Model Uncertainty and Robustness: Entropy Coherent and Entropy Convex Measures of Risk

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1. Introduction

Sign conventions used in this talk:

- Random variables represent payoffs of financial positions. Positive realizations represent gains.
- A risk measure represents a negative valuation.

Convex Measures of Risk

- Convex measures of risk (Föllmer and Schied, 2002, Fritelli and Rosazza Gianin, 2002, and Heath and Ku, 2004) are characterized by the axioms of monotonicity, translation invariance and convexity.
- They can (under additional assumptions on the space of random variables and on continuity properties of the risk measure) be represented in the form

$$\rho(X) = \sup_{Q \in \mathcal{Q}} \{\mathrm{E}_Q[-X] - \alpha(Q)\},$$

  where $\mathcal{Q}$ is a set of probability measures and α is a penalty function defined on $\mathcal{Q}$.
- With

$$\alpha(Q) = \begin{cases} 0, & \text{if } Q \in \mathcal{Q}; \\ \infty, & \text{otherwise}; \end{cases}$$

  we obtain the subclass of coherent measures of risk, represented in the form

$$\rho(X) = \sup_{Q \in M \subset \mathcal{Q}} \mathrm{E}_Q[-X].$$

Variational Preferences

- A rich paradigm for decision-making under ambiguity is the theory of variational preferences (Maccheroni, Marinacci and Rustichini, 2006).
- An economic agent evaluates the payoff of a choice alternative (financial position) X according to

$$U(X) = \inf_{Q \in \mathcal{Q}} \left\{ \mathrm{E}_Q \left[u(X) \right] + \alpha(Q) \right\},$$

  where $u : \mathbb{R} \to \mathbb{R}$ is an increasing function, $\mathcal{Q}$ is a set of probability measures and α is an ambiguity index defined on $\mathcal{Q}$.

Multiple Priors Preferences

- A special case of interest is that of multiple priors preferences (Gilboa and Schmeidler, 1989), obtained by considering

$$U(X) = \inf_{Q \in \mathcal{Q}} \left\{ \mathrm{E}_Q \left[u(X) \right] + \bar{I}_M(Q) \right\},$$

  where $\bar{I}_M$ is the ambiguity index that is zero if $Q \in M$ and ∞ otherwise.
- Gilboa and Schmeidler (1989) and Maccheroni, Marinacci and Rustichini (2006) established preference axiomatizations of these theories, generalizing Savage (1954) in the framework of Anscombe and Aumann (1963).
- The representation of Gilboa and Schmeidler (1989), also referred to as maxmin expected utility, was a decision-theoretic foundation of the classical decision rule of Wald (1950) — see also Huber (1981) — that had long seen little popularity outside (robust) statistics.

Interpretation

- The function u, referred to as a utility function, represents the agent's attitude towards wealth.
- The set $\mathcal{Q}$ represents the set of priors held by agents.
- Under multiple priors preferences, the degree of ambiguity is reflected by the multiplicity of the priors.
- Under general variational preferences, the degree of ambiguity is reflected by the multiplicity of the priors and the esteemed plausibility of the prior as reflected in the ambiguity index (or penalty function).

Homothetic Preferences

- Recently, Chateauneuf and Faro (2010) and, slightly more generally, Cerreia-Vioglio *et al.* (2008) axiomatized a multiplicative analog of variational preferences, referred to henceforth as homothetic preferences.
- It is represented as

$$U(X) = \inf_{Q \in \mathcal{Q}} \left\{ \beta(Q) \mathrm{E}_Q \left[u(X) \right] \right\},$$

  with $\beta : \mathcal{Q} \to [0, \infty]$.
- It also includes multiple priors as a special case ($\beta(Q) \equiv 1$).

