Dissecting and Deciphering European Option Prices using Closed-Form Series Expansion

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Aug 30, 2011

Background

- Continuous-time diffusion models are developed to capture the dynamics of assets:

$$dX_t = \mu(t, X_t)dt + \sigma(t, X_t)dW_t + J_t dN_t$$

- A European call option is one of the first derivatives that are priced in closed-form within this framework. [Black and Scholes (1973)]
- This paper systematically develops a new option pricing method.

Review of Prior Work on Option Pricing Methods

- Closed-Form Pricing Formulas
  - Log-Normal Class: Black-Scholes-Merton
    [Black and Scholes (1973), Merton (1976), Black (1976)]
  - Bessel Process Class: CIR, CEV
    [Cox (1975), Cox et al. (1976, 1985), Goldenberg (1991)]
  - Fourier Transform: Levy Process, Heston Model, Affine Model
    [Heston(1993), Bakshi and Madan(1999), Bates(1996), Scott(1997), Carr and Madan(1998), Duffie, Singleton and Pan(2000)]
- Numerical Methods
  - Monte Carlo Simulations
    [Boyle(1977)]
  - Numerical Solutions to PDE
    [Schwartz(1977)]

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- Closed-Form Expansions - This Paper
- Numerical Methods
  - Monte Carlo Simulations
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Review of Prior Work on Closed-Form Expansions

1. Density or Likelihood Expansion
   - Diffusion, Multivariate Jump Diffusion, Inhomogeneous
     [Aït-Sahalia (1999, 2002, 2008), Yu (2007), Egorov et al. (2003)]
   - Related Works and Applications
     [Jensen and Poulsen (2002), Hurn et al. (2007), Stramer and Yan (2007), Bakshi et al. (2006), Aït-Sahalia and Kimmel (2007, 2009), Bakshi and Ju (2005), Kimmel et al. (2007)]
2. Expansion for Bond Prices
   - Analytical Series [Kimmel (2009, 2010)]
3. Asymptotic Expansion of Option Prices
   - Fail to converge
   - Inappropriate for statistical inference
4. Option Price Expansion around Black-Scholes
   [Kristensen and Mele (2010)]

Why This Approach?

- Independent of special model structure
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  - Explain how parameters are translated into option prices
  - Relative importance of each component
  - Model comparison

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  - Done once and for all
  - Two or three terms are enough
  - Greeks, comparative statics, etc
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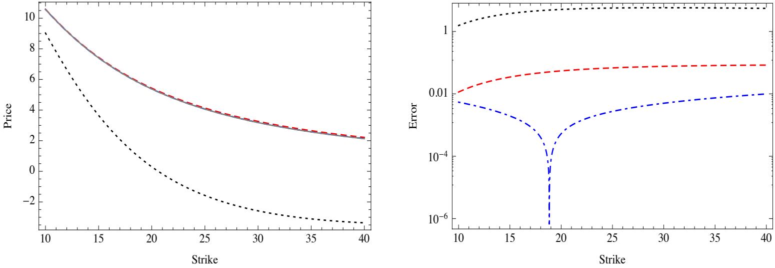
What can be Obtained

CEV Model: $dX_t = (r - \delta)X_t dt + \sigma X_t^{\gamma} dW_t^Q$



Note: The black dotted line, red dashed line and blue dotted-dash line illustrate the $O(\Delta^{1/2})$, $O(\Delta^{3/2})$ and $O(\Delta^{5/2})$ order approximations respectively. The grey line denotes the true prices. Y-axis of the right panel is on a logarithmic scale. The parameters are: $\sigma = 0.2, r = 4\%, \delta = 0.01, x = 20, \Delta = 1$, and $\gamma = 1.4$.

