Multivariate option pricing models: some extensions of the αVG model

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Time change in finance

- time-changed Brownian motion $W(T(t))$ in finance: first proposed by Clark (1973) $\Rightarrow$ Business clock $T(t)$ (positive stochastic process) quantifies the information arrival rate
- Motivation: information flow directly affects evolution of the price: when low amount of available information, slow trading and price process evolves slowly
- time change T: characteristics
  - no loss of information over time $\Rightarrow$ non decreasing process
  - amount of new information independent on the amount of information previously released $\Rightarrow$ independent increments
  - amount of released information during $[t, t + dt]$ only depends on the length of that interval dt $\Rightarrow$ stationary increments

$\Rightarrow T \equiv$ **subordinator** (non-decreasing Lévy process)

Lévy processes

Definition

$X = \{X_t, t \geq 0\}$ defined on $(\Omega, \mathcal{F}, \mathbb{P})$ is a **Lévy process** if

- X starts at 0: $X_0 = 0$ a.s.;
- X has independent increments;
- X has stationary increments.

$\Rightarrow X_{t+s} - X_s \sim$ infinitely divisible distribution and has $(\phi(u))^t$ as CF, where $\phi(u)$ is the CF of X_1.

Definition

Lévy-Khintchine representation *of an infinitely divisible distribution CF:*

$$\log(\phi_X(u)) = i\gamma u - \frac{\sigma^2}{2}u^2 + \int_{-\infty}^{+\infty} \left(\exp(iux) - 1 - iux\mathbf{1}_{|x|<1}\right) \nu(dx)$$

where $\gamma \in \mathbb{R}$, $\sigma^2 \geq 0$ *and* ν *is a measure on* $\mathbb{R}\setminus\{0\}$ *satisfying* $\int_{-\infty}^{+\infty} \inf\{1, x^2\}\, \nu(dx) < \infty$

Example: the VG process as time-changed Brownian motion (BM)

The Gamma process

- **CF** of Gamma(a, b) with $a > 0$, $b > 0$:

$$\phi_{\text{Gamma}}(u; a, b) = \left(1 - \frac{\mathrm{i}u}{b}\right)^{-a}$$

- **Scaling property**: if $X \sim$ Gamma(a, b) then
  $cX \sim \text{Gamma}(a, b/c), c > 0 \quad (1)$
  $X_t \sim \text{Gamma}(at, b) \quad (2)$
- If $X_i \sim$ Gamma(a_i, b) are N independent r.v. then
  $\sum_{i=1}^{N} X_i \sim \text{Gamma}\left(\sum_{i=1}^{N} a_i, b\right) \quad (3)$

Example: the VG process as time-changed Brownian motion (Cont)

The VG process

- $\mathsf{VG}(\sigma, \nu, \theta)$ process $\equiv$ Gamma time-changed BM with drift

$$X_t^{\mathbf{VG}}(\sigma, \nu, \theta) = \theta G_t + \sigma W_{G_t}$$

  where $G = \{G_t, t \geq 0\} \sim \mathbf{Gamma}(t/\nu, 1/\nu)$ process and $W = \{W_t, t \geq 0\}$ is a standard Brownian motion
- **CF** of $\mathbf{VG}(\sigma, \nu, \theta)$ with $\sigma > 0$, $\nu > 0$, $\theta \in \mathbb{R}$:

$$\phi_{\mathsf{VG}}(u; \sigma, \nu, \theta) = \left(1 - \mathrm{i} u\theta\nu + \frac{u^2\sigma^2\nu}{2}\right)^{\frac{-1}{\nu}}$$

	$\mathbf{VG}(\sigma, \nu, \theta)$	$\mathbf{VG}(\sigma, \nu, 0)$
mean	θ	0
variance	$\sigma^2 + \nu\theta^2$	σ^2
skewness	$\frac{\theta\nu\left(3\sigma^2+2\nu\theta^2\right)}{\left(\sigma^2+\nu\theta^2\right)^{\frac{3}{2}}}$	0
kurtosis	$3\left(1 + 2\nu - \frac{\nu\sigma^4}{(\sigma^2+\nu\theta^2)^2}\right)$	$3(1+\nu)$

Multivariate Lévy models as time changed BM

1. multivariate BM subordinated by an univariate time change (Madan and Senata, 1990).
   Unique business clock $\Rightarrow$ independence can not be captured
2. αVG model (Semeraro, 2008):
   extend multivariate time changed BM by considering a subordinator $=$ sum of idiosyncratic and common Gamma subordinators:
   $$G_t^{(i)} = X_t^{(i)} + \alpha_i Z_t$$
   Original model: constraints on the Gamma subordinator parameters such that $G_1^{(i)} \sim$ **Gamma**. CFs: independent of common subordinator settings $\Rightarrow$ calibration requires multivariate derivatives
3. multivariate Lévy two factors models (Luciano and Semeraro, 2010):
   extend the αVG model to other subordinator distributions

Some extensions of the αVG model

1. **relax the constraints** imposed on the subordinator parameters in the original model
2. consider **Sato processes** instead of Lévy processes