Measuring 'Risk' (in the broad sense)

- To measure the 'risk' related to a financial position X, the theories of variational and homothetic preferences sketched above would lead to the definition of a loss functional $L(X) = -U(X)$, satisfying

$$L(X) = \sup_{Q \in \mathcal{Q}} \left\{ \mathrm{E}_Q \left[\phi(-X) \right] - \alpha(Q) \right\} \quad \text{and}$$
$$L(X) = \sup_{Q \in \mathcal{Q}} \left\{ \beta(Q) \mathrm{E}_Q \left[\phi(-X) \right] \right\},$$

  respectively, where $\phi(x) = -u(-x)$.

- One could, then, look at the amount of capital one needs to hold in response to the position X, i.e., the negative certainty equivalent of X, denoted by m_X, satisfying $L(-m_X) = \phi(m_X) = L(X)$, or equivalently,

$$m_X = \phi^{-1} \left(\sup_{Q \in \mathcal{Q}} \left\{ \mathrm{E}_Q \left[\phi(-X) \right] - \alpha(Q) \right\} \right) \quad \text{and}$$
$$m_X = \phi^{-1} \left(\sup_{Q \in \mathcal{Q}} \left\{ \beta(Q) \mathrm{E}_Q \left[\phi(-X) \right] \right\} \right).$$

Variational and Homothetic Preferences vs. Convex Measures of Risk

- Compare

$$m_X = \phi^{-1}\left(\sup_{Q\in\mathcal{Q}} \{\mathrm{E}_Q[\phi(-X)] - \alpha(Q)\}\right) \text{ and}$$

$$m_X = \phi^{-1}\left(\sup_{Q\in\mathcal{Q}} \{\beta(Q)\mathrm{E}_Q[\phi(-X)]\}\right)$$

to

$$\rho(X) = \sup_{Q\in\mathcal{Q}} \{\mathrm{E}_Q[-X] - \alpha(Q)\}.$$

- Question: find sufficient and necessary conditions.

Multiple Priors Preferences vs. Convex Measures of Risk

- Compare

$$m_X = \phi^{-1}\Big(\sup_{Q \in M \subset \mathcal{Q}} \mathrm{E}_Q [\phi(-X)] \Big),$$

to

$$\rho(X) = \sup_{Q \in \mathcal{Q}} \{ \mathrm{E}_Q [-X] - \alpha(Q) \}.$$

- Question: find sufficient and necessary conditions.

Question Rephrased [1]

- In other words, we consider

$$\rho(X) = \phi^{-1}(\bar{\rho}(-\phi(-X))),$$

  with

$$\bar{\rho}(X) = \sup_{Q \in M \subset \mathcal{Q}} \mathrm{E}_Q[-X].$$

- Preferences of Gilboa and Schmeidler (1989).
- We also consider

$$\rho(X) = \phi^{-1}(\bar{\rho}(-\phi(-X))),$$

  with

$$\bar{\rho}(X) = \sup_{Q \in \mathcal{Q}} \{\mathrm{E}_Q[-X] - \alpha(Q)\}.$$

- Preferences of Maccheroni, Marinacci and Rustichini (2006).
- In the latter case, negative certainty equivalents are invariant under translation of u (or ϕ).
- Traditionally (in the models of Savage, 1954, and Gilboa and Schmeidler, 1989), negative certainty equivalents are invariant under both translation and positive multiplication of u (or ϕ).

Question Rephrased [2]

- We consider in addition

$$\rho(X) = \phi^{-1}(\bar{\rho}(-\phi(-X))),$$

with

$$\bar{\rho}(X) = \sup_{Q \in M \subset \mathcal{Q}} \beta(Q) \mathrm{E}_Q [-X] \,,$$

with $\beta : M \rightarrow [0, 1]$.

- Preferences of Chateauneuf and Faro (2010).
- With $\bar{\rho}$ as given above, negative certainty equivalents are invariant under positive multiplication of u (or ϕ); complementary case.
- $\beta : M \rightarrow [0, 1]$ can be viewed as a discount factor; $\bar{\rho}$ seems natural.
- Includes multiple priors preferences as a special case.
- Recall question: find sufficient and necessary conditions under which ρ (not $\bar{\rho}$) is a convex risk measure.