Behind the Screen

CEV Model Expansion

Closed form expansion coefficients for a vanilla call option price:

$$\Psi(\Delta,x) = \Phi\left(\frac{C^{(-1)}(x)}{\sqrt{\Delta}}\right)\sum_{k=0}^{\infty} B^{(k)}(x)\Delta^k + \sqrt{\Delta}\phi\left(\frac{C^{(-1)}(x)}{\sqrt{\Delta}}\right)\sum_{k=0}^{\infty} C^{(k)}(x)\Delta^k$$

$$B^{(k)}(x) = \frac{(-1)^k}{k!}(x\delta^k - Kr^k),\ k \geq 0$$

$$C^{(-1)}(x) = -\frac{K^{1-\gamma} - x^{1-\gamma}}{\sigma - \gamma\sigma}$$

$$C^{(0)}(x) = \frac{K^{\gamma}(K-x)x^{\gamma}(-1+\gamma)\sigma}{K^{\gamma}x - Kx^{\gamma}},\ \text{if}\ x \neq K;\ \text{or}\ K^{\gamma}\sigma,\ \text{if}\ x = K.$$

$$C^{(1)}(x) = \frac{(Kx)^{\gamma}(-1+\gamma)\sigma}{(-K^{\gamma}x + Kx^{\gamma})^3}\Bigg(K^{1+2\gamma}rx^2 + K^3rx^{2\gamma} - K^{2\gamma}x^3\delta - K^2x(2r(Kx)^{\gamma} + x^{2\gamma}\delta)$$
$$+e^{\frac{(Kx)^{-2\gamma}(K^{2\gamma}x^2 - K^2x^{2\gamma})(r-\delta)}{2(-1+\gamma)\sigma^2}} K^{1+\frac{3\gamma}{2}}x^{5\gamma/2}(-1+\gamma)\sigma^2 - e^{\frac{(Kx)^{-2\gamma}(K^{2\gamma}x^2 - K^2x^{2\gamma})(r-\delta)}{2(-1+\gamma)\sigma^2}}$$
$$K^{5\gamma/2}x^{1+\frac{3\gamma}{2}}(-1+\gamma)\sigma^2$$
$$-x(Kx)^{2\gamma}(-1+\gamma)^2\sigma^2 + K(Kx)^{\gamma}(2x^2\delta + (Kx)^{\gamma}(-1+\gamma)^2\sigma^2)\Bigg),\ \text{if}\ x \neq K;\ \text{or}$$
$$\frac{K^{-2-\gamma}}{24\sigma}\left(12K^4(r-\delta)^2 - 12K^{2+2\gamma}(r+\delta)\sigma^2 + K^{4\gamma}(-2+\gamma)\gamma\sigma^4\right),\ \text{if}\ x = K.$$

Derivative Pricing 101

- Consider a derivative that pays $f(X_T)$ at maturity T:
  - Its price $\Psi(\Delta, x; \theta)$ satisfies the Feymann-Kac PDE:

$$(-\frac{\partial}{\partial \Delta} + \mathcal{L} - r)\Psi(\Delta, x; \theta) = 0$$
$$\text{with} \quad \Psi(0, x; \theta) = f(x)$$

  where the operator is defined as

$$\mathcal{L} = \mu(x;\theta)\frac{\partial}{\partial x} + \frac{1}{2}\sigma(x;\theta)^2\frac{\partial}{\partial x^2}$$

  - Its price also has the Feymann-Kac representation:

$$\Psi(\Delta, x; \theta) = e^{-r\Delta}E^Q(f(X_T)|X_t = x; \theta)$$
$$= e^{-r\Delta}\int f(s)p_X(s|x, \Delta; \theta)ds$$

How to Expand Option Prices?

- Bottom-Up Approach - Hermite Polynomials
  - Construct the expansion of transition density.
  - Calculate the conditional expectation.
- Top-Down Approach - Lucky Guess
  - Postulate an expansion of the option price.
  - Plug it into the pricing PDE and verify.

Closed-Form Expansion of Options

Bottom-Up Approach

- Expansion Strategies:

1. Variable Transformations from $X \overset{\gamma}{\to} Y \to Z$, such that Z is sufficiently "close to" normal.
2. Expand the density of Z around normal using Hermite Polynomials $\{H_j\}$.
3. Calculate conditional expectation.

