Lévy processes and the financial crisis: can we design a more effective deposit protection?

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Background and objective of the work

Deposit Guarantee Schemes (DGSs) are financial institutions whose main aim is to provide a safety net for depositors so that, if a credit institution fails, they will be able to recover their bank deposits up to a certain limit.

The choice of the appropriate size of funds DGSs should set aside is a core topic.

OBJECTIVE: develop a procedure to define a target level for the fund.

The approach is applied to a sample of Italian banks.

Outline

- Description of Deposit Guarantee Schemes.
- Methodology to build the loss distribution:
  a. Estimate banks' default probabilities using CDS spreads;
  b. Draw realizations of the asset value process and compute the corresponding default times;
  c. Evaluate the corresponding losses.
- Results.

Outline

- **Description of Deposit Guarantee Schemes.**
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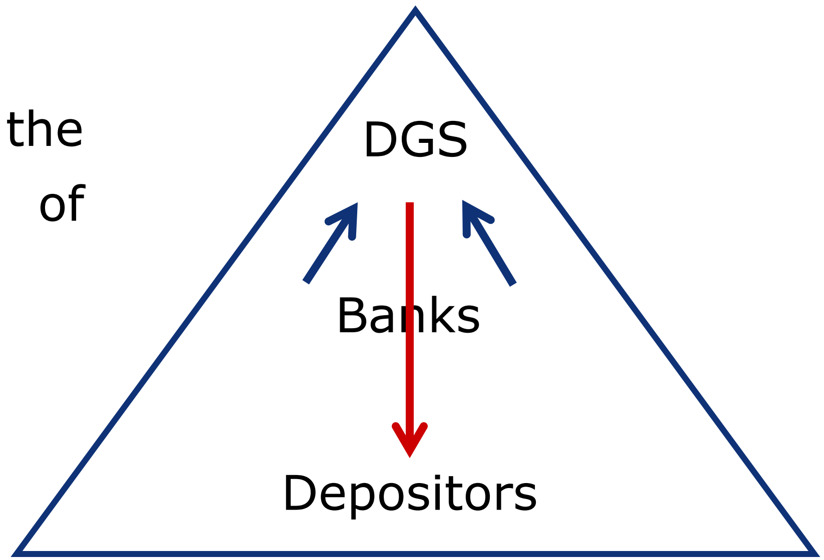
Deposit Guarantee Schemes

HOW DOES A DGS WORK?

Banks pay contributions to DGS to fill up the fund. The DGS employs the fund in case of payout to reimburse depositors.

Important to choose an optimal fund size.



KEY CONCEPTS

- Level of coverage: level of protection granted to deposits in case of failure
- Eligible deposits: deposits entitled to be reimbursed by DGS
- Covered deposits: amount of deposits obtained from eligible when applying the level of coverage

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How to build the loss distribution

The procedure to choose the target level relies upon the loss distribution.

MAIN STEPS:

1. Estimate banks' default probabilities from CDS spreads market data and from financial indicators and calibrate the default intensities of the default time distributions;

2. Draw realizations of the asset value process (firm-value approach);

3. From asset values' draws compute the corresponding default times;

4. Evaluate the corresponding losses.

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Loss distribution: estimate default probabilities (1)

Default intensity parameters λ_i of the default time τ_i distributions. ← Derived from default probabilities, which are estimated from CDS spreads.

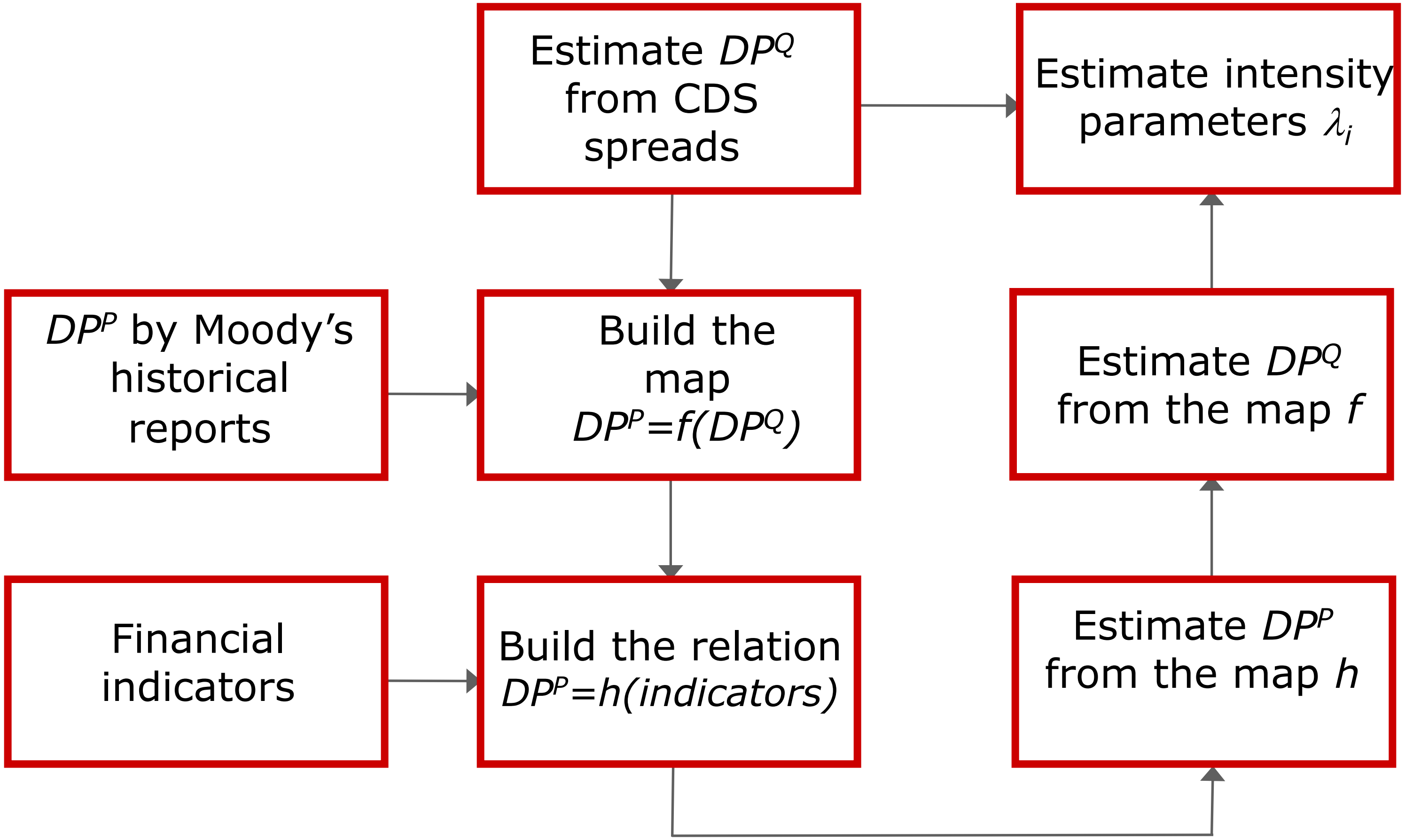
PROBLEM: there is a small sample of banks underlying a CDS contract.