Results [1]

The contribution of this paper is twofold.

- First we derive precise connections between risk measurement under the theories of variational, homothetic and multiple priors preferences on the one hand and risk measurement using convex measures of risk on the other.
- This is, despite the vast literature on both paradigms, a hitherto open problem.
- In particular, we identify two subclasses of convex risk measures that we call entropy coherent and entropy convex measures of risk, and that include all coherent risk measures.
- We show that, under technical conditions, negative certainty equivalents under variational, homothetic, and multiple priors preferences are translation invariant if and only if they are convex, entropy convex, and entropy coherent measures of risk, respectively.

Results [2]

- It entails that convex, entropy convex and entropy coherent measures of risk induce linear or exponential utility functions in the theories of variational, homothetic and multiple priors preferences.
- We show further that, under a normalization condition, this characterization remains valid when the condition of translation invariance is replaced by requiring convexity.
- The mathematical details in the proofs of these characterization results are delicate.

Results [3]

- These connections suggest two new subclasses of convex risk measures: entropy coherent and entropy convex measures of risk, and our second contribution is to study their properties.
- We show that they satisfy many appealing properties.
- We prove various results on the dual conjugate function for entropy coherent and entropy convex measures of risk. We show in particular that, quite exceptionally, the dual conjugate function can explicitly be identified under some technical conditions.
- We also study entropy coherent and entropy convex measures of risk under the assumption of distribution invariance. Due to their convex nature, a feature that singles out entropy convex measures of risk in the class of negative certainty equivalents under homothetic preferences, we can obtain explicit representation results in this setting.
- Some financial applications and examples of these risk measures are also provided, explicitly utilizing some of the representation results derived.

Outline

2. Entropic Measures of Risk [1]

We fix a probability space $(\Omega, \mathcal{F}, P)$ and fix a scalar $\gamma \in [0, \infty]$. Let $X \in L^{\infty}$. Define

$$e_{\gamma}(X) := \gamma \log \left(\mathrm{E} \left[\exp \left(-\frac{X}{\gamma} \right) \right] \right).$$

- Entropic measures of risk (exponential premiums) emerge in various paradigms.

Entropic Measures of Risk [2]

- Note that for every given X, the mapping $\gamma \to e_\gamma(X)$ is increasing.
- As is well-known (Csiszár, 1975),

$$e_\gamma(X) = \sup_{\bar{P} \ll P} \left\{ \mathrm{E}_{\bar{P}}\left[-X\right] - \gamma H(\bar{P}|P) \right\},$$

where $H(\bar{P}|P)$ is the relative entropy, i.e.,

$$H(\bar{P}|P) = \begin{cases} \mathrm{E}_{\bar{P}}\left[\log\left(\frac{d\bar{P}}{dP}\right)\right], \text{ if } \bar{P} \ll P; \\ \infty, \text{ otherwise.} \end{cases}$$

The relative entropy is also known as the Kullback-Leibler divergence; it measures the distance between the distributions $\bar{P}$ and P.

Two Interpretations

1. Kullback-Leibler. The parameter γ may be viewed as measuring the degree of trust the agent puts in the reference measure P. If $\gamma = 0$, then $e_0(X) = -\operatorname{ess\,inf} X$, which corresponds to a maximal level of distrust; in this case only the zero sets of the measure P are considered reliable. If, on the other hand, $\gamma = \infty$, then $e_\infty(X) = -\mathrm{E}[X]$, which corresponds to a maximal level of trust in the measure P.
2. Exponential utility. An economic agent with a CARA (exponential) utility function $u(x) = 1 - e^{-\frac{x}{\gamma}}$ computes the (negative) certainty equivalent or applies the (negative) equivalent utility principle to the payoff X with respect to the reference measure P.