The αVG model

$$\mathbf{S}_t = \begin{pmatrix} S_t^{(1)} \\ S_t^{(2)} \\ \dots \\ S_t^{(N)} \end{pmatrix} = \begin{pmatrix} \frac{S_0^{(1)} \exp\left((r-q_1)t+Y_t^{(1)}\right)}{\mathbb{E}\left[\exp\left(Y_t^{(1)}\right)\right]} \\ \frac{S_0^{(2)} \exp\left((r-q_2)t+Y_t^{(2)}\right)}{\mathbb{E}\left[\exp\left(Y_t^{(2)}\right)\right]} \\ \dots \\ \frac{S_0^{(N)} \exp\left((r-q_N)t+Y_t^{(N)}\right)}{\mathbb{E}\left[\exp\left(Y_t^{(N)}\right)\right]} \end{pmatrix}$$

$$\mathbf{Y}_t = \begin{pmatrix} Y_t^{(1)} \\ Y_t^{(2)} \\ \dots \\ Y_t^{(N)} \end{pmatrix} = \begin{pmatrix} \theta_1 G_t^{(1)} + \sigma_1 W^{(1)}_{G_t^{(1)}} \\ \theta_2 G_t^{(2)} + \sigma_2 W^{(2)}_{G_t^{(2)}} \\ \dots \\ \theta_N G_t^{(N)} + \sigma_N W^{(N)}_{G_t^{(N)}} \end{pmatrix} \quad \mathbf{G}_t = \begin{pmatrix} G_t^{(1)} \\ G_t^{(2)} \\ \dots \\ G_t^{(N)} \end{pmatrix} = \begin{pmatrix} X_t^{(1)} + \alpha_1 Z_t \\ X_t^{(2)} + \alpha_2 Z_t \\ \dots \\ X_t^{(N)} + \alpha_N Z_t \end{pmatrix}$$

where $W^{(i)}$'s are independent standard BM, $\alpha_i > 0$,
$Z_1 \sim \mathbf{Gamma}(c_1, c_2)$, $c_1, c_2 > 0$ and $X_1^{(i)} \sim \mathbf{Gamma}(a_i, b_i)$, $a_i, b_i > 0$ are independent r.v. and are independent of the $W^{(i)}$'s.

The αVG model: CF's

- **CF of $\mathbf{Y}_t$:**

$$\phi_{\mathbf{Y}}(\mathbf{u},t) = \mathbb{E}\left[\exp(\mathrm{i}\mathbf{u}'\mathbf{Y}_t)\right] = \prod_{i=1}^{N}\phi_{X_1^{(i)}}\left(u_i\theta_i + \mathrm{i}\frac{1}{2}\sigma_i^2 u_i^2, t\right)$$

$$\phi_{Z_1}\left(\sum_{i=1}^{N}\alpha_i\left(u_i\theta_i + \mathrm{i}\frac{1}{2}\sigma_i^2 u_i^2\right), t\right)$$

- **marginal CF's**

$$\phi_{Y^{(i)}}(u,t) = \left(1 - \mathrm{i}\frac{u\theta_i + \mathrm{i}\frac{1}{2}\sigma_i^2 u^2}{b_i}\right)^{-a_i t}\left(1 - \mathrm{i}\frac{\alpha_i}{c_2}\left(u\theta_i + \mathrm{i}\frac{1}{2}\sigma_i^2 u^2\right)\right)^{-c_1 t}$$

- $c_2 = 1$ (by space scaling property (1))
- On average, business time grows as the real time: $\mathbb{E}\left[G_t^{(i)}\right] = t \Rightarrow \frac{a_i}{b_i} = 1 - \alpha_i\frac{c_1}{c_2}$ (4) $\Rightarrow$ $\mathbf{b_i}\left(\mathbf{1} - \alpha_{\mathbf{i}}\frac{\mathbf{c_1}}{\mathbf{c_2}}\right) > \mathbf{0}$ **(5)**

The αVG model: correlation

$$\rho_{ij} = \frac{\mathrm{Cov}\left(Y_t^{(i)}, Y_t^{(j)}\right)}{\sqrt{\mathrm{Var}\left(Y_t^{(i)}\right)\mathrm{Var}\left(Y_t^{(j)}\right)}}$$

where

$$\begin{cases} \mathrm{Cov}\left(Y_t^{(i)}, Y_t^{(j)}\right) & = \theta_i\theta_j\alpha_i\alpha_j\frac{c_1}{c_2^2}t \\ \mathrm{Var}\left(Y_t^{(i)}\right) & = \left(\theta_i^2\left(\frac{a_i}{b_i^2} + \alpha_i^2\frac{c_1}{c_2^2}\right) + \sigma_i^2\left(\frac{a_i}{b_i} + \alpha_i\frac{c_1}{c_2}\right)\right)t \end{cases}$$