Details

- For simplicity: do binary option with payoff $f(x) = 1_{\{x>K\}}$.
- Equivalent to expanding the cumulative distribution function.

Closed-Form Expansion of Binary Options

Bottom-Up Approach

- **Theorem:** There exists $\bar{\Delta} > 0$ (could be ∞), such that for every $\Delta \in (0, \bar{\Delta})$, the following sequence

$$\Psi^{(J)}(\Delta, x) = e^{-r\Delta}\Big(\Phi\big(\frac{\gamma(x)-\gamma(K)}{\sqrt{\Delta}}\big) + \phi\big(\frac{\gamma(x)-\gamma(K)}{\sqrt{\Delta}}\big)\sum_{j=0}^{J}\eta_{j+1}(\Delta, \gamma(x)) H_j\big(\frac{\gamma(x)-\gamma(K)}{\sqrt{\Delta}}\big)\Big) \longrightarrow \Psi(\Delta, x)$$

  uniformly in x over any compact set in D_X, where $\Psi(\Delta, x)$ solves the Feymann-Kac equation with initial condition $\Psi(0, x) = 1_{\{x>K\}}$ for any $K > 0$. ▸ Details
- **Caveat:** General case is doable but cumbersome! - Use Top-down approach.

Closed-Form Expansion of Options

Top-Down Approach

- Postulate the right form and plug it into the equation.
  - How about this?

$$\Psi(\Delta, x) = \sum_{k=0}^{\infty} f_k(x)\Delta^k$$

  - $f_0(x)$ is non-smooth, e.g. $1_{\{x>K\}}$, ...does not work.
  - Alternative forms?

$$\Psi(\Delta, x) = h(\Delta, x) + g(\Delta, x) \sum_{k=0}^{\infty} f_k(x)\Delta^k$$

  - $h(\Delta, x) \equiv 0$, $g(\Delta, x) \to 1_{\{x>K\}}$, as $\Delta \to 0$? Or
  - $h(\Delta, x) \to 1_{\{x>K\}}$, $g(\Delta, x) \to 0$, as $\Delta \to 0$?
- How to make a lucky guess?

Closed-Form Expansion of Options

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  - $h(\Delta, x) \to 1_{\{x>K\}}$, $g(\Delta, x) \to 0$, as $\Delta \to 0$?
- How to make a lucky guess? - You know it when you see it.

Closed-Form Expansion of Binary Options

Top-Down Approach

- **Postulate:**

$$\Psi(\Delta, x) = e^{-r\Delta}\Big(\Phi(\frac{C^{(-1)}(x)}{\sqrt{\Delta}}) + \sqrt{\Delta}\phi(\frac{C^{(-2)}(x)}{\sqrt{\Delta}})\sum_{j=0}^{\infty} C^{(k)}(x)\Delta^k\Big)$$

- **Verify:**

$$C^{(-1)}(x) = \int_K^x \frac{1}{\sigma(s)}ds, \qquad C^{(-2)}(x) = \frac{1}{2}\Big(\int_K^x \frac{1}{\sigma(s)}ds\Big)^2$$

For $k \geq -1$,

$$C^{(k+1)}(x)\Big(\frac{1}{2} + (k+1) + \mathcal{L}C^{(-2)}(x)\Big) + \sigma^2(x)\frac{dC^{(k+1)}(x)}{dx}\frac{dC^{(-2)}(x)}{dx} = \mathcal{L}C^{(k)}(x)$$

- The two approaches agree with each other.

Extensions

Jump Diffusion Models

- Jump Diffusion Models

$$dX_t = \mu(t, X_t)dt + \sigma(t, X_t)dW_t^Q + J_t dN_t$$

where jumps are of finite activity with intensity $\lambda(x;\theta)$ and jump size density $\nu(z;\theta)$.