↓

SOLUTION: study a relation between risk indicators and default probability and use this relation to enlarge the sample.

Attention to the difference between risk-neutral and historical default probabilities!

Loss distribution: estimate default probabilities (2)



DP^Q: risk-neutral default probability

DP^P: historical default probability

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Loss distribution: estimate default probabilities.

Step 1 – Credit Default Swaps

At this stage we make use of the 2006 daily 5Y CDS spreads of 40 EU banks.

We assume the default time of the *i*-th bank τ_i to be exponentially distributed with intensity parameter λ_i.

The term structure of the cumulative risk-neutral default probability:

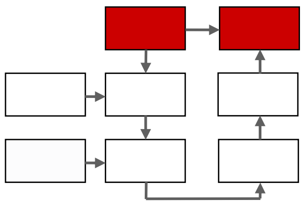
$$p_i(t) = 1 - e^{-\lambda_i t}$$

$$DP_i^Q = 1 - e^{-\lambda_i}$$

CDS spread:

$$c_i = (1 - R_i)\lambda_i$$

Recovery rate R_i=40%



Loss distribution: estimate default probabilities.
Step 2 (a) – Map between PB measures $DP^P = f(DP^Q)$

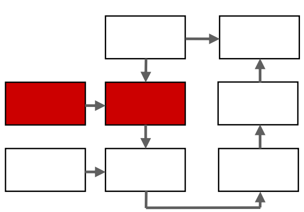
<u>GOAL</u>: build a one-to-one relation between 1-year DP^Q and DP^P. Associate every rating class with a DP^Q and a DP^P.

Rating	Aaa	Aa1	Aa2	Aa3	A1	A2	A3
DP^Q	0.0975%	0.1196%	0.1265%	0.1558%	0.1976%	0.3053%	0.4957%
DP^P	0.0022%	0.0038%	0.0067%	0.0116%	0.0201%	0.0348%	0.0604%

DP^P (historical DP): from statistics on average cumulative default rates.

DP^Q (risk-neutral DP): consider all banks belonging to a common rating class, the DP^Q is the average of all banks' DP^Q_i.

$$DP^Q_{\text{Aaa}} = \frac{1}{n_{\text{Aaa}}} \sum_{i \in \text{Aaa}} DP^Q_i$$



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Loss distribution: estimate default probabilities.
Step 2 (b) – Map between PB measures $DP^P = f(DP^Q)$