$\Rightarrow$ time independent correlations

The original αVG model

Original αVG model:

impose $\mathbf{b_i} = \frac{c_2}{\alpha_i}$ $\forall \mathbf{i}$ such that

$G_1^{(i)} \overset{(3)}{\sim} \text{Gamma}(a_i + c_1, \frac{c_2}{\alpha_i}) \overset{(4)}{\equiv} \text{Gamma}(\frac{c_2}{\alpha_i}, \frac{c_2}{\alpha_i}) \Rightarrow$

$$\phi_{Y^{(i)}}(u,t) = \left(1 - \mathrm{i}\frac{\alpha_i}{c_2}\left(u\theta_i + \mathrm{i}\frac{1}{2}\sigma_i^2 u^2\right)\right)^{-\frac{c_2}{\alpha_i}t}$$

$$\Rightarrow Y_1^{(i)} \sim \mathbf{VG}\left(\sigma_i, \frac{\alpha_i}{c_2}, \theta_i\right)$$

$\Rightarrow$ **CFs independent of the common subordinator setting**

$$\rho_{ij} = \frac{\theta_i \theta_j \alpha_i \alpha_j}{\sqrt{\left(\frac{\theta_i^2}{b_i} + \sigma_i^2\right)\left(\frac{\theta_j^2}{b_j} + \sigma_j^2\right)}} c_1 \propto c_1.$$

The generalized αVG model

Generalized αVG model:

relax the constraints on the b_i's $\Rightarrow$

$$\begin{aligned}\phi_{Y^{(i)}}(u,t) &= \left(1 - \mathrm{i}\frac{u\theta_i + \mathrm{i}\frac{1}{2}\sigma_i^2 u^2}{b_i}\right)^{-a_i t} \left(1 - \mathrm{i}\frac{\alpha_i}{c_2}\left(u\theta_i + \mathrm{i}\frac{1}{2}\sigma_i^2 u^2\right)\right)^{-c_1 t} \\ &= \left(\phi_{Y^{(i)}}(u,1)\right)^t\end{aligned}$$

$\Rightarrow$ **CFs depend on the whole parameter set**

$Y_t^{(i)} \sim$ Lévy process (but not necessarily VG)

Towards the Sato two factors models

Extend the two factors Lévy model to the class of Sato processes:

- Sato processes typically lead to a significantly better fit of option prices in both the strike and time to maturity dimensions in the univariate case (Carr, Geman, Madan and Yor, 2007)

Sato processes

Definition

X is **self-decomposable** *if $X \overset{d}{=} cX + X_c, \forall 0 < c < 1$, where X_c is independent of X.*

Self-decomposable distributions are infinitely divisible distributions with a Lévy-Khintchine representation

$$\log \Phi_X(u) = i\gamma u - \frac{\sigma^2}{2} u^2 + \int_{-\infty}^{+\infty} \left(\exp(iux) - 1 - iux\mathbf{1}_{|x|<1}\right) \frac{h(x)}{|x|} dx$$

where $h(x) \geq 0$ is decreasing for $x > 0$ and increasing for $x < 0$. The probability law of the Sato process at time t is obtained by scaling the self-decomposable law of X at unit time:

$$X_t \overset{d}{=} t^\gamma X,$$

where γ = self-similar exponent. Sato processes have independent but time inhomogeneous increments.

The Sato VG process

- From the Lévy density of the VG process,

$$\nu_{\mathbf{VG}}(dx) = \begin{cases} \frac{C\exp(Gx)}{|x|}dx & x < 0, \\ \frac{C\exp(-Mx)}{|x|}dx & x > 0, \end{cases}$$

  the VG probability law at unit time is self-decomposable for all acceptable VG parameter sets $\{\sigma, \nu, \theta\}$.

- CF of VG Sato process at time t:

$$X_t^{\text{VG Sato}}(\sigma, \nu, \theta, \gamma) = t^\gamma X_1^{\text{VG}}(\sigma, \nu, \theta)$$

$$\begin{aligned} \phi_{\mathbf{VG\ Sato}}(u, t; \sigma, \nu, \theta, \gamma) &= \phi_{\mathbf{VG}}(u, 1; t^\gamma\sigma, \nu, t^\gamma\theta) \\ &= \left(1 - \mathrm{i}u\nu\theta t^\gamma + \frac{\sigma^2\nu t^{2\gamma}u^2}{2}\right)^{\frac{-1}{\nu}}. \end{aligned}$$

The Sato αVG model

$$\mathbf{Y}_t = \begin{pmatrix} Y_t^{(1)} \\ Y_t^{(2)} \\ \dots \\ Y_t^{(N)} \end{pmatrix} = \begin{pmatrix} \theta_1 t^{\gamma_1} G^{(1)} + \sigma_1 t^{\gamma_1} W^{(1)}_{G^{(1)}} \\ \theta_2 t^{\gamma_2} G^{(2)} + \sigma_2 t^{\gamma_2} W^{(2)}_{G^{(2)}} \\ \dots \\ \theta_N t^{\gamma_N} G^{(N)} + \sigma_N t^{\gamma_N} W^{(N)}_{G^{(N)}} \end{pmatrix}$$

$$\mathbf{G} = \begin{pmatrix} G^{(1)} \\ G^{(2)} \\ \dots \\ G^{(N)} \end{pmatrix} = \begin{pmatrix} X^{(1)} + \alpha_1 Z \\ X^{(2)} + \alpha_2 Z \\ \dots \\ X^{(N)} + \alpha_N Z \end{pmatrix}$$

where $W^{(i)}$'s are independent standard BM, $\alpha_i > 0$, $Z \sim$ **Gamma**(c_1, c_2), $c_1, c_2 > 0$ and $X^{(i)} \sim$ **Gamma**(a_i, b_i), $a_i, b_i > 0$ are independent random variables and are independent of the $W^{(i)}$'s.