- The PDE becomes:

$$0 = -\frac{\partial \Psi(\Delta, x)}{\partial \Delta} + \mu(x)\frac{\partial \Psi(\Delta, x)}{\partial x} + \frac{1}{2}\sigma^2(x)\frac{\partial^2 \Psi(\Delta, x)}{\partial x^2} - r(x)\Psi(\Delta, x) + \lambda(x)\int_{-\infty}^{\infty}\Big(\Psi(\Delta, x+z) - \Psi(\Delta, x)\Big)\nu(x, z)dz$$

with initial condition:

$$\Psi(0, x) = f(x)$$

Postulate the Expansion

Jump Diffusion Models

- By Bayes' Rule, we have

$$p(y|x,\Delta;\theta) = \sum_{k=0}^{\infty} p(y|x, N_\Delta = k;\theta) \cdot p(N_\Delta = k|x;\theta)$$

- Also, Poisson process indicates

$$p(N_\Delta = 0|x;\theta) = O(1)$$
$$p(N_\Delta = 1|x;\theta) = O(\Delta)$$
$$p(N_\Delta \geq 2|x;\theta) = o(\Delta)$$

- Postulate the following form:

$$\Psi(\Delta, x) = \Phi\left(\frac{C^{(-1)}(x)}{\sqrt{\Delta}}\right) \sum_{k=0}^{\infty} B^{(k)}(x)\Delta^k + \Delta^{\frac{1}{2}} \phi\left(\frac{C^{(-1)}(x)}{\sqrt{\Delta}}\right) \sum_{k=0}^{\infty} C^{(k)}(x)\Delta^k + \sum_{k=1}^{\infty} D^{(k)}(x)\Delta^k$$

Coefficients

Implications

Jump Diffusion Models

- **Remark:** for any vanilla call option under jump diffusion models, the option price can be expanded as

$$\Psi(\Delta, x) = \Phi\left(\Delta^{-\frac{1}{2}} \int_K^x \frac{1}{\sigma(s)} ds\right)\left((x-K) + B^{(1)}(x)\Delta\right) + D^{(1)}(x)\Delta + (x-K)\left(\int_K^x \frac{1}{\sigma(s)} ds\right)^{-1} \phi\left(\Delta^{-\frac{1}{2}} \int_K^x \frac{1}{\sigma(s)} ds\right)\Delta^{\frac{1}{2}} + O(\Delta^{\frac{3}{2}})$$

- Volatility determines the leading terms, followed by jumps and drift part which affect the first order terms.
- Possible to separate price contributions made by each part.

Summary of Models

with Brownian Leading Terms

- Depends on the Model
  - 1-D Diffusion Models
  - 1-D Jump Diffusion Models (Finite Activity Only)
  - Time-inhomogeneous Models
  - Certain Multivariate Models (No Stochastic Volatility)
- and Payoff Structure
  - No Path Dependent
  - No American Option

The Influence of Stochastic Interest Rate

Stock: CEV + Interest Rate: CIR

- How does stochastic interest rate affect option prices?

$$\begin{aligned} dX_t &= r_t X_t dt + \sigma X_t^{3/2} dW_t^Q, \quad E(dW_t^Q dB_t^Q) = 0 \\ dr_t &= \beta(\alpha - r_t) + \kappa \sqrt{r_t} dB_t^Q \quad \text{v.s.} \quad r_t = \alpha \end{aligned}$$

The Influence of Stochastic Interest Rate

Stock: CEV + Interest Rate: CIR

- How does stochastic interest rate affect option prices? $O(\Delta^{5/2})$

$$
\begin{aligned}
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\end{aligned}
$$