Rating	Aaa	Aa1	Aa2	Aa3	A1	A2	A3
DP^Q	0.0975%	0.1196%	0.1265%	0.1558%	0.1976%	0.3053%	0.4957%
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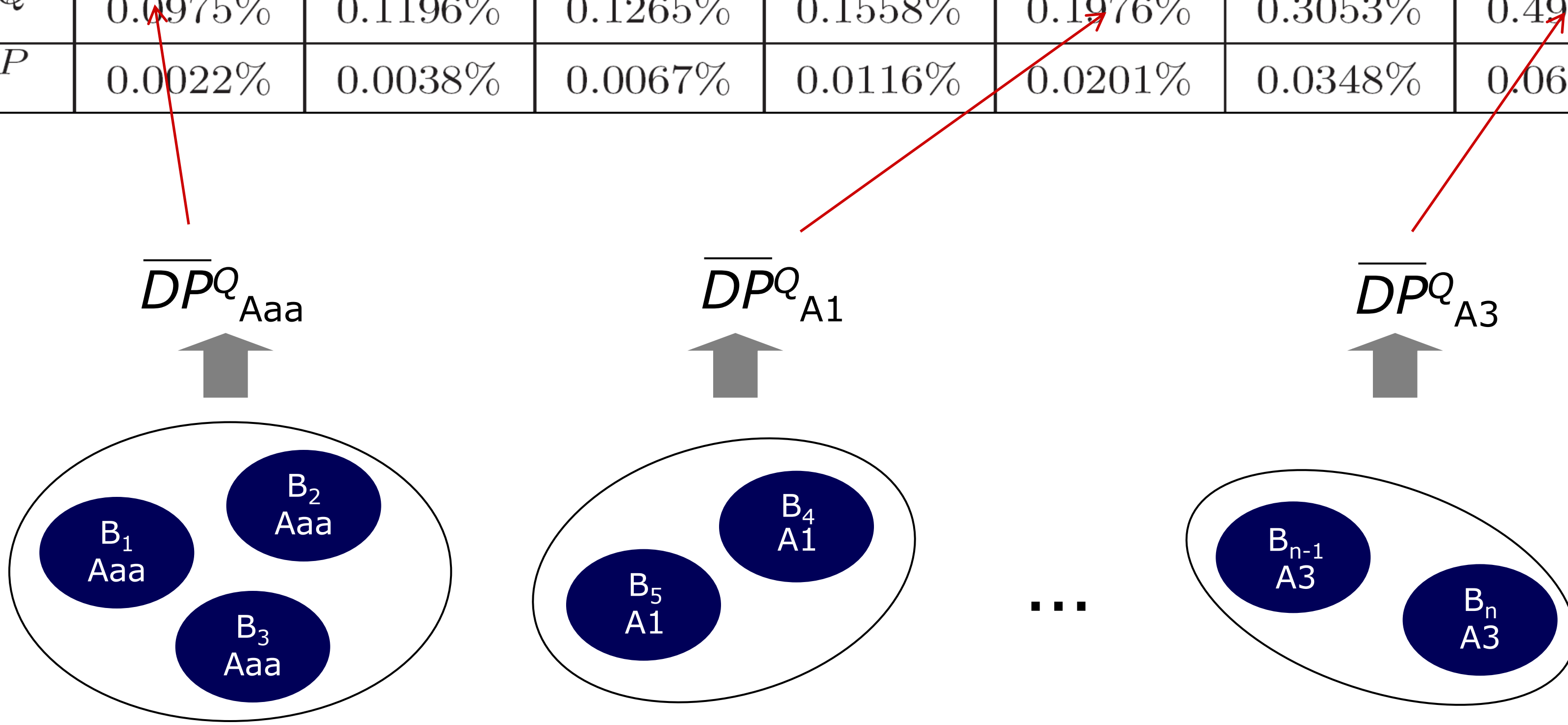
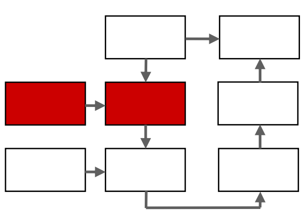
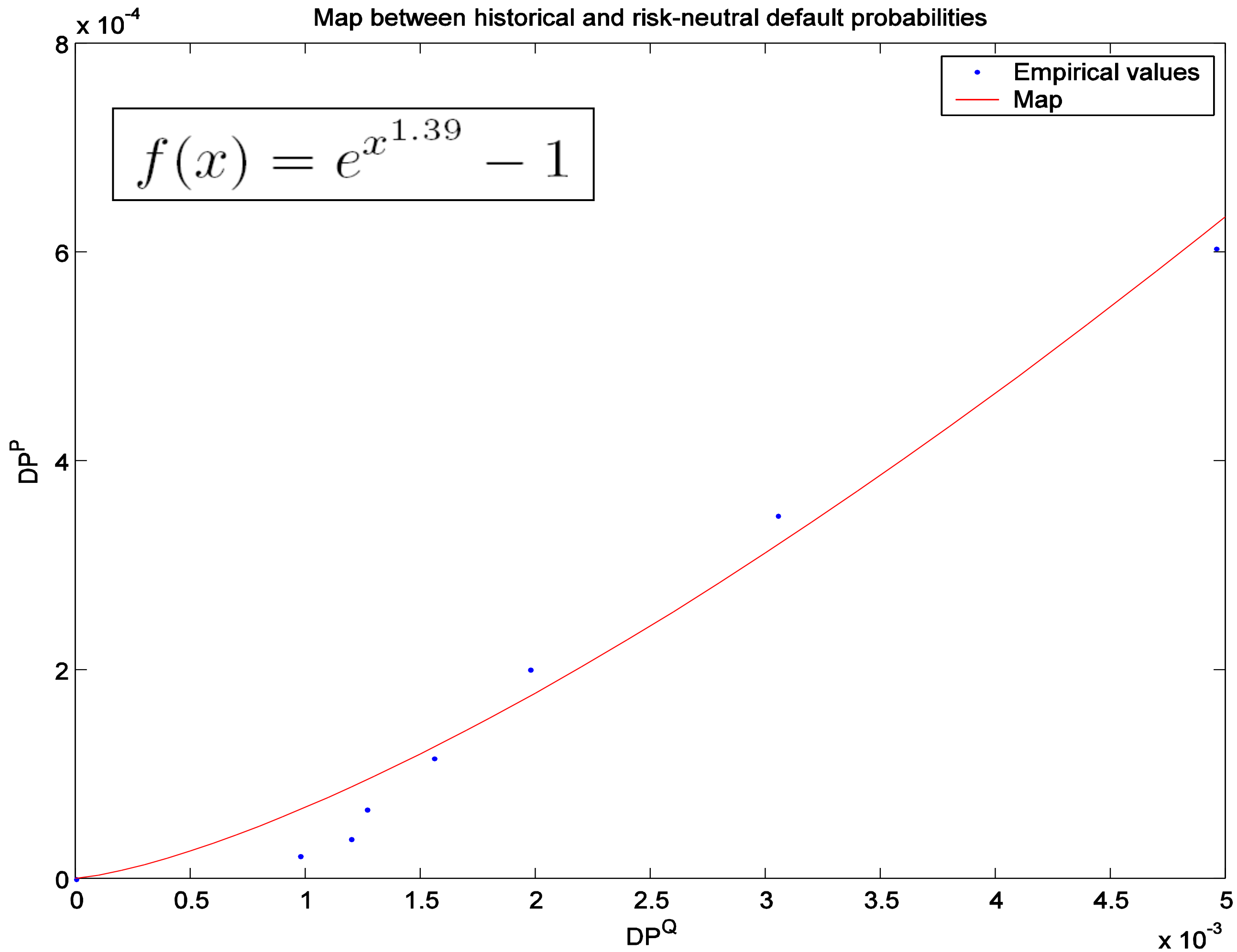
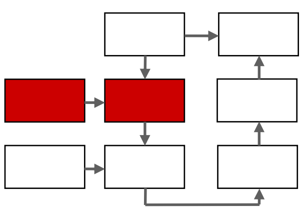


Sample of *n* banks with CDS



Loss distribution: estimate default probabilities.
Step 2 (c) – Map between PB measures $DP^P = f(DP^Q)$



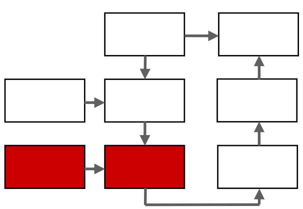


Loss distribution: estimate default probabilities.
Step 3 – Linear model *DP^P=h(financial indicators)*

GOAL: estimate a relationship between historical default probabilities and the financial indicators.

$$DP^P = \mathbb{X}\beta + \epsilon$$

ROAA	Excess capital/RWA
Liquid assets/customer & short term funding	Excess capital/total assets
Net Loans/customer & short term funding	Loan loss provisions/net interest revenue
Cost to income	Loan loss provisions/ operating income



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Loss distribution: estimate default probabilities.

Steps 4 and 5 – Estimate *DP^P* and *DP^Q* for the banks' sample

At this stage we turn to the banks' sample.

Using the relationship *h(financial indicators)*, we estimate the DP^P;

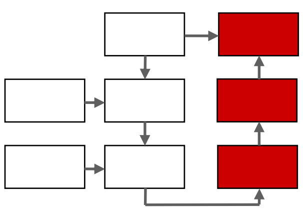
From the DP^P we estimate DP^Q by inverting the map *f.*

We assume the default time of the *i*-th bank τ_i to be exponentially distributed with intensity parameter λ_i.

$$DP_i^Q = 1 - e^{-\lambda_i}$$

The term structure of the cumulative risk-neutral default probability:

$$\lambda_i = -\ln\left(1 - DP_i^Q\right)$$



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Loss distribution: asset-value processes realizations (1)

DEFAULT'S DEFINITION:

a bank goes into default when its asset value falls below a certain threshold.

