < \frac{N_i+1}{2}, \qquad i = 1, \ldots, M-1$$



  - Advantage: Only *WIP* must be monitored.

- **WF Improvability Indicator**: The system is practically WF-unimprovable if each buffer is close to being half-full.
- This indicator holds for CT-improvability as well (in Markovian and non-Markovian systems).

1.1 Improvability of Open Serial Lines (cont)

- **WF Continuous Improvement Procedure**:

(1) By calculations off-line (using the aggregation procedure (4.30) and formula (4.37)) or by measurements on the factory floor, evaluate the average buffer occupancy, WIP_i, $i = 1, \ldots, M-1$.

(2) Determine the buffer for which $|WIP_i - (N_i + 1)/2|$ is the largest. Assume this is buffer k.

(3) If $WIP_k - (N_k + 1)/2$ is positive, re-allocate a sufficiently small amount of work, ϵp_k, from m_k to m_{k+1}; if $WIP_k - (N_k + 1)/2$ is negative, re-allocate ϵp_{k+1} from m_{k+1} to m_k (observing, of course, the constraint $p_k p_{k+1} = const$).

(4) Return to step (1).

(5) Continue this process until $\max_i |WIP_i - (N_i + 1)/2|$ is sufficiently small.

1.1 Improvability of Open Serial Lines (cont)

- **Theorem:** A Bernoulli line is unimprovable w.r.t. BC if and only if

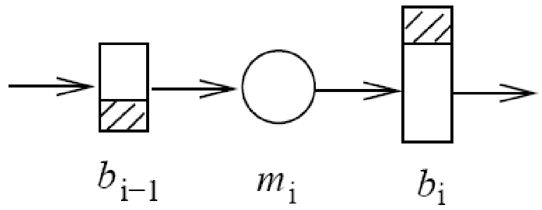
$$\min_{i=1,\ldots,M} p_i \left(\min \left\{ \frac{p_i^f}{p_i^b}, \frac{p_i^b}{p_i^f} \right\} \right)$$

is maximized over all sequences $N_1', \ldots, N_{M-1}'$ such that $\sum_{i=1}^{M-1} N_i' = N^*$.

- **Numerical Fact:** A production line with any model of machine reliability is practically BC-umimprovable if

$$\max_{i=2,\ldots,M-1} |WIP_{i-1} - (N_i - WIP_i)|$$

is minimized over all sequences $N_1', \ldots, N_{M-1}'$.



1.1 Improvability of Open Serial Lines (cont)

- **BC Continuous Improvement Procedure**:

(1) By calculations off-line (using (4.30) and (4.37)) or by measurements on the factory floor, evaluate the average occupancy of each buffer, WIP_i, $i = 1$, ..., $M-1$.

(2) Determine the buffer for which $|WIP_i - (N_{i+1} - WIP_{i+1})|$, $i = 1, \ldots, M-2$, is the largest. Assume this is buffer k.

(3) If $WIP_k - (N_{k+1} - WIP_{k+1})$ is positive, transfer a unit of capacity from b_k to b_{k+1}; if $WIP_k - (N_{k+1} - WIP_{k+1})$ is negative, re-allocate a unit of capacity from b_{k+1} to b_k.

(4) Return to step (1).

(5) Continue this process until arriving at a limit cycle and choose the buffer capacity allocation on the limit cycle, which maximizes PR.

1.1 Improvability of Open Serial Lines (cont)

- **Example:** $M = 11$; $p_i = 0.8$, $i \neq 6$; $p_6 = 0.6$. Total buffer capacity 24.
  - BC-Continuous Improvement Procedure leads to

    $N_1 = 1$, $N_2 = N_3 = N_4 = 2$, $N_5 = 5$,

    $N_6 = 4$, $N_7 = N_8 = N_9 = N_{10} = 2$.

    Resulting throughput $\widehat{PR} = 0.5843$.

  - Goldratt ("The Goal") allocation

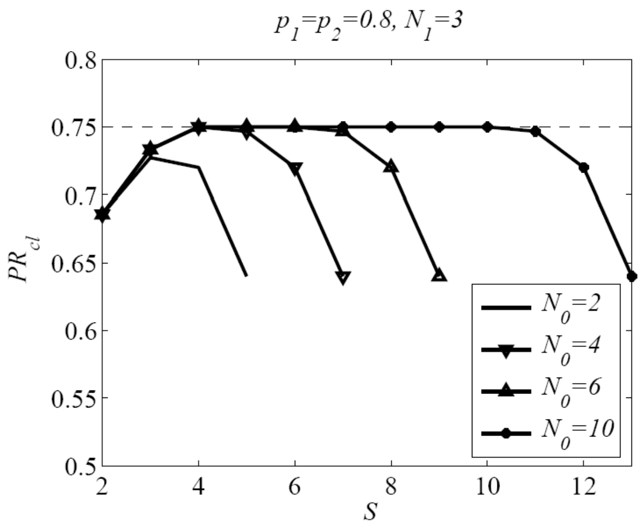
    $N_i = 1$, $i \neq 5$,

    $N_5 = 17$.

    Resulting throughput $\widehat{PR} = 0.426$ (27% less).

1.2 Improvability of Closed Serial Lines

- Closed line performance as a function of S and N_0.



- **Definition:** A closed line is:
  - S^+-improvable if PR can be increased by adding a carrier;
  - S^--improvable if PR can be increased by removing a carrier.