$$
\begin{aligned}
\Psi(\Delta, x, r) &= \Phi\left(\frac{C^{(-1)}(x,r)}{\sqrt{\Delta}}\right)\sum_{k=0}^{\infty} B^{(k)}(x,r)\Delta^k + \sqrt{\Delta}\phi\left(\frac{C^{(-1)}(x,r)}{\sqrt{\Delta}}\right)\sum_{k=0}^{\infty} C^{(k)}(x,r)\Delta^k \\
B^{(0)}(x,r) &= x - K, \quad B^{(1)}(x,r) = Kr \\
B^{(2)}(x,r) &= \frac{-K\left(r^2 + r\beta - \alpha\beta\right)}{2} \\
C^{(-1)}(x,r) &= \frac{1}{\sigma}\left(\frac{2}{\sqrt{K}} - \frac{2}{\sqrt{x}}\right) \\
C^{(0)}(x,r) &= \frac{1}{2}\left(K\sqrt{x}\sigma + \sqrt{K}x\sigma\right) \\
C^{(1)}(x,r) &= -\frac{1}{8\left(\sqrt{K}-\sqrt{x}\right)^2}\left(-2e^{\frac{r(K-x)}{Kx\sigma^2}}K^{7/4}x^{7/4}\sigma^3 + K^{3/2}x\sigma\left(-4r + x\sigma^2\right)\right. \\
&\quad \left. + K^2\sqrt{x}\sigma\left(4r + x\sigma^2\right)\right)
\end{aligned}
$$

The Effect of Mean-Reversion - SQR Model

- How does mean reversion affect option prices?

$$dV_t = \beta(\alpha - V_t)dt + \sigma V_t^{1/2} dW_t^Q$$

- Consider a binary option with payoff $1_{\{v>K\}}$:

$$\Psi_1^{(1)}(\Delta, v) = \Phi\Big(\frac{2(\sqrt{v} - \sqrt{K})}{\sigma\sqrt{\Delta}}\Big) + \sqrt{\Delta}\phi\Big(\frac{2(\sqrt{v} - \sqrt{K})}{\sigma\sqrt{\Delta}}\Big) C^{(0)}(v)$$

where

$$C^{(0)}(v) = \frac{\left(-1 + e^{\frac{(-K+v)\beta}{\sigma^2}} K^{-\frac{1}{4}+\frac{\alpha\beta}{\sigma^2}} v^{\frac{1}{4}-\frac{\alpha\beta}{\sigma^2}}\right)\sigma}{2\left(\sqrt{K} - \sqrt{v}\right)}$$

- The dominating $O(1)$ term reflects the effect of moneyness.
- The $O(\sqrt{\Delta})$ term measures 1st order mean reversion effect.
- Indistinguishable from DMR model.

$$d\alpha_t = \gamma(\alpha_0 - \alpha_t)dt + \kappa\sqrt{\alpha_t} dB_t^Q$$

The Impact of Jumps - Gaussian Jumps

- Benchmark Merton's Jump

$$\frac{dX_t}{X_t} = (r - (m-1)\lambda)dt + \sigma dW_t^Q + (e^J - 1)dN_t$$

- Similarly, we have

$$\Psi(\Delta, x) = \Phi\left(\frac{C^{(-1)}(x)}{\sqrt{\Delta}}\right)\sum_{k=0}^{\infty} B^{(k)}(x)\Delta^k + \sqrt{\Delta}\phi\left(\frac{C^{(-1)}(x)}{\sqrt{\Delta}}\right)\sum_{k=0}^{\infty} C^{(k)}(x)\Delta^k + \sum_{k=1}^{\infty} D^{(k)}(x)\Delta^k$$

- First order contribution by jumps: $O(\Delta)$.

$$\underbrace{mx\lambda\left(\Phi\left(\frac{\log(\frac{x}{K}) + \log(m) + \frac{1}{2}\nu^2}{\nu}\right) - \Phi\left(\frac{\log(\frac{x}{K})}{\sigma\sqrt{\Delta}}\right)\right)}_{\text{asset-or-nothing portion}}$$

$$- K\underbrace{\lambda\left(\Phi\left(\frac{\log(\frac{x}{K}) + \log(m) - \frac{\nu^2}{2}}{\nu}\right) - \Phi\left(\frac{\log(\frac{x}{K})}{\sigma\sqrt{\Delta}}\right)\right)}_{\text{cash-or-nothing portion}} \geq 0$$