Asset value: generic one-factor Lévy model (ρ=70%)

$$A_i(t) = X_\rho + X_{1-\rho}^{(i)}$$

- One-factor Gaussian model
- One-factor Shifted Gamma Lévy model

A default occurs if:

$$A_i(t) \leq K_i(t)$$

$$K_i(t) = F_{X_1}^{(-1)}(p_i(t))$$

and thus the default times τ_i are

$$\tau_i = p_i^{(-1)}(F_{X_1}(A_i)) = -\frac{\ln(1 - F_{X_1}(A_i))}{\lambda_i}$$

Loss distribution: asset-value processes realizations (2)

Asset value

One-factor Gaussian model

$$A_i(t) = \sqrt{\rho}Y + \sqrt{1-\rho}X_i$$

The default times τ_i are

$$\tau_i = p_i^{(-1)}\left(\Phi(A_i)\right) = -\frac{\ln(1-\Phi(A_i))}{\lambda_i}$$

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Loss distribution: asset-value processes realizations (3)

Asset value

One-factor Shifted Gamma Lévy model

$$A_i(t) = X_\rho + X_{1-\rho}^{(i)}$$

- $X_\rho = \sqrt{a}\rho - G_\rho$ and $X_{1-\rho}^{(i)} = \sqrt{a}(1-\rho) - G_{1-\rho}$ are independent Shifted Gamma random variables;
- $G = \{G_u, u \geq 0\}$ is a unit-variance Gamma process such that $a > 0 \;\; b = \sqrt{a}$ and $\mathbb{E}[G_1] = \sqrt{a} \;\; var(G_1) = 1$

The default times τ_i are

$$\tau_i = p_i^{(-1)}\left(F_{X_1}(A_i)\right) = -\frac{\ln(F_\Gamma(\sqrt{a} - A_i))}{\lambda_i}$$

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Loss distribution: generating the loss distribution

For every bank, check if the default time τ_i is smaller than 1Y.

If this is the case, there will be a loss attributable to bank *i* equal to:

$$\boxed{L_i = EAD_i * (1 - 40\%)}$$

where EAD_i is the amount of covered deposits by bank *i.*

The total loss hitting the Fund is estimated by aggregating individual bank losses.

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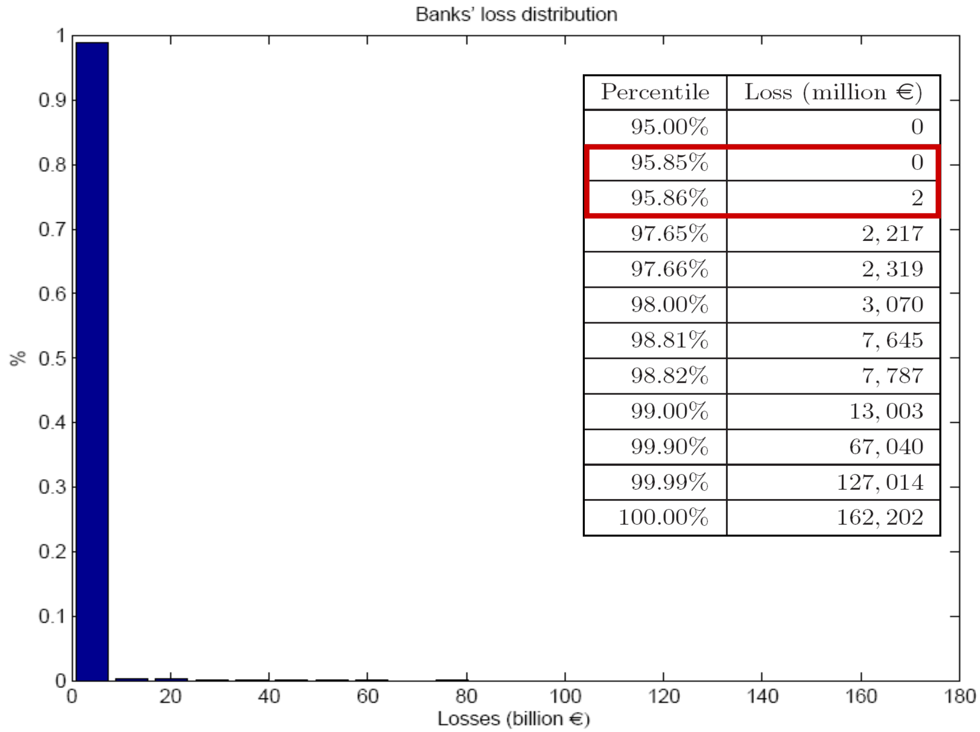
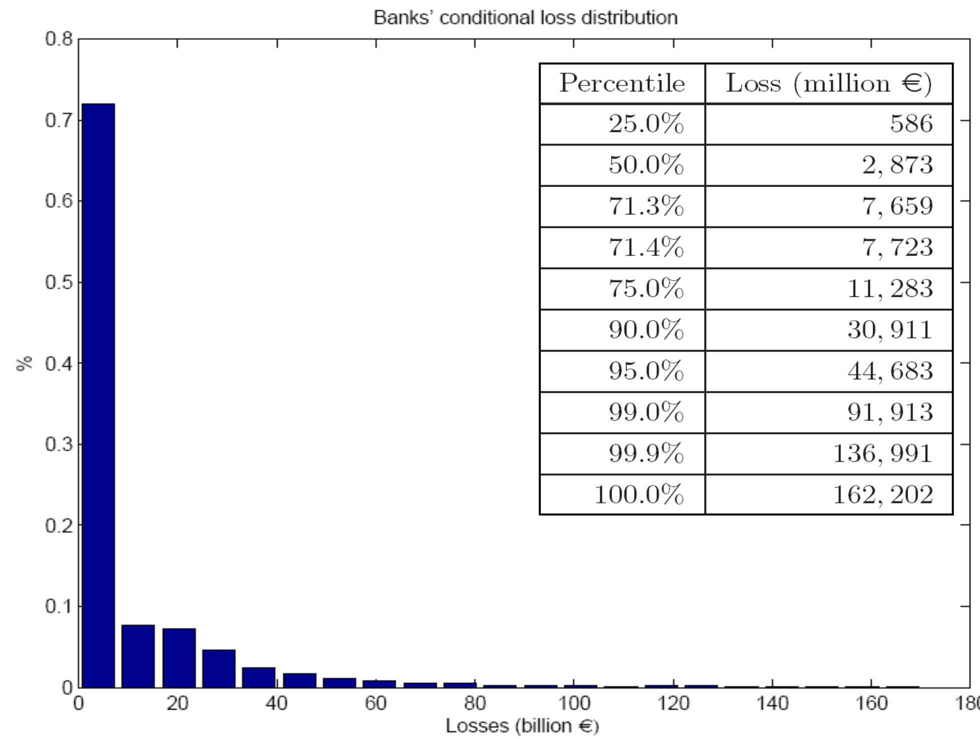
- **Results.**