Other Reference Measure

In certain situations the agent could consider other reference measures $Q \ll P$. Then we define the entropy $e_{\gamma,Q}$ with respect to Q as

$$e_{\gamma,Q}(X) = \gamma \log \left(\mathrm{E}_Q \left[\exp \left(\frac{-X}{\gamma} \right) \right] \right).$$

Entropy Coherence and Entropy Convexity

Definition
We call a mapping $\rho : L^\infty \to \mathbb{R}$ γ-entropy coherent, $\gamma \in [0, \infty]$, if there exists a set $M \subset \mathcal{Q}$ such that

$$\rho(X) = \sup_{Q \in M} e_{\gamma,Q}(X).$$

It will be interesting to consider as well a more general class of risk measures:

Definition
The mapping $\rho : L^\infty \to \mathbb{R}$ is γ-entropy convex, $\gamma \in [0, \infty]$, if there exists a penalty function $c : \mathcal{Q} \to [0, \infty]$ with $\inf_{Q \in \mathcal{Q}} c(Q) = 0$, such that

$$\rho(X) = \sup_{Q \in \mathcal{Q}} \{e_{\gamma,Q}(X) - c(Q)\}.$$

Again Two Interpretations [1]

Suppose that the agent is only interested in downside tail risk and considers Tail-Value-at-Risk ($TV@R$) defined by

$$TV@R^{\alpha}(X) = \frac{1}{\alpha} \int_0^{\alpha} V@R^{\lambda}(X) d\lambda, \quad \alpha \in]0,1].$$

It is well-known that

$$TV@R^{\alpha}(X) = \sup_{Q \in M_{\alpha}} \mathrm{E}_Q[-X],$$

where M_{α} is the set of all probability measures $Q \ll P$ such that $\frac{dQ}{dP} \leq \frac{1}{\alpha}$.

Again Two Interpretations [2]

The economic agent may, however, not fully trust the probabilistic model of X under P, hence under Q. Therefore, for every fixed Q, the agent considers the supremum over all measures absolutely continuous with respect to Q, where measures that are 'close' to Q are esteemed more plausible than measures that are 'distant' from Q. This leads to a risk measure ρ given by

$$\rho(X) = \sup_{\bar{P} \ll Q} \sup_{Q \in M_\alpha} \{\mathrm{E}_{\bar{P}}[-X] - \gamma H(\bar{P}|Q)\} = \sup_{\bar{P} \ll P} \sup_{Q \in M_\alpha} \{\mathrm{E}_{\bar{P}}[-X] - \gamma H(\bar{P}|Q)\}$$

$$= \sup_{Q \in M_\alpha} \sup_{\bar{P} \ll P} \{\mathrm{E}_{\bar{P}}[-X] - \gamma H(\bar{P}|Q)\} = \sup_{Q \in M_\alpha} e_{\gamma,Q}(X),$$

where we have used in the second and last equalities that $H(\bar{P}|Q) = \infty$ if $\bar{P}$ is not absolutely continuous with respect to Q.

Again Two Interpretations [3]

The definition of entropy convexity (whence the special case of entropy coherence as well) can also be motivated using microeconomic theory, as follows:

- An economic agent with a CARA (exponential) utility function $u(x) = 1 - e^{-\frac{x}{\gamma}}$ computes the certainty equivalent to the payoff X with respect to the reference measure P.
- The agent is, however, uncertain about the probabilistic model under the reference measure, and therefore takes the infimum over all probability measures Q absolutely continuous with respect to P, where the penalty function $c(Q)$ represents the esteemed plausibility of the probabilistic model under Q.
- The robust certainty equivalent thus computed is precisely $-\rho(X)$.