The Sato αVG model (Cont)

- **CF of $\mathbf{Y}_t$:**

$$\phi_{\mathbf{Y}}(\mathbf{u},t)=\mathbb{E}\left[\exp(\mathbf{i}\mathbf{u}'\mathbf{Y}_t)\right] = \prod_{i=1}^{N}\phi_{X^{(i)}}\left(u_i\theta_i t^{\gamma_i}+\mathbf{i}\frac{1}{2}\sigma_i^2 t^{2\gamma_i}u_i^2\right) \phi_{Z_1}\left(\sum_{i=1}^{N}\alpha_i\left(u_i\theta_i t^{\gamma_i}+\mathbf{i}\frac{1}{2}\sigma_i^2 t^{2\gamma_i}u_i^2\right)\right).$$

- **marginal CF's**

$$\phi_{Y^{(i)}}(u,t)=\left(1-\mathbf{i}\frac{u\theta_i t^{\gamma_i}+\mathbf{i}\frac{1}{2}\sigma_i^2 t^{2\gamma_i}u^2}{b_i}\right)^{-a_i}\left(1-\mathbf{i}\frac{\alpha_i}{c_2}\left(u\theta_i t^{\gamma_i}+\mathbf{i}\frac{1}{2}\sigma_i^2 t^{2\gamma_i}u^2\right)\right)^{-c_1}$$

- $c_2=1$
- $\mathbb{E}\left[G^{(i)}\right]=1\Rightarrow \frac{a_i}{b_i}=1-\alpha_i\frac{c_1}{c_2}\Rightarrow b_i\left(1-\alpha_i\frac{c_1}{c_2}\right)>0$

The Sato αVG model: correlations

$$\rho_{ij} = \frac{\text{Cov}\left(Y_t^{(i)}, Y_t^{(j)}\right)}{\sqrt{\text{Var}\left(Y_t^{(i)}\right)\text{Var}\left(Y_t^{(j)}\right)}}$$

where

$$\text{Cov}\left(Y_t^{(i)}, Y_t^{(j)}\right) = \theta_i\theta_j\alpha_i\alpha_j\frac{c_1}{c_2^2}t^{\gamma_i+\gamma_j}$$

and

$$\text{Var}\left[Y_t^{(i)}\right] = \left(\theta_i^2\left(\frac{a_i}{b_i^2} + \alpha_i^2\frac{c_1}{c_2^2}\right) + \sigma_i^2\left(\frac{a_i}{b_i} + \alpha_i\frac{c_1}{c_2}\right)\right)t^{2\gamma_i}$$

$$\Rightarrow \rho_{ij}^{\text{Sato}} = \rho_{ij}^{\text{Lévy}}$$

The original Sato αVG model

- **Original Sato αVG model**:

  impose $\mathbf{b_i = \frac{c_2}{\alpha_i}}$ $\forall \mathbf{i}$ such that $G^{(i)} \sim \text{Gamma}(b_i, b_i) \Rightarrow$

$$\phi_{Y^{(i)}}(u,t) = \left(1 - \mathrm{i}\frac{u\theta_i t^{\gamma_i} + \mathrm{i}\frac{1}{2}\sigma_i^2 t^{2\gamma_i} u^2}{b_i}\right)^{-b_i}$$

$$\Rightarrow Y_t^{(i)} \sim \textbf{VG Sato}\left(\sigma_i, \frac{1}{b_i}, \theta_i, \gamma_i\right)$$

$\Rightarrow$ **CFs independent of the common subordinator setting**

$$\rho_{ij} = \frac{\theta_i \theta_j \alpha_i \alpha_j}{\sqrt{\left(\frac{\theta_i^2}{b_i} + \sigma_i^2\right)\left(\frac{\theta_j^2}{b_j} + \sigma_j^2\right)}} c_1 \propto c_1.$$

The generalized Sato αVG model

- **Generalized Sato αVG model**:
  relax the constraints on the b_i's $\Rightarrow$

$$\phi_{Y^{(i)}}(u,t) = \left(1 - \mathrm{i}\frac{u\theta_i t^{\gamma_i} + \mathrm{i}\frac{1}{2}\sigma_i^2 t^{2\gamma_i} u^2}{b_i}\right)^{-a_i} \left(1 - \mathrm{i}\frac{\alpha_i}{c_2}\left(u\theta_i t^{\gamma_i} + \mathrm{i}\frac{1}{2}\sigma_i^2 t^{2\gamma_i} u^2\right)\right)^{-c_1}$$

$\Rightarrow$ **CFs depend on the whole parameter set**

The generalized Sato αVG model

Under the generalized Sato model, $Y_t^{(i)} \sim$ Sato

Proof.