1.2 Improvability of Closed Serial Lines (cont)

- **Numerical Fact:** A closed line is
  - S^+-improvable if

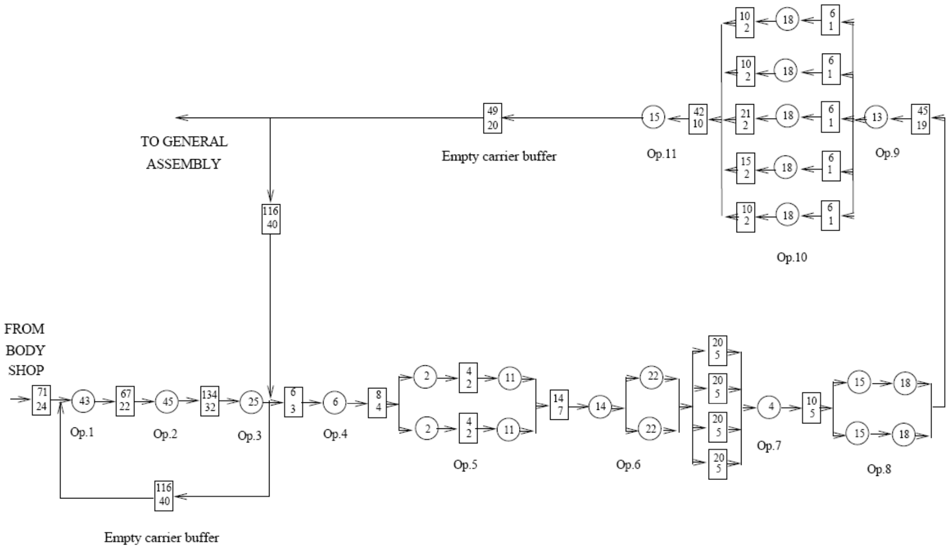
$$I = \sum_{i=1}^{M} ST_i - \sum_{i=1}^{M} BL_i > 0,$$

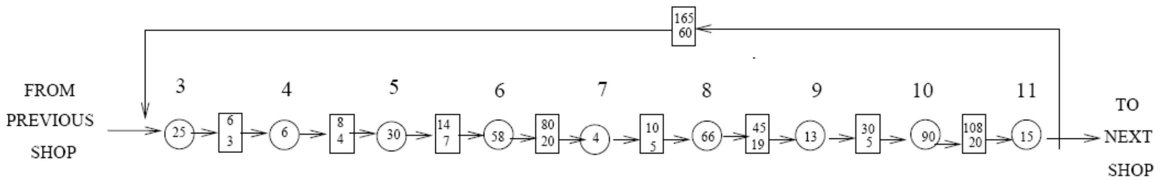
  - S^--improvable if

$$I = \sum_{i=1}^{M} ST_i - \sum_{i=1}^{M} BL_i < 0,$$

1.2 Improvability of Closed Serial Lines (cont)

- **Case study:** Automotive paint shop





1.2 Improvability of Closed Serial Lines (cont)

- Model validation

	Month 1	Month 2	Month 3	Month 4	Month 5
Est. TP	52.59	51.54	52.77	51.92	52.64
Actual TP	53.50	43.81	51.27	54.28	55.89
Error	-1.70%	17.64%	2.93%	-4.35%	-5.81%

1.2 Improvability of Closed Serial Lines (cont)

- Continuous improvement with respect to *S*

Step #	S	I	TP
0	8	1.3534	52.59
1	9	0.7217	57.00
2	10	0.5216	58.36
3	11	0.4297	58.93
4	12	0.3926	59.16
5	13	0.3780	59.24
6	14	0.3735	59.27
7	15	0.3711	59.28
8	16	0.3707	59.29

- Implemented on the factory floor

2 UNCONSTRAINED IMPROVABILITY
2.1 Definitions

- **Definition:**
  - m_i, $i \in \{1,\ldots, M\}$, is the bottleneck machine (BN-m) in a Bernoulli line if
$$\frac{\partial PR}{\partial p_i} > \frac{\partial PR}{\partial p_j}, \qquad \forall j \neq i.$$
  - m_i, $i \in \{1,\ldots, M\}$, is the bottleneck (BN-m) in a serial line with continuous time model of machine reliability if
$$\frac{\partial TP}{\partial c_i} > \frac{\partial TP}{\partial c_j}, \qquad \forall j \neq i.$$
- **Definition:** b_i, $i \in \{1,\ldots, M-1\}$, is the bottleneck buffer (BN-b) if
$$PR(p_1,\ldots,p_M, N_1,\ldots,N_i+1,\ldots,N_{M-1}) > PR(p_1,\ldots,p_M, N_1,\ldots,N_j+1,\ldots,N_{M-1}), \quad \forall j \neq i.$$

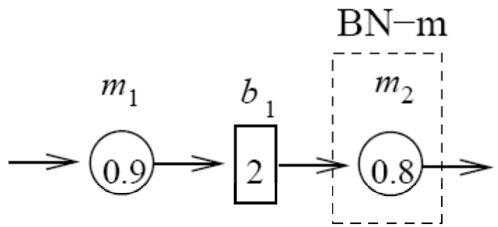
2.2 Two-machine Lines

- **Theorem:**

$$\frac{\partial PR}{\partial p_1} > \frac{\partial PR}{\partial p_2} \quad \text{if and only if} \quad BL_1 < ST_2 \;;$$

$$\frac{\partial PR}{\partial p_2} > \frac{\partial PR}{\partial p_1} \quad \text{if and only if} \quad BL_1 > ST_2 \;;$$

- Graphical representation:



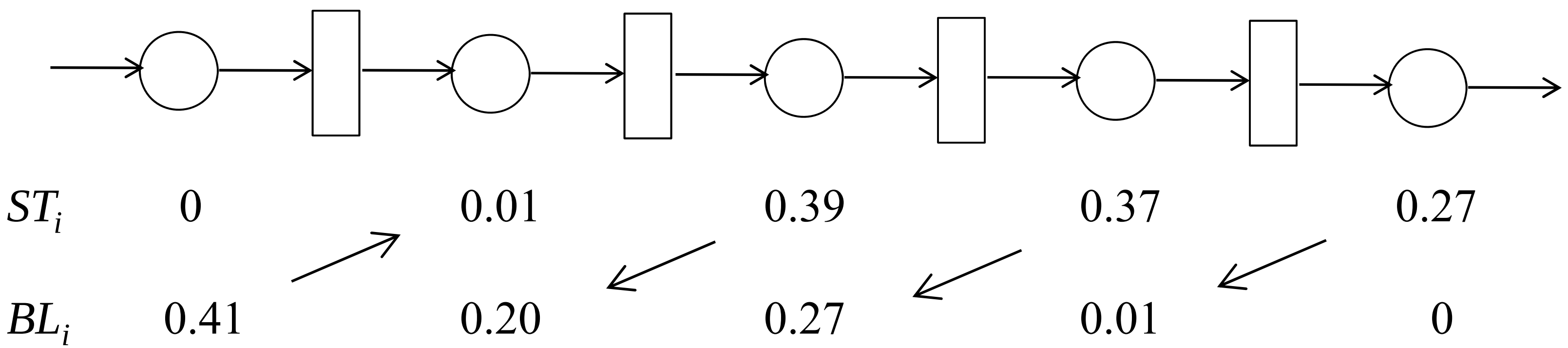
ST_i	0	0.0215
BL_i	0.1215	0

2.3 *M*-machine Lines

- **Arrow assignment rule**:

$$BL_i > ST_{i+1} \rightarrow \text{arrow from } m_i \text{ to } m_{i+1}$$

$$BL_i < ST_{i+1} \rightarrow \text{arrow from } m_{i+1} \text{ to } m_i$$



2.3 *M*-machine Lines (cont)

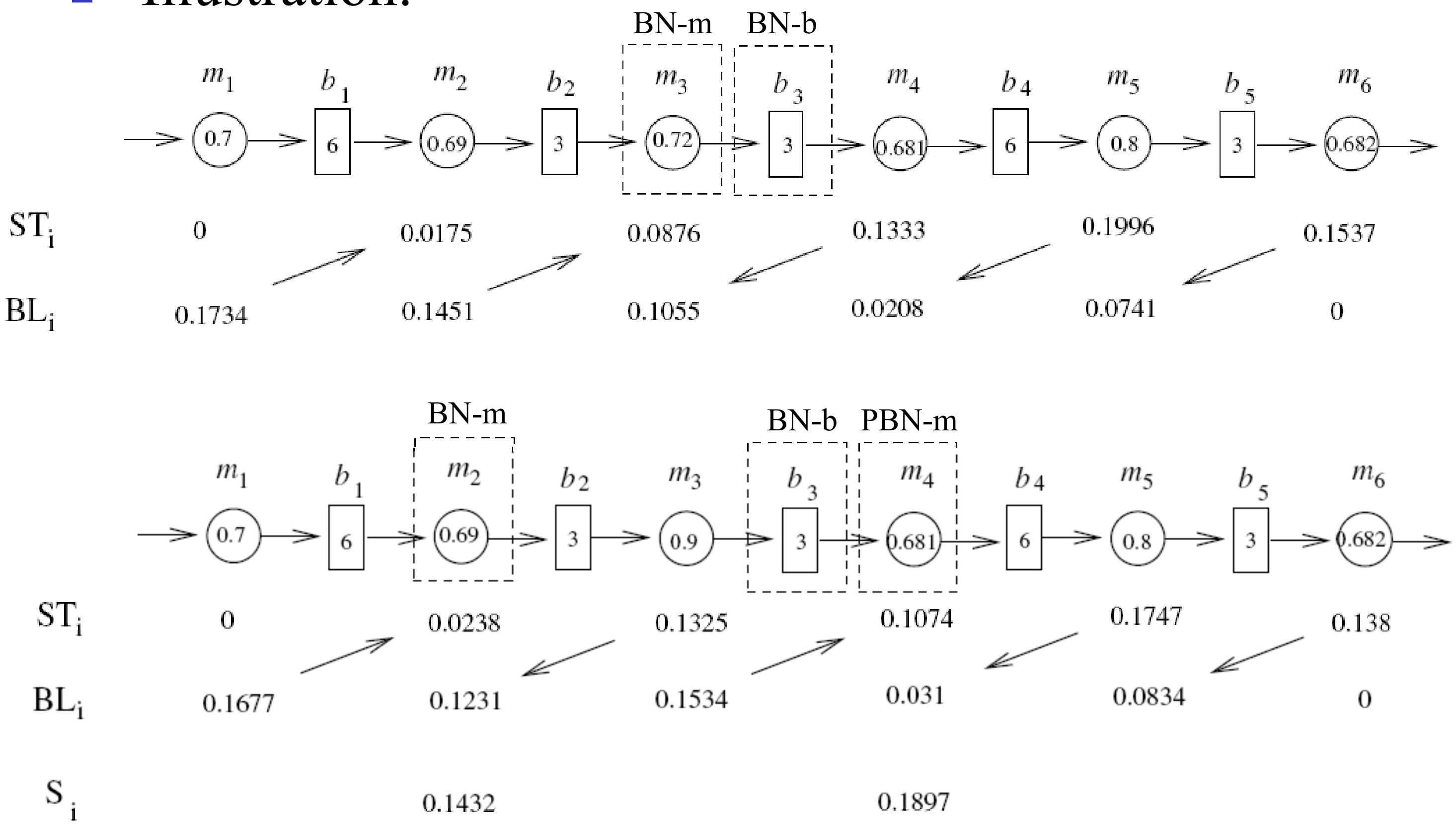
- **Bottleneck Indicator**:
  - A single machine with no emanating arrows is the BN-m.
  - If there are multiple machines with no emanating arrows, the one with the largest severity is the BN-m:

$$
\begin{aligned}
S_i &= |ST_{i+1} - BL_i| + |BL_{i-1} - ST_i|, \;\; i = 2, \ldots, M-1, \\
S_1 &= |ST_2 - BL_1|, \\
S_M &= |BL_{M-1} - ST_M|.
\end{aligned}
$$

  - BN-b is one of the buffers surrounding the BN-m.

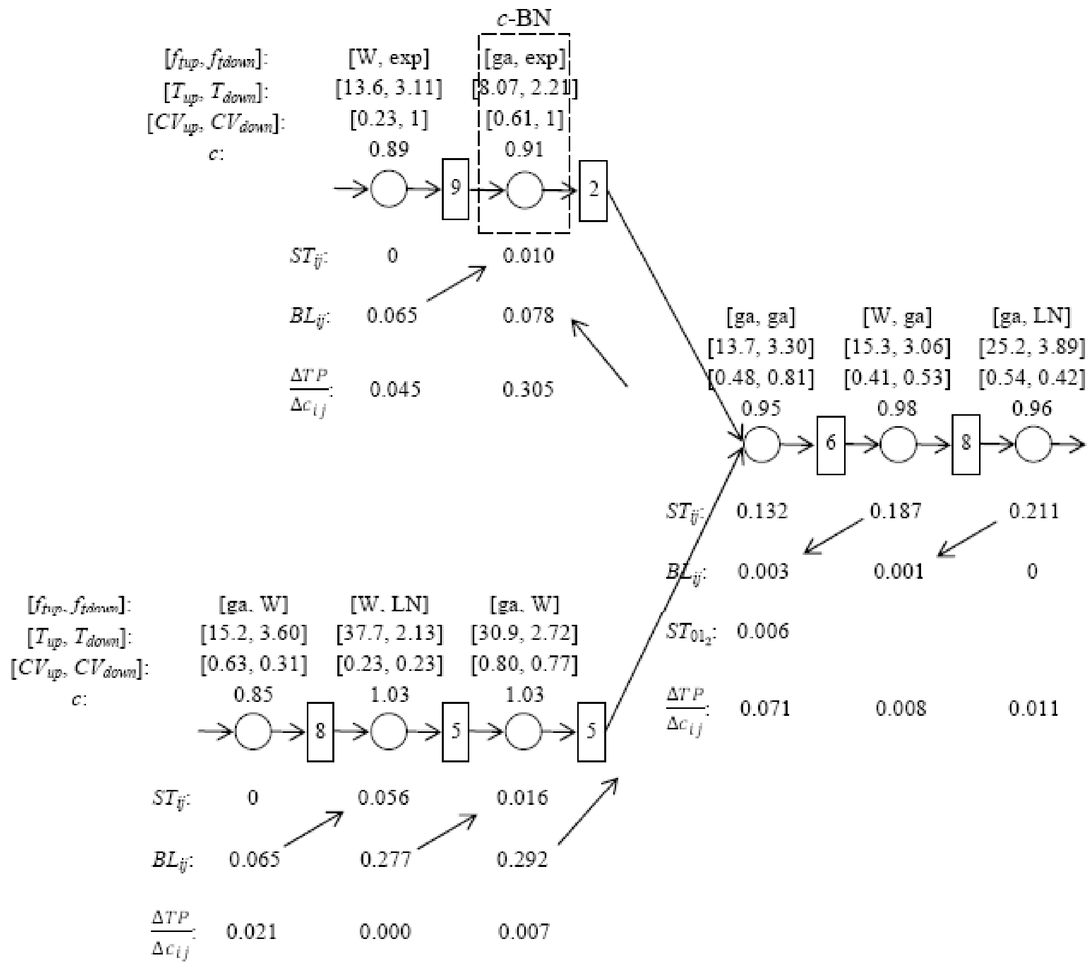
2.3 *M*-machine Lines (cont)

- Illustration:



2.3 *M*-machine Lines (cont)

- Illustration: Assembly system



2.3 *M*-machine Lines (cont)