A Basic Duality Result

Define

$$\rho^*(Q) = \sup_{X \in L^\infty} \{e_{\gamma,Q}(X) - \rho(X)\}$$

and

$$\rho^{**}(X) = \sup_{Q \ll P} \{e_{\gamma,Q}(X) - \rho^*(Q)\}.$$

Then the following result holds:

Lemma
A normalized mapping ρ is γ-entropy convex if and only if $\rho^{**} = \rho$. Furthermore, ρ^* is the minimal penalty function.

Subdifferential

- We define the subdifferential of ρ by

$$\partial\rho(X) = \{Q \in \mathcal{Q} | \rho(X) = \mathrm{E}_Q[-X] - \alpha(Q)\}.$$

  We say that ρ is subdifferentiable if for every $X \in L^\infty$ we have $\partial\rho(X) \neq \emptyset$.

- For a γ-entropy convex function ρ we define by

$$\partial_{entropy}\rho(X) = \{Q^* \in \mathcal{Q} | \rho(X) = e_{\gamma,Q^*}(X) - c(Q^*)\}$$

  the entropy subdifferential. Furthermore, if for every $X \in L^\infty$, $\partial_{entropy}\rho(X) \neq \emptyset$, then we say that ρ is entropy subdifferentiable.

3. Characterization Results [1]

Recall the first question asked in the Introduction (slide 11). Answer:

Theorem
Suppose that the probability space is rich. Let ϕ be a strictly increasing and continuous function satisfying $0 \in closure(Image(\phi))$, $\phi(\infty) = \infty$ and $\phi \in C^3(]\phi^{-1}(0), \infty[)$.

Then the following statements are equivalent:

(i) $\rho(X) = \phi^{-1}(\bar{\rho}(-\phi(-X)))$ *is translation invariant and the subdifferential of* $\bar{\rho}$ *is always nonempty.*

(ii) ρ *is γ-entropy coherent with $\gamma \in]0, \infty]$, and the entropy subdifferential is always nonempty.*

Characterization Results [2]

Recall the question asked in the Introduction (slide 12). Answer:

Theorem

Suppose that the probability space is rich. Let ϕ be a strictly increasing and continuous function satisfying $0 \in closure(Image(\phi))$, $\phi(\infty) = \infty$ and $\phi \in C^3(]\phi^{-1}(0), \infty[)$.

Then the following statements are equivalent:

(i) $\rho(X) = \phi^{-1}(\bar{\rho}(-\phi(-X)))$ *is translation invariant and the subdifferential of $\bar{\rho}$ is always nonempty.*

(ii) ρ *is γ-entropy convex with $\gamma \in \mathbb{R}^+$ or ρ is ∞-entropy coherent, and the entropy subdifferential is always nonempty.*

Remark 1

- The case that ρ is entropy convex corresponds to ρ being the negative certainty equivalent of $\bar{\rho}(X) = \sup_{Q \in M} \beta(Q) \mathrm{E}_Q[-X]$, where $\beta : M \to [0, 1]$ can be viewed as a discount factor, and with ϕ being linear (implying $\beta(Q) \equiv 1$) or exponential.
- In this case, every model Q is discounted by a factor $\beta(Q)$ corresponding to its esteemed plausibility.
- If $\beta(Q) = 1$ for all $Q \in M$, we are back in the framework of Gilboa-Schmeidler.
- However, if there exists a $Q \in M$ such that $\beta(Q) < 1$, ρ is entropy convex with $\gamma \in \mathbb{R}^+$ but not entropy coherent.

Remark 2

- Recall the definition of entropy convexity (slide 21).
- As $e_{\infty,Q}(X) = \mathrm{E}_Q\left[-X\right]$, ρ is a convex risk measure if and only if it is ∞-entropy convex.
- As we will see later, however, with $\gamma < \infty$, not every convex risk measure is γ-entropy convex.
- This is important: we have seen that, under some technical conditions, negative certainty equivalents under homothetic preferences are translation invariant if and only if they are γ-entropy convex with $\gamma \in \mathbb{R}^+$ or ∞-entropy coherent, ruling out the general ∞-entropy convex case.
- Translation invariant negative certainty equivalents under homothetic preferences do not span the class of convex risk measures.