Results: banks' loss distributions
One-factor Gaussian model



Percentile	Loss (million €)
95.00%	0
95.85%	0
95.86%	2
97.65%	2, 217
97.66%	2, 319
98.00%	3, 070
98.81%	7, 645
98.82%	7, 787
99.00%	13, 003
99.90%	67, 040
99.99%	127, 014
100.00%	162, 202

Probability that at least one bank defaults:4.15%



Percentile	Loss (million €)
25.0%	586
50.0%	2, 873
71.3%	7, 659
71.4%	7, 723
75.0%	11, 283
90.0%	30, 911
95.0%	44, 683
99.0%	91, 913
99.9%	136, 991
100.0%	162, 202

<u>Sample</u>: 51 IT banks, accounting for 60% of IT eligible deposits and for 43% of total assets as of 2006.

<u>Monte Carlo iterations</u>: 100 000 runs.

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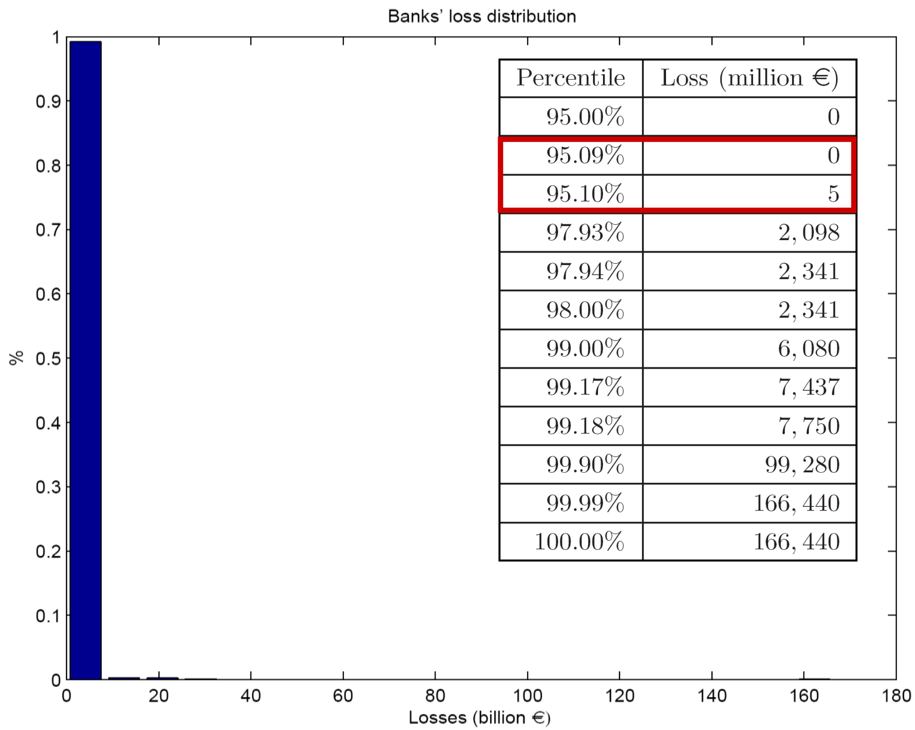
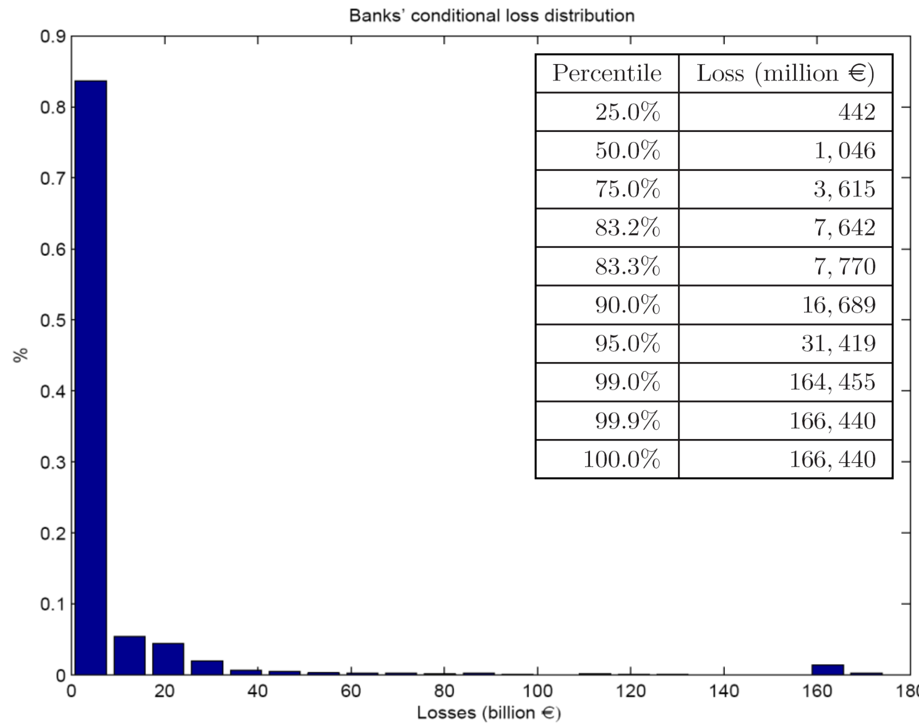
Results: banks' loss distributions

One-factor Shifted Gamma Lévy model



Percentile	Loss (million €)
95.00%	0
95.09%	0
95.10%	5
97.93%	2,098
97.94%	2,341
98.00%	2,341
99.00%	6,080
99.17%	7,437
99.18%	7,750
99.90%	99,280
99.99%	166,440
100.00%	166,440

Probability that at least one bank defaults: 4.91%



Percentile	Loss (million €)
25.0%	442
50.0%	1,046
75.0%	3,615
83.2%	7,642
83.3%	7,770
90.0%	16,689
95.0%	31,419
99.0%	164,455
99.9%	166,440
100.0%	166,440

<u>Sample</u>: 51 IT banks, accounting for 60% of IT eligible deposits and for 43% of total assets as of 2006.