- **Justification**:
  - **Hypothesis:**

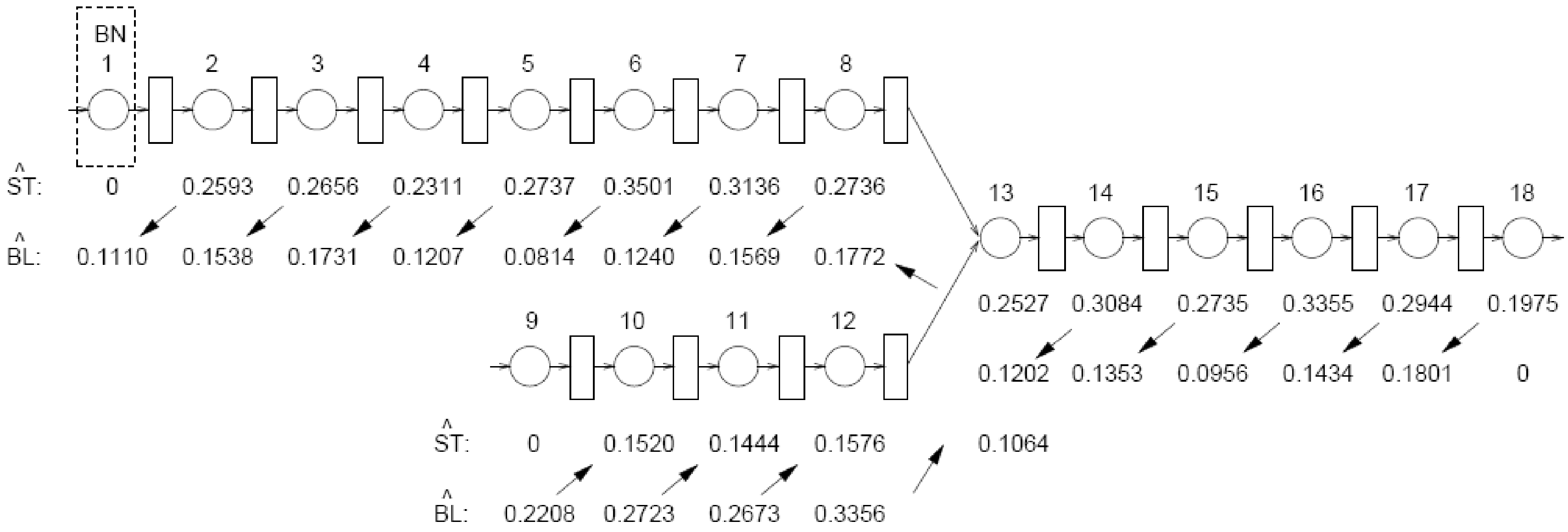
$BL_{j-1} > ST_j$ implies $\epsilon_{j1} := P_{j-1}(0) \approx Q(p^f_{j-1}, p^b_j, N_{j-1}) \ll 1,$

$BL_j < ST_{j+1}$ implies $\epsilon_{j2} := (1 - p^b_{j+1})P_j(N_j) \approx Q(p^b_{j+1}, p^f_j, N_j) \ll 1.$

  - **Lemma:** For any $0 < \epsilon_0 \ll 1$, there exists *N**, such that if $N_j > N^*$, then $\epsilon = \max(\epsilon_{j1}, \epsilon_{j2}) < \epsilon_0.$

  - **Theorem:** Under the Hypothesis, BN-m is downstream of m_j if $\widehat{BL}_j > \widehat{ST}_{j+1}$ and upstream of m_j if $\widehat{BL}_{j-1} > \widehat{ST}_j$.

2.4 Application: Ignition Assembly System



2.4 Application: Ignition Assembly System (cont)

- Analysis of buffering potency:

Month	May	June	July	Aug.	Sept.	Oct.
Bottleneck	Op.1	Op.13	Op.13	Op.13	Op.13	Op.1
Machines with the smallest isolation PR	Op.4	Op.4	Op.4	Op.11	Op.4	Op.14

  - Conclusion: Buffering is not potent.
- Recommendations:
  - Increasing the capacity of buffer conveyor from 19 to 40 carriers leading to 9% throughput improvement;
  - Eliminating starvations of first and blockages of last operations leading to 7-17% throughput improvement.
- Both recommendations have been implemented on the factory floor.

3. LEANNESS

3.1. Parametrization

- Assume that

$$T_{up,i} =: T_{up}, \quad T_{down,i} =: T_{down}, \text{ i.e., } e = \frac{T_{up}}{T_{up} + T_{down}}; \quad N_i =: N$$

- Normalizations:
  - *Level of buffering* – capacity of the buffer in units of downtime:

$$k := \frac{N}{T_{down}}.$$

  - *Line efficiency*:

$$E := \frac{PR}{PR_\infty}.$$

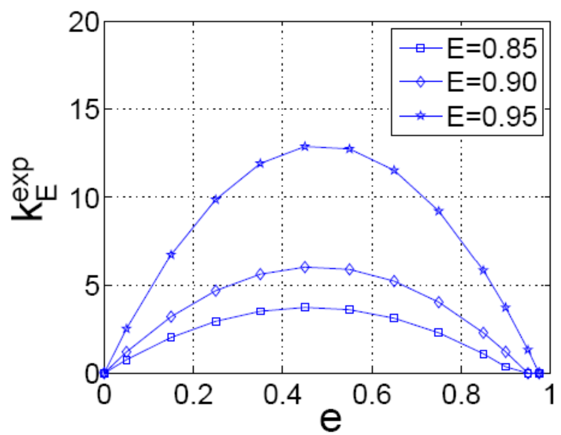
- **Definition:** *Lean level of buffering* (k_E) – the minimum level of buffering necessary and sufficient to ensure E.