The Impact of Jumps - Asymmetric Double Exponential Jumps

- Kou's Jump Diffusion

$$d\log(X_t) = \mu dt + \sigma dW_t^Q + JdN_t$$

where the jump has double exponential distribution:

$$\nu(z) = p \cdot \eta_1 e^{-\eta_1 z} 1_{\{z \geq 0\}} + q \cdot \eta_2 e^{\eta_2 z} 1_{\{z<0\}}$$

- Similarly, we have

$$\Psi(\Delta, x) = \Phi\left(\frac{C^{(-1)}(x)}{\sqrt{\Delta}}\right) \sum_{k=0}^{\infty} B^{(k)}(x)\Delta^k + \sqrt{\Delta}\phi\left(\frac{C^{(-1)}(x)}{\sqrt{\Delta}}\right) \sum_{k=0}^{\infty} C^{(k)}(x)\Delta^k$$
$$+ \left(1 - \Phi\left(\frac{C^{(-1)}(x)}{\sqrt{\Delta}}\right)\right) \sum_{k=1}^{\infty} D^{(k)}(x)\Delta^k$$

- First order contribution by jumps: $O(\Delta)$.

$$\lambda K\left(\frac{q}{1+\eta_2}\left(\frac{K}{x}\right)^{\eta_2} \Phi\left(\frac{\log(\frac{x}{K})}{\sigma\sqrt{\Delta}}\right) + \frac{p}{-1+\eta_1}\left(\frac{x}{K}\right)^{\eta_1}\left(1 - \Phi\left(\frac{\log(\frac{x}{K})}{\sigma\sqrt{\Delta}}\right)\right)\right) \geq 0$$

The Impact of Jumps - Self-Exciting Jumps

- Hawkes' Jump Diffusion

$$\begin{aligned} d\log X_t &= \mu dt + \sigma dW_t^Q + JdN_t \\ d\lambda_t &= \alpha(\lambda_\infty - \lambda_t)dt + \beta dN_t \end{aligned}$$

- The PDE is

$$-\frac{\partial\Psi(\Delta,x,\lambda)}{\partial\Delta} + (r-(m-1)\bar{\lambda})x\frac{\partial\Psi(\Delta,x,\lambda)}{\partial x} + \frac{1}{2}\sigma^2x^2\frac{\partial^2\Psi(\Delta,x,\lambda)}{\partial x^2} - r\Psi(\Delta,x,\lambda) + \alpha(\lambda_\infty-\lambda)\frac{\partial\Psi(\Delta,x,\lambda)}{\partial\lambda} + \lambda\int_{-\infty}^{\infty}\Big(\Psi(\Delta,xe^z,\beta+\lambda) - \Psi(\Delta,x,\lambda)\Big)\nu(z)dz = 0$$

- Again, we have

$$\begin{aligned} \Psi(\Delta,x,\lambda) =& \Phi\left(\frac{C^{(-1)}(x,\lambda)}{\sqrt{\Delta}}\right)\sum_{k=0}^{\infty}B^{(k)}(x,\lambda)\Delta^k + \sum_{k=1}^{\infty}D^{(k)}(x,\lambda)\Delta^k \\ &+ \sqrt{\Delta}\phi\left(\frac{C^{(-1)}(x,\lambda)}{\sqrt{\Delta}}\right)\sum_{k=0}^{\infty}C^{(k)}(x,\lambda)\Delta^k \end{aligned}$$

The Impact of Jumps - Self-Exciting Jumps

The Role of β - Contagion Parameter

1. Will self-exciting jumps replace Brownian to become the leading term? i.e. $O(1)$? - No.

2. Will β come into play once the first jump occurs? i.e. $O(\Delta^2)$? -No.

- β appears on the order of $O(\Delta)$.
- $\mu = r - \frac{1}{2}\sigma^2 - (m-1)\frac{\alpha}{\alpha-\beta}\lambda_\infty$

Concluding Remarks

This paper proposes a series expansion, which

- Enlarges the class of models that have closed-form formulas
- Translates mode structure into option prices
- Offers insight on how model parameters affect option prices

Future work includes cases with

- Stochastic Volatility
- Infinite Activity Jumps