$$\phi_{Y^{(i)}}(u,1) = \left(1 - \mathrm{i}\frac{u\theta_i + \mathrm{i}\frac{1}{2}\sigma_i^2 u^2}{b_i}\right)^{-a_i} \left(1 - \mathrm{i}\frac{\alpha_i}{c_2}\left(u\theta_i + \mathrm{i}\frac{1}{2}\sigma_i^2 u^2\right)\right)^{-c_1}$$

$$= \phi_{\mathbf{VG}}\left(u; \sqrt{\frac{a_i}{b_i}}\sigma_i, \frac{1}{a_i}, \frac{a_i}{b_i}\theta_i\right) \phi_{\mathbf{VG}}\left(u; \sqrt{\frac{c_1}{c_2}\alpha_i}\sigma_i, \frac{1}{c_1}, \frac{c_1}{c_2}\alpha_i\theta_i\right)$$

$\Rightarrow Y_1^{(i)} \stackrel{d}{=} U_1^{(i)} + U_2^{(i)}$, where $U_1^{(i)} \sim \mathbf{VG}\left(\sqrt{\frac{a_i}{b_i}}\sigma_i, \frac{1}{a_i}, \frac{a_i}{b_i}\theta_i\right)$ and $U_2^{(i)} \sim \mathbf{VG}\left(\sqrt{\frac{c_1}{c_2}\alpha_i}\sigma_i, \frac{1}{c_1}, \frac{c_1}{c_2}\alpha_i\theta_i\right)$ are independent VG r.v $\Rightarrow Y_1^{(i)}$'s are self decomposable since they are the sum of 2 self decomposable r.v. □

Calibration: original αVG model I

Original αVG model:
dissociate the calibration of the univariate option surfaces and the calibration of the correlations (Leoni and Schoutens 2008, Luciano and Semeraro 2010):

1. simultaneous calibration of each option surface:

$$\text{MRMSE} = \sum_{i=1}^{N} \frac{\text{RMSE}^{(i)}}{N} = \sum_{i=1}^{N} \frac{1}{N} \sqrt{\frac{\sum_{j=1}^{M^{(i)}} \left(P_j^{(i)} - \hat{P}_j^{(i)} \right)^2}{M^{(i)}}}$$

   where
   - $N \equiv$ number of underlying stocks
   - $M^{(i)} \equiv$ number of quoted options for the ith stock
   - $P_j^{(i)} \equiv$ jth market option price of the ith stock
   - $\hat{P}_j^{(i)} \equiv$ jth model option price of the ith stock

   $\Rightarrow \equiv N$ univariate option surface calibrations since CF's do not share any common parameter

Calibration: original αVG model II

2. calibrate parameters which do not influence any marginal CF, i.e. c_1 on market implied or historical stock correlations (Moving Window or EWMA technique):

$$\text{RMSE}^{\rho} = \sqrt{\frac{1}{\frac{N^2-N}{2}} \sum_{i,j\neq i}^{N} (\rho_{ij} - \hat{\rho}_{ij})^2}$$

BUT: (5) $\Rightarrow$ $c_1 < \frac{1}{\max_i \alpha_i}$ AND only 1 parameter to fit $\frac{N^2-N}{2}$ correlations

Calibration: generalized αVG model

Generalized αVG model

- Univariate option surfaces only: minimize

$$\sum_i \frac{\text{RMSE}^{(i)}}{N} = \sum_i \frac{\text{RMSE}^{(i)}(\theta_i, \sigma_i, \alpha_i, b_i | c_1)}{N}$$

and repeat for different values of $c_1 \Rightarrow$ MRMSE*.

- Univariate option surfaces + correlation:

$$\text{MRMSEJ} = \sum_{i=1}^{N} \frac{\text{RMSE}^{(i)}}{N} + \alpha^\rho \text{MRMSE}^* \sqrt{\frac{1}{\frac{N^2-N}{2}} \sum_{j,k \neq j}^{N} \left(\rho_{jk} - \hat{\rho}_{jk}\right)^2}$$

where MRMSE* = optimal value of MRMSE given by the calibration of the option surfaces and $\alpha^\rho \geq 0$ specifies the relative importance of the correlation matching

Numerical study: data I

- basket = 4 of the 10 largest (market cap) S&P500 components which satisfy:

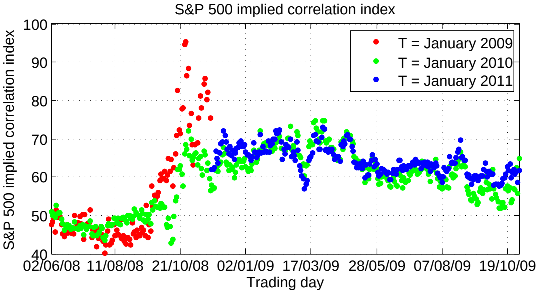
$$D_i \leq K\left(1 - \exp(-r(t_{i+1} - t_i))\right), \quad i = 1, 2, \ldots, n-1$$