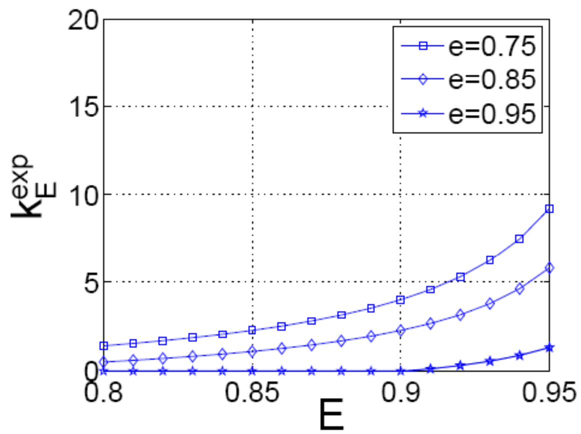
3.2 Lean Buffering in Two- and Three-machine Lines

- **Theorem:**

$$k_E^{exp}(M=2) = \begin{cases} \frac{2e(E-e)}{1-E}, & if\ e < E, \\ 0, & otherwise. \end{cases}$$

$$k_E^{exp}(M=3) = \begin{cases} \frac{e(1+\sqrt{E})(e+e\sqrt{E}-2)}{2(1-\sqrt{E})} \ln\left(\frac{1-e\sqrt{E}}{(1-e)(1+\sqrt{E})}\right), & if\ e < \sqrt{E}, \\ 0, & otherwise. \end{cases}$$





3.2 Lean Buffering in *M*-machine Lines

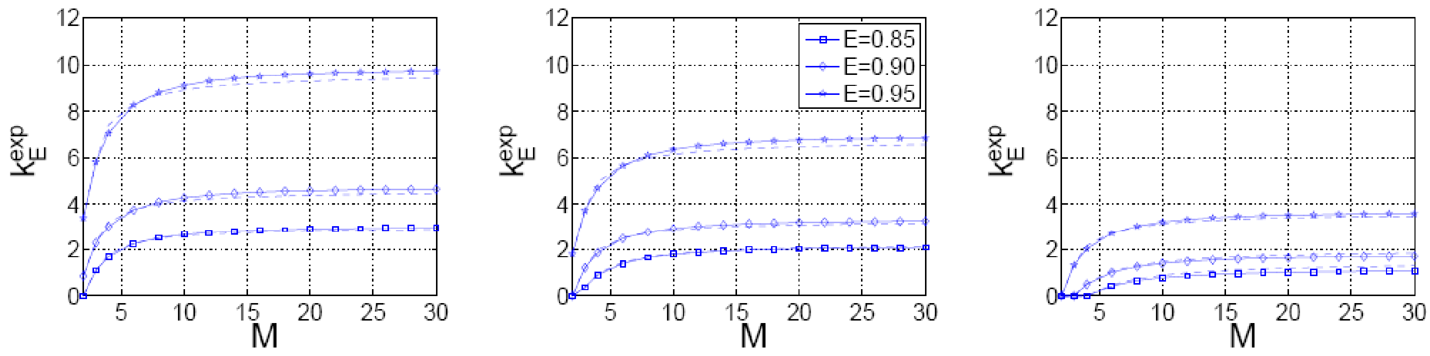
- **Theorem:**

$$k_E^{exp}(M>3)=\begin{cases}\frac{e(2-Q)(2e-eQ-2)}{2Q}\ln\left(\frac{E-eE+eEQ-1+e-2eQ+eQ^2+Q}{(1-e-Q+eQ)(E-1)}\right), & \\ & if\ e<E^{\frac{1}{M-1}}, \\ 0, & otherwise,\end{cases}$$

where $Q=Q(\lambda_{M-2}^f,\mu_{M-2}^f,\lambda_{M-1}^b,\mu_{M-1}^b,N_E)$

- Approximation of Q:

$$\widehat{Q}=1-E^{\frac{1}{2}\left[1+\left(\frac{M-3}{M-1}\right)^{M/4}\right]}+\left(E^{\frac{1}{2}\left[1+\left(\frac{M-3}{M-1}\right)^{M/4}\right]}-E^{\frac{M-2}{M-1}}\right)\exp\left\{-\left(\frac{E^{\frac{1}{M-1}}-e}{1-\sqrt{E}}\right)\right\}.$$



(a) $e=0.85$ (b) $e=0.9$ (c) $e=0.95$

3.2 Lean Buffering in *M*-machine Lines (cont)

- **Rule-of-thumb**: For exponential lines with $M \geq 10$, LLB can be selected as follows:

e	$E = 0.85$	$E = 0.90$	$E = 0.95$
0.85	3.4	5	9.8
0.90	2.7	3.9	7.2
0.95	1.6	2.4	4.3

  - Q: Is a buffer of capacity $N = 1000$ lean or not?
  - A: It depends: If $T_{down} = 1000$, the buffer is too lean (since $k = 1$). If $T_{down} = 10$, the buffer is very much not lean (since $k = 100$).

4 TRANSIENTS
4.1 Formulation

- Two-machine Bernoulli line with identical machines:

  $x(n)$: the probability that the buffer contains x parts in slot *n*

  $y[n] = [PR(n), WIP(n)]^T$

- The dynamics are described by:

$$x(n+1) = Ax(n), \quad ||x(n)||_1 = 1, \tag{25}$$

$$y(n) = Cx(n) = \begin{bmatrix} 0 & 1 & p & \cdots & p \\ 0 & 1 & 2 & \cdots & N \end{bmatrix} x(n). \tag{26}$$

4.1 Formulation (cont)

- Let λ_i, $i = 1.\ldots.n.$ be the eigenvalue of matrix A,

$$1 = \lambda_0 > \lambda_1 > |\lambda_2| \geq \ldots \geq |\lambda_N|.$$

Then, the transients of $x(n)$ are characterized by λ_1 .