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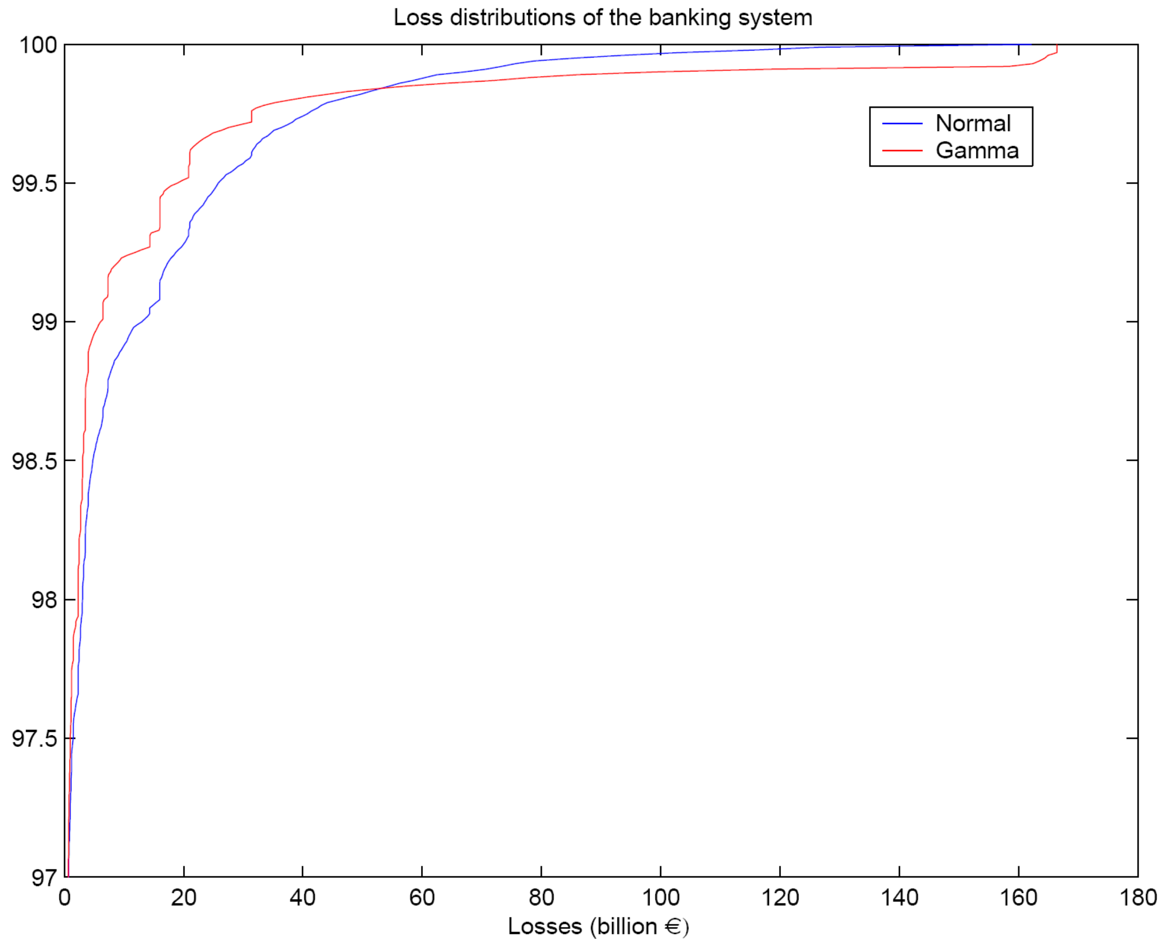
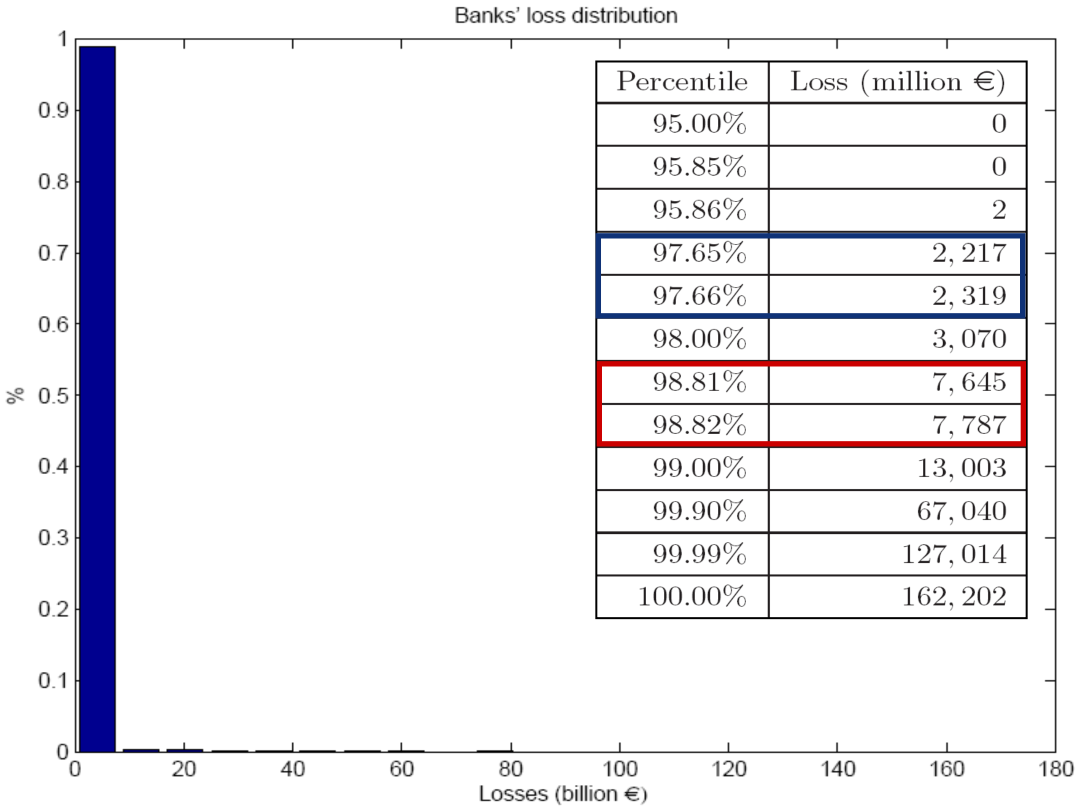
Results: banks' loss distributions
Comparisons



Results: DGS loss distribution
One-factor Gaussian model



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98.82%	7,787
99.00%	13,003
99.90%	67,040
99.99%	127,014
100.00%	162,202

DGS Proposal: target fund 2% of eligible deposits = € 7.7 billion.