  and

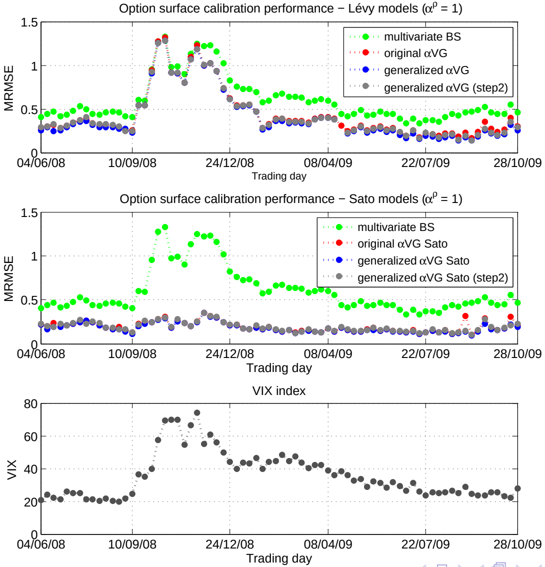
$$D_n \leq K\left(1 - \exp(-r(T - t_n))\right)$$

  where D_i = dividend at ith ex-dividend dates t_i with $t_0 < t_i < \ldots < t_n < T$
- implied correlations: inferred from the S&P500 implied correlation index (= measure of the expected average correlation of price return): $\rho^{\text{CBOE}} \sim \rho\left(\frac{S^{(i)}_{T_\rho}}{S^{(i)}_{t_0}}, \frac{S^{(j)}_{T_\rho}}{S^{(j)}_{t_0}}\right)$
- infer $\rho\left(Y^{(i)}_{T_\rho}, Y^{(j)}_{T_\rho}\right)$ by Taylor series expansions

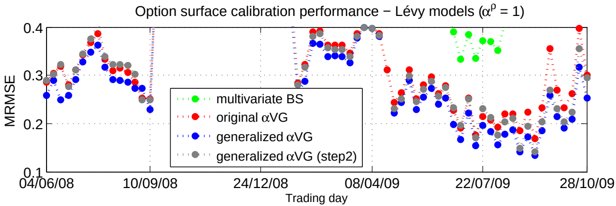
Numerical study: data II

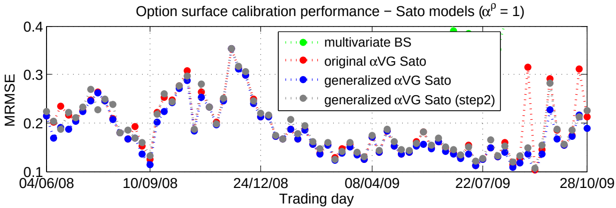


The αVG models: calibration performance (univariate option surfaces) I

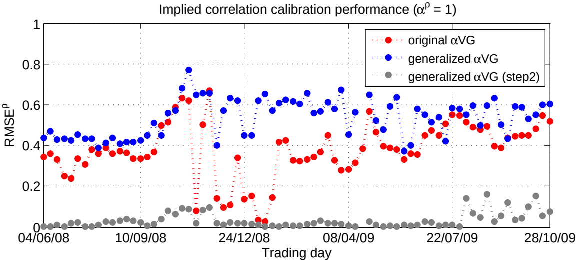


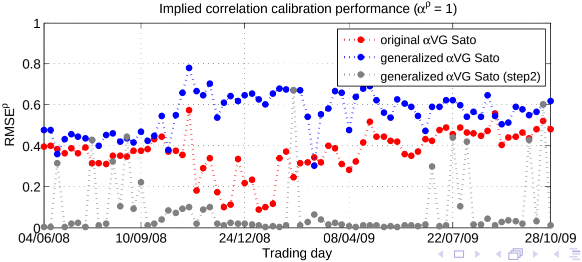
The αVG models: calibration performance (univariate option surfaces) II