- $$PR(n) = PR_{ss}\left[1 + \frac{\tilde{C}_{11}}{\tilde{C}_{10}}\tilde{x}_1(0)\lambda_1(n) + \ldots + \frac{\tilde{C}_{1N}}{\tilde{C}_{10}}\tilde{x}_N(0)\lambda_N(n)\right], \quad (27)$$

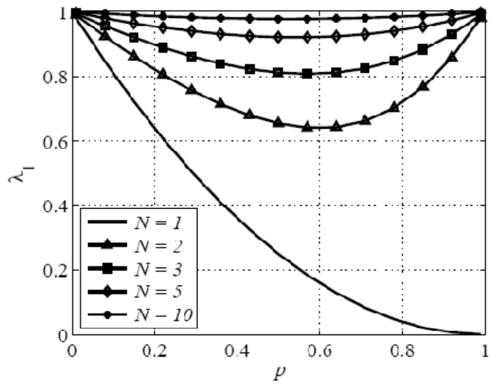
$$WIP(n) = WIP_{ss}\left[1 + \frac{\tilde{C}_{21}}{\tilde{C}_{20}}\tilde{x}_1(0)\lambda_1(n) + \ldots + \frac{\tilde{C}_{2N}}{\tilde{C}_{20}}\tilde{x}_N(0)\lambda_N(n)\right], \quad (28)$$

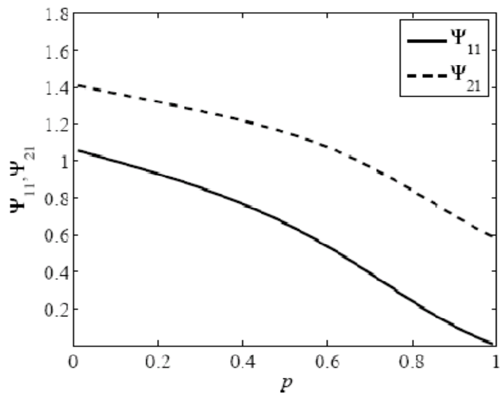
$$\Psi_{11} = \left|\frac{\tilde{C}_{11}}{\tilde{C}_{10}}\right|, \quad \Psi_{21} = \left|\frac{\tilde{C}_{21}}{\tilde{C}_{20}}\right|, \quad (29)$$

Thus, the dynamics of *PR* and *WIP* are characterized by both λ_1 and Ψ_{11}, Ψ_{21} .

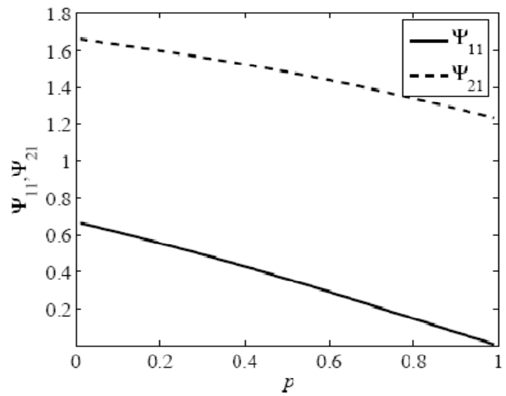
4.2 Results

Numerical Fact 5 *For two-machine Bernoulli line with identical machines, the behavior of* λ_1 *and* Ψ_{11} *and* Ψ_{21} *as functions of* p *and* N *is shown in Figures 12 and 13, respectively.*