Default probability = 1.2%

IT DGS virtual fund: 0.8% of covered deposits = € 2.22 billion

Default probability = 2.4%

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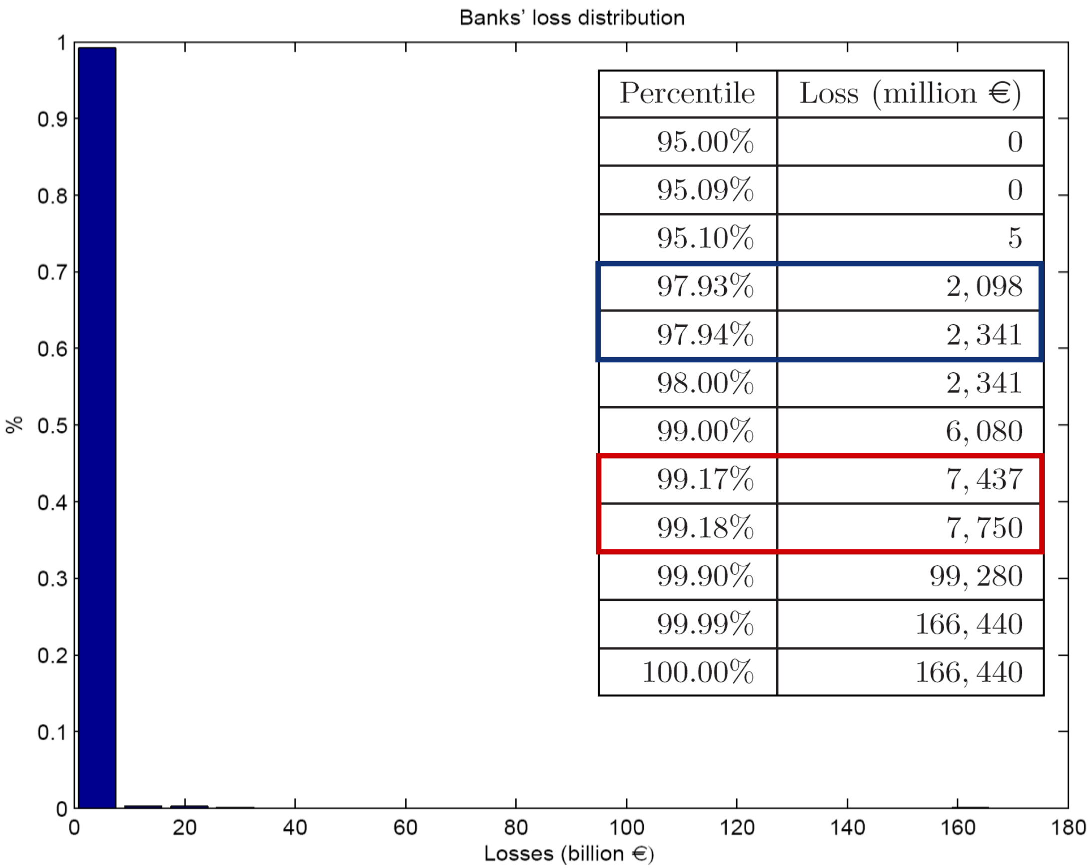
Results: DGS loss distribution
One-factor Shifted Gamma Lévy model



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99.99%	166,440
100.00%	166,440

DGS Proposal: target fund 2% of eligible deposits = € 7.7 billion.