Characterization Results [3]

Recall the second question asked in the Introduction (slide 11). Answer:

Theorem

Suppose that the probability space is rich. Let ϕ be a strictly increasing and convex function with $\phi \in C^3(\mathbb{R})$ and either $\phi(-\infty) = -\infty$ or $\lim_{x\to\infty} \phi(x)/x = \infty$.

Then the following statements are equivalent:

(i) $\rho(X) = \phi^{-1}(\bar{\rho}(-\phi(-X)))$ *is translation invariant and the subdifferential of* $\bar{\rho}$ *is always nonempty.*

(ii) ρ *is a convex risk measure and the subdifferential is always nonempty.*

Another Characterization Result

Reconsider the question asked in the Introduction (slide 12). Another answer:

Theorem

Suppose that the probability space is rich. Let ϕ *be a strictly increasing and continuous function satisfying* $0 \in closure(Image(\phi))$, $\phi(\infty) = \infty$ *and* $\phi \in C^3(]\phi^{-1}(0), \infty[)$.

Then the following statements are equivalent:

(i) $\rho(X) = \phi^{-1}(\bar{\rho}(-\phi(-X)))$ *is convex,* $\rho(m) = -m$ *for all* $m \in \mathbb{R}$ *and the subdifferential of* $\bar{\rho}$ *is always nonempty.*

(ii) ρ *is* γ*-entropy convex with* $\gamma \in \mathbb{R}^+$ *or* ρ *is* ∞*-entropy coherent, and the entropy subdifferential is always nonempty.*

4. Duality Results [1]

Recall that if ρ is a convex risk measure then (under additional continuity assumptions) there exists a unique $\alpha : \mathcal{Q} \to \mathbb{R} \cup \{\infty\}$, referred to as the dual conjugate of ρ, such that the following dual representation holds:

$$\rho(X) = \sup_{Q \in \mathcal{Q}} \Big\{ \mathrm{E}_Q [-X] - \alpha(Q) \Big\},$$

with

$$\alpha(Q) = \sup_{X \in L^\infty} \Big\{ \mathrm{E}_Q [-X] - \rho(X) \Big\}.$$

Duality Results [2]

Theorem

Suppose that ρ is γ-entropy convex with penalty function c. Then:

(i) *The dual conjugate of ρ, is given by the largest convex and lower-semicontinuous function α being dominated by* $\inf_{Q\in\mathcal{Q}}\{\gamma H(\bar{P}|Q)+c(Q)\}$.

(ii) *If c is convex and lower-semicontinuous, then α is the largest lower-semicontinuous function being dominated by* $\inf_{Q\in\mathcal{Q}}\{\gamma H(\bar{P}|Q)+c(Q)\}$.

(iii) *If c is convex and lower-semicontinuous, and satisfies additional integrability conditions (see paper), then the conjugate dual*

$$\alpha(\bar{P})=\min_{Q\in\mathcal{Q}}\{\gamma H(\bar{P}|Q)+c(Q)\}.$$

5. Distribution Invariant Representation [1]

- Let

$$\Psi = \{\psi : [0,1] \to [0,1] \\ \quad |\psi \text{ is concave, right-continuous at zero with } \psi(0+) = 0 \text{ and } \psi(1) = 1\}.$$

- For $\psi \in \Psi$ and $X \in L^\infty$ we define $\mathrm{E}_\psi[X] := \int X d\psi(P)$.
- Furthermore, we define

$$e_{\gamma,\psi}(X) := \gamma \log \left(\mathrm{E}_\psi \left[\exp\left\{ \frac{-X}{\gamma} \right\} \right] \right) =: e_{\gamma,\psi(P)}(X).$$

Distribution Invariant Representation [2]

Theorem

Suppose that ρ is γ-entropy convex. Then the following statements are equivalent:

(i) ρ *is distribution invariant.*

(ii) $\rho(X) = \sup_{\psi \in \Psi} \{e_{\gamma,\psi}(X) - (\rho^*)'(\psi)\}$ *with*
$(\rho^*)'(\psi) = \sup_{X \in L^\infty} \{e_{\gamma,\psi}(X) - \rho(X)\}$.