$N = 2$

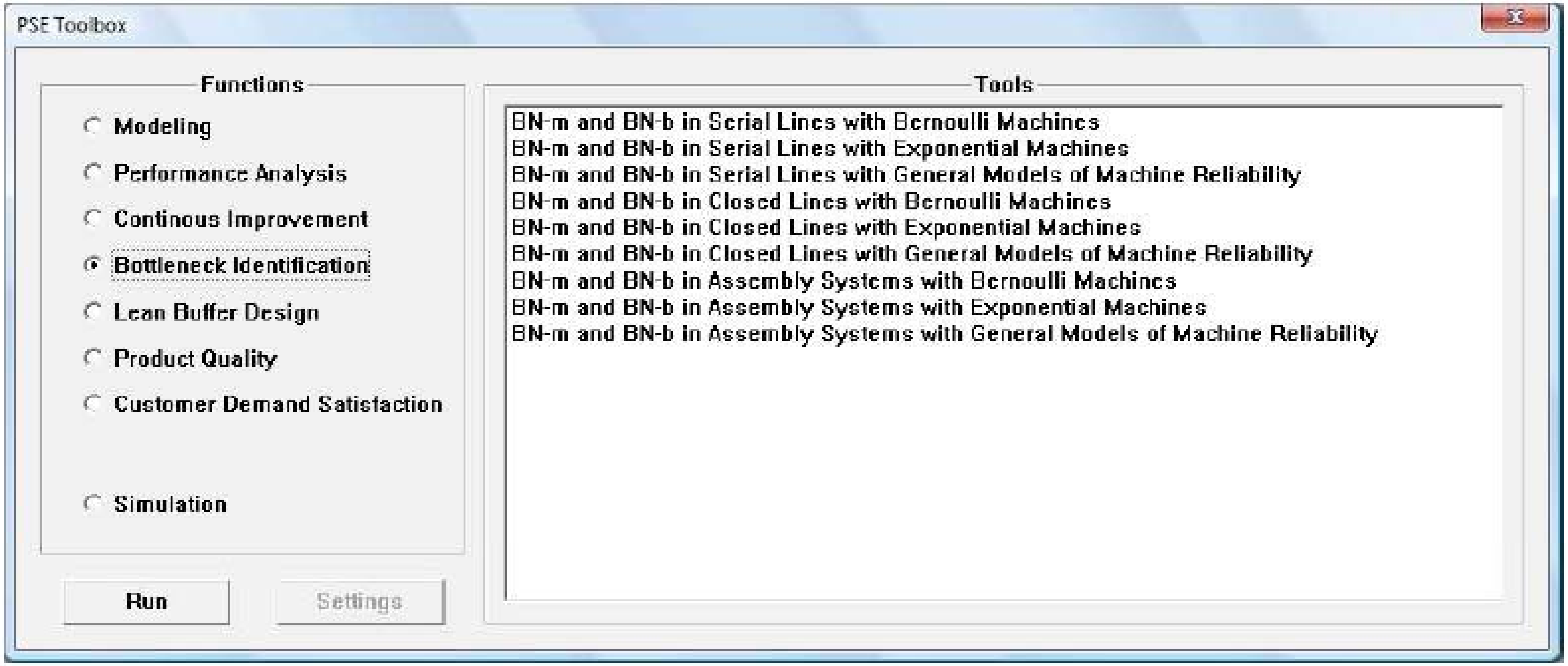


$N = 5$

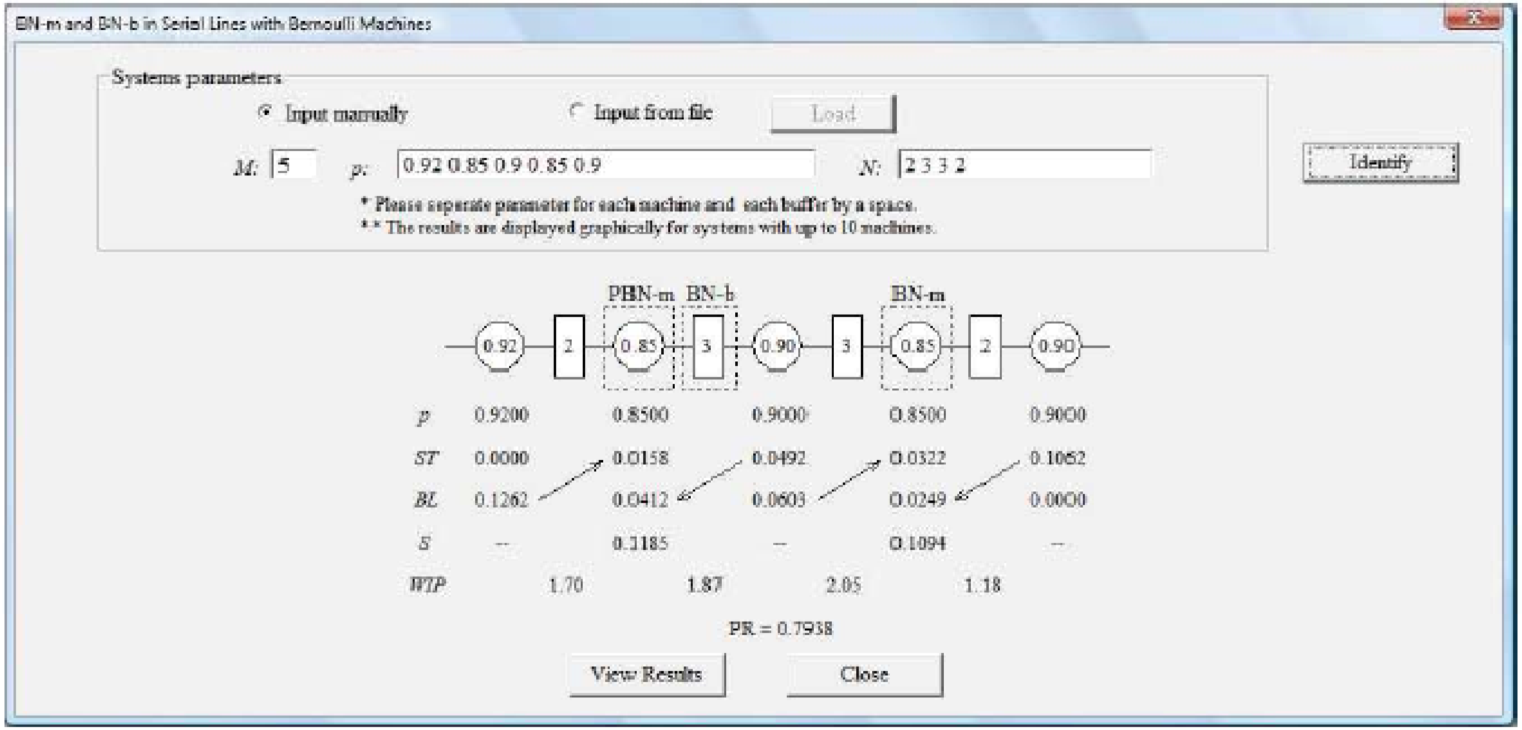
- Thus, for efficient machines, the transients of x_i and *WIP* are slow, while the transients of *PR* are fast.

5 PSE TOOLBOX

5.1 Functions



5.2 Example of tools



6 SUMMARY

- PSE provides first-principle-based methods for solving most production management problems arising on the factory floor.
- To apply these methods, one has to make measurements.
- Optimality may not be as conducive for applications as improvability.
- The main results of PSE are due to the following circumstances:
  - Markovian systems can be investigated analytically.
  - Non-Markovian systems, due to the filtering properties afforded by buffering, can be investigated using the Markovian results.
  - The sensitivity of throughput to machine parameters can be translated into blockages and starvations.
- This leads to a conclusion that, paraphrasing A. Einstein,

Production systems are complex… but not evil.

Acknowledgements

- The PSE research, applications, and *PSE Toolbox* design have been carried out jointly with my Ph.D. students:

J.-T.Lim, F. Top, D. Jacobs, C.-T. Kuo,

S.-Y. Chiang, J. Li, E. Enginarlar and L. Zhang






- To all of them, as well as to NSF that supported this work for over 25 years and to numerous industrial partners, our deepest gratitude and thanks.