Default probability = 0.83%

IT DGS virtual fund: 0.8% of covered deposits = € 2.22 billion

Default probability = 2.07%

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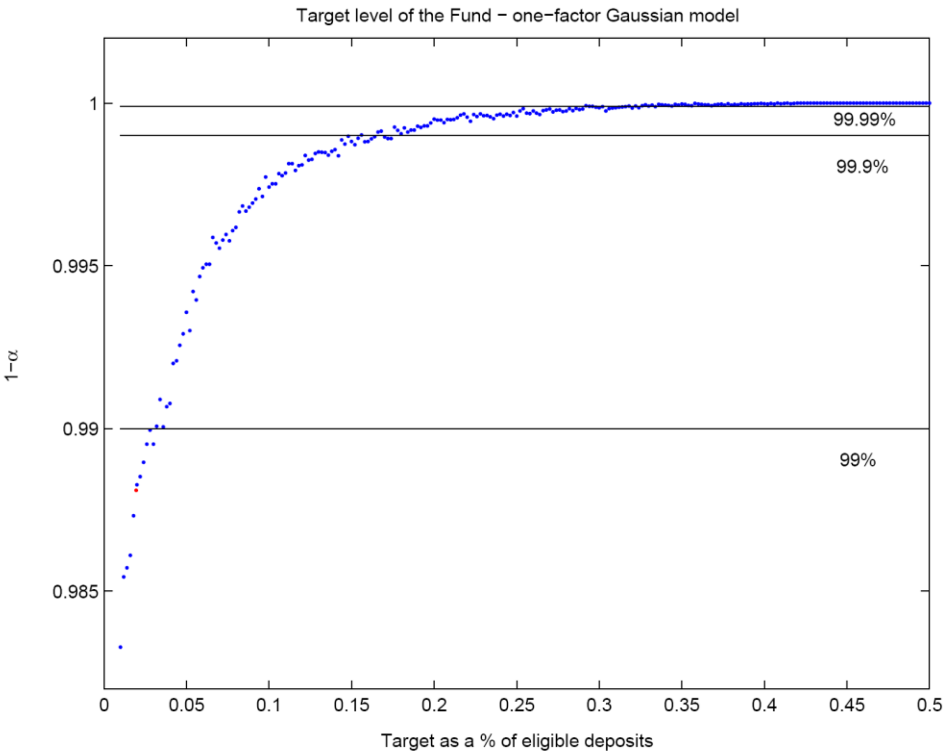
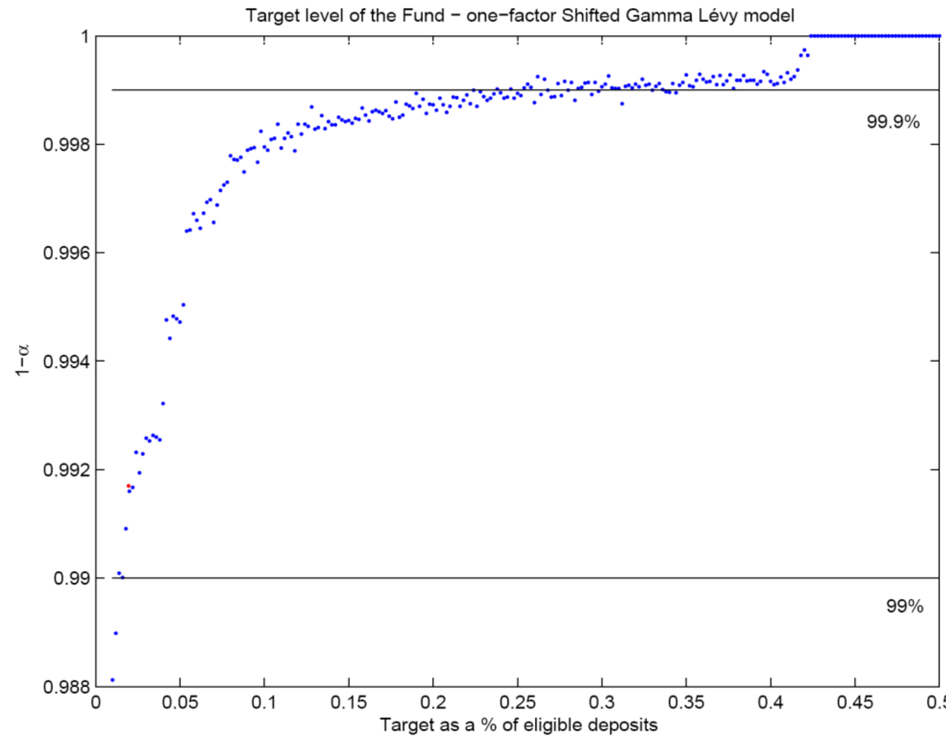
Optimum target size

The simulation procedure can be used to choose the optimal target level such that it can cover up to a desired percentage of losses.





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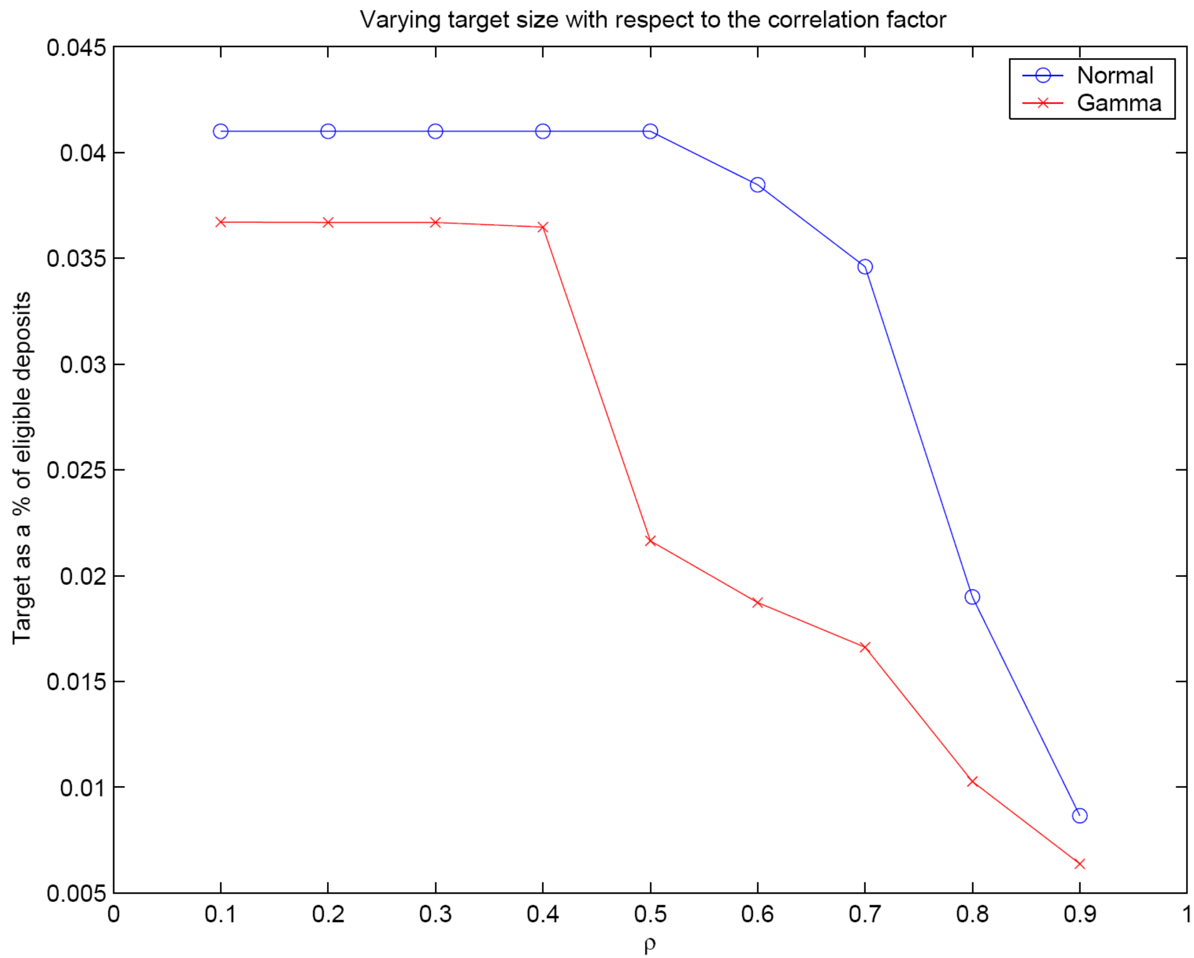
Sensitivity to results



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