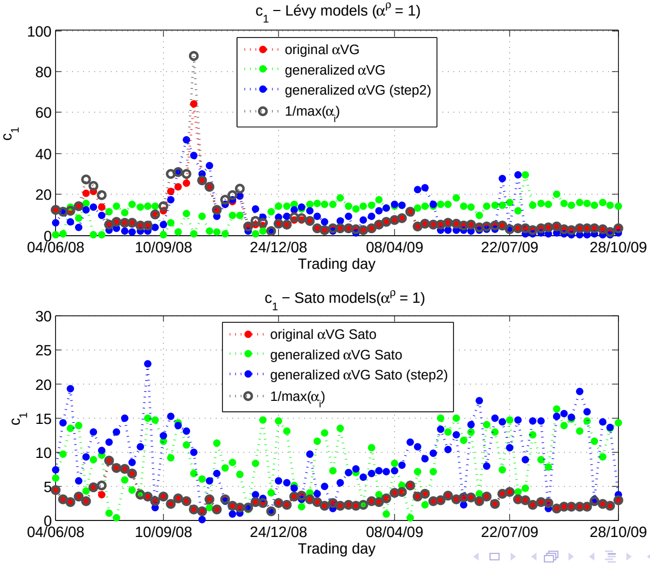


The αVG models: calibration performance (correlations) I

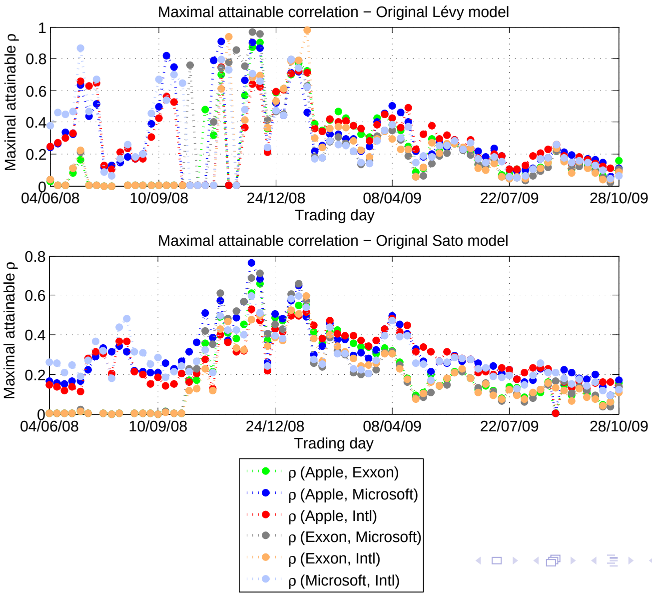




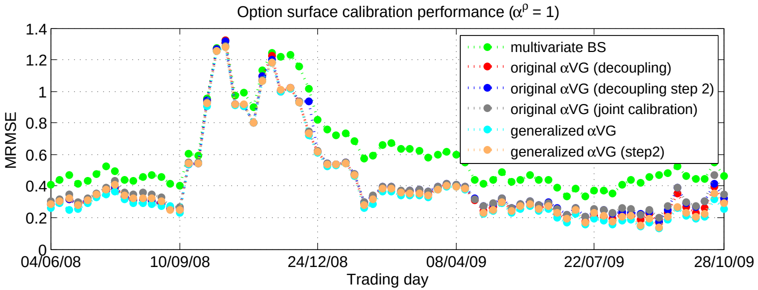
The αVG models: calibration performance (correlations) II

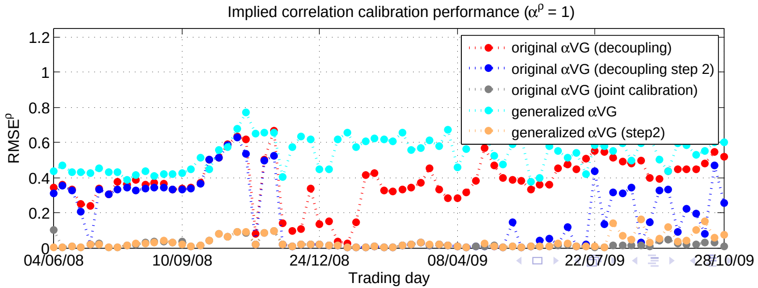


The αVG models: calibration performance (correlations) III



Alternative calibrations of the original model





Choice of α^ρ

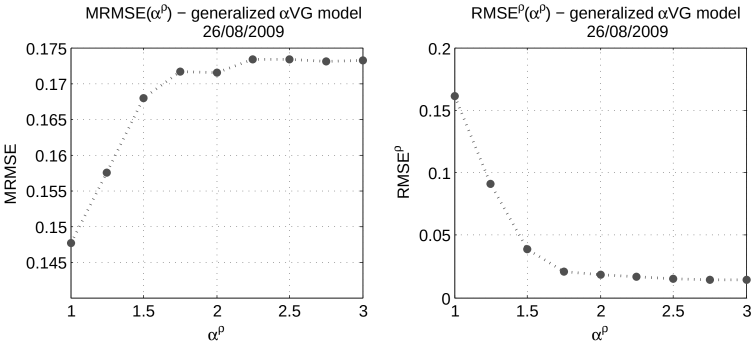


Figure: Influence of α^ρ for the generalized αVG model.

Multivariate exotic options: Rainbow options

- Worst-of call

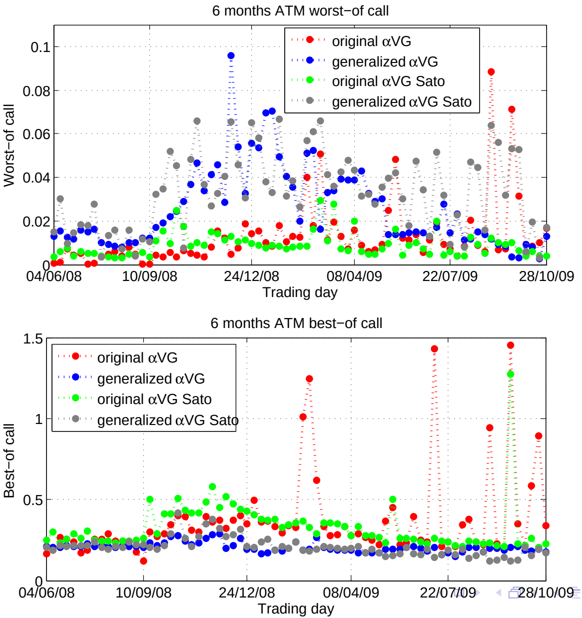
$$\mathrm{WC} = \exp(-rT)\mathbb{E}_{\mathbb{Q}} \max\left(\min\left(\frac{S_T^{(1)} - S_0^{(1)}}{S_0^{(1)}}, \frac{S_T^{(2)} - S_0^{(2)}}{S_0^{(2)}}, \frac{S_T^{(3)} - S_0^{(3)}}{S_0^{(3)}}, \frac{S_T^{(4)} - S_0^{(4)}}{S_0^{(4)}}\right), 0\right)$$