6.1 Risk Sharing

- Suppose that there are two economic agents A and B measuring risk using a general entropy convex measure of risk ρ^A and ρ^B with $\gamma^A, \gamma^B \in \mathbb{R}^+$.
- Let $\bar{\rho} = -\rho$, $\bar{e}_{\gamma,Q} = -e_{\gamma,Q}$ and $\bar{c} = -c$.
- Suppose that A owns a financial payoff X^A and B owns a financial payoff X^B.
- We solve explicitly the problem of optimal risk sharing given by

$$R^{A,B}(X^A, X^B) = \sup_{F \in L^\infty} \{\bar{\rho}^A(X^A - F + \Pi^F) + \bar{\rho}^B(X^B + F - \Pi^F)\}$$
$$= \sup_{\bar{F} \in L^\infty} \{\bar{\rho}^A(X^A + X^B - \bar{F}) + \bar{\rho}^B(\bar{F})\} =: \bar{\rho}^A \Box \bar{\rho}^B(X^A + X^B),$$

where Π^F is the agreed price of the financial derivative (risk transfer) F and where we have set $\bar{F} := F + X^B$.

- In particular, under technical conditions (see paper), the optimal risk sharing is attained in the derivative $F^* = \frac{\gamma^B}{\gamma^A + \gamma^B} X^A - \frac{\gamma^A}{\gamma^A + \gamma^B} X^B$.

6.2 Portfolio Optimization and Indifference Valuation [1]

- Let F be a bounded contingent claim.
- Consider a Brownian-Poisson setting: we assume that the financial market consists of a bond with interest rate zero and $n \leq d$ stocks. The price process of stock i evolves according to

$$\frac{dS_t^i}{S_{t-}^i} = b_t^i dt + \sigma_t^i dW_t + \int_{\mathbb{R}^{d'} \setminus \{0\}} \tilde{\beta}_t^i(x) \tilde{N}_p(dt, dx), \qquad i = 1, \ldots, n,$$

where b^i (σ^i, $\tilde{\beta}^i$) are $\mathbb{R}$ ($\mathbb{R}^d$, $\mathbb{R}$)-valued predictable and uniformly bounded stochastic processes.

Portfolio Optimization and Indifference Valuation [2]

- Using BSDEs, we solve explicitly the following optimization problem:

$$\hat{V}^{\gamma}(x) := \sup_{\pi \in \tilde{\mathcal{A}}} \inf_{Q \in M} -\gamma \log \left(\mathrm{E}_Q \left[\exp \left\{ -\frac{1}{\gamma} \left(x + \int_0^T \pi_t \frac{dS_t}{S_{t-}} + F \right) \right\} \right] \right),$$

  where x is the initial wealth, the process π_t^i describes the amount of money invested in stock i at time t, and M is a set of measures equivalent to P.
- Note the generality: robust, constraints and jumps.

5. Conclusions

- We have introduced two new classes of risk measures: entropy coherent and entropy convex measures of risk.
- We have demonstrated that convex, entropy convex and entropy coherent measures of risk emerge as translation invariant certainty equivalents under variational, homothetic and multiple priors preferences, respectively, and induce linear or exponential utility functions in these paradigms.
- A variety of representation and duality results as well as some applications and examples have made explicit that entropy coherent and entropy convex measures of risk satisfy many appealing properties.
- The theory developed in this paper is of a static nature. In future research we intend to develop its dynamic counterpart.