- Best-of call

$$\mathrm{BC} = \exp(-rT)\mathbb{E}_{\mathbb{Q}} \max\left(\max\left(\frac{S_T^{(1)} - S_0^{(1)}}{S_0^{(1)}}, \frac{S_T^{(2)} - S_0^{(2)}}{S_0^{(2)}}, \frac{S_T^{(3)} - S_0^{(3)}}{S_0^{(3)}}, \frac{S_T^{(4)} - S_0^{(4)}}{S_0^{(4)}}\right), 0\right)$$

$\Rightarrow$ when ρ increases $\Rightarrow$ asset prices tend to be the same $\Rightarrow$ minimum of asset prices increases and maximum decreases

Rainbow options: model risk



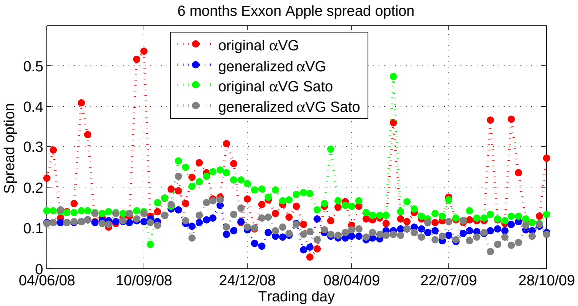
Multivariate exotic options: Spread options

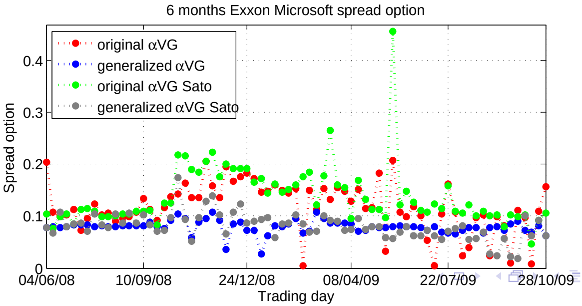
- Spread options

$$\text{Spread} = \exp(-rT)\mathbb{E}_{\mathbb{Q}} \max\left(\frac{S_T^{(1)}}{S_0^{(1)}} - \frac{S_T^{(2)}}{S_0^{(2)}}, 0\right)$$

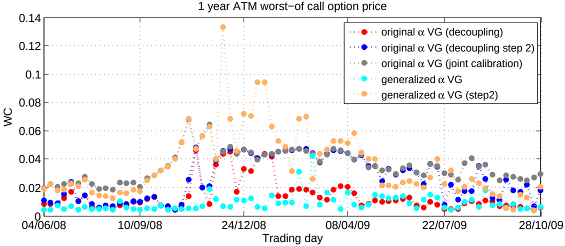
$\Rightarrow$ when ρ increases $\Rightarrow$ asset prices tend to be the same $\Rightarrow$ spread decreases

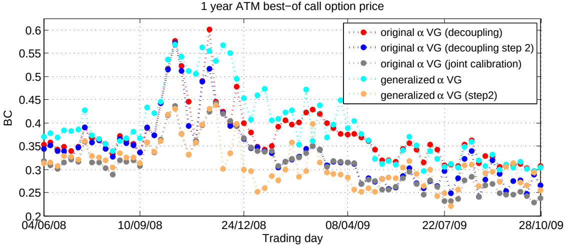
Spread options: model risk





Alternative calibrations of the original model: impact on rainbow option prices





The generalized αVG model: conclusions

- generalized αVG model: CFs which remain of Lévy type but become dependent on the whole parameter set $\Rightarrow$
  - market-implied calibration does not require anymore the existence of a liquid market for multivariate derivatives
  - volatility (trading activity) depends on both the idiosyncratic and common subordinators $\Rightarrow$ in line with empirical evidence of the presence of both individual and common business clocks
- generalized model: better fit of univariate option surfaces except during high volatility regime periods + correlation goodness of fit significantly improved by performing a second calibration (penalty term assessing the correlation goodness of fit)
- shortfall of the decoupling calibration procedure for the original αVG model: condition that the business time grows on average as the calendar time implies an upper bound on the common parameter c_1 which is a function of the α_i's $\Rightarrow$ decoupling calibration limits severely admissible value range of c_1

The αVG Sato models: Conclusions

- Sato processes $\Rightarrow$ significantly better fit of univariate option surfaces (especially during high volatility regime)
- Similar correlation fit than the αVG models
- Test higher values of α^{ρ} to improve the correlation fit
- Significantly different multivariate exotic option prices under the original and generalized models

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Thank you for your attention