Tenor Specific Pricing

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Motivation

- Observing that pure discount curves are now based on a variety of tenors giving rise to tenor specific zero coupon bond prices, the question is raised on how to construct tenor specific prices for all financial contracts.

- Noting that in conic finance one has the law of two prices, bid and ask, that are nonlinear functions of the random variables being priced, dynamically consistent sequences of such prices are related to the theory of nonlinear expectations.

- The latter theory is closely connected to solutions of backward stochastic difference equations.

- The drivers for these stochastic difference equations are modeled using concave distortions implementing risk charges for local tenor specific risks.

- It is observed that tenor specific prices given by the mid quotes of bid and ask converge to the risk neutral price as the tenor is decreased and liquidity increased.

- The greater liquidity of lower tenors may lead to an increase or decrease in prices depending on whether the lower liquidity of a higher tenor has a mid quote above or below the risk neutral value.

- Generally for contracts with a large upside and a bounded downside the prices fall with liquidity while the opposite is the case for contracts subject to a large downside and a bounded upside.

Tenor Specific Yield Curves

- Most banks post the crisis of 2008 construct pure discount curves using as base instruments fixed income contracts like certificates of deposit, forward rate agreements, futures contracts, and swaps to build discount curves at a variety of tenors, with the most popular ones being the OIS curve for the daily tenor, followed by tenors of $1, 3, 6$ and 12 months.

- By way of an example we present in Figure (1) the gap in basis points between the pure discount price of maturity t on a tenor above OIS and the OIS price on December 15 2010. The price gap is almost 200 basis points near a ten year maturity.
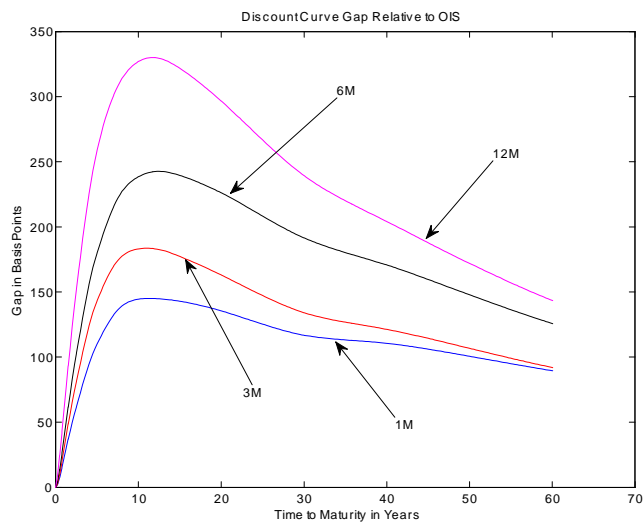
Figure 1: Zero coupon bond prices at tenors of one, three, six and twelve months less the OIS price in basis points.
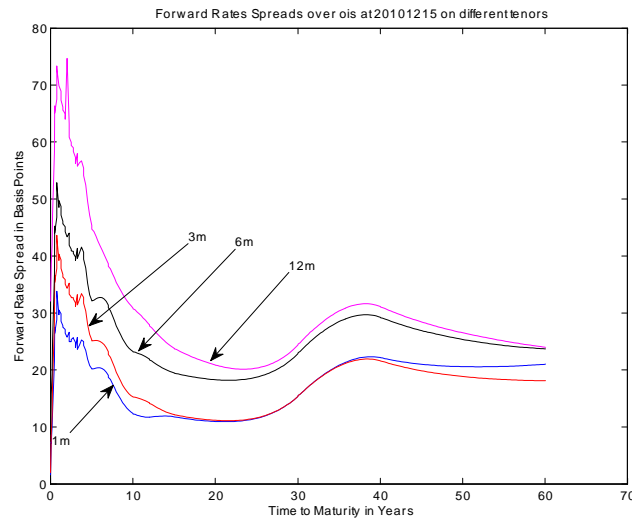
Figure 2: Forward Rate spreads at various tenors over OIS forward rates.

- From this data one may also construct the spread between forward rates on the higher tenors and the OIS forward rate. Figure (2) presents a graph of these spreads at various maturities. The spread in the forward rates reach up to 70 basis points.

- We have to ask ourselves what these prices are and what is their basis.

- A possibility is that the differences are credit related, but the instruments employed are quite varied with multiple counterparties and it is unclear that the biases built in are purely credit related.

- For example, Eberlein, Madan and Schoutens (2011) show using a joint model of credit and liquidity that the Lehman default was a liquidity event for the remaining banks and not a credit issue.

- Certainly lower tenors represent a greater liquidity so might the difference be to some extent due to this enhanced liquidity.

- How does liquidity expressed via a lower trading tenor theoretically affect prices?

Theoretical Tenor Specific Pricing

- We develop the theory for tenor specific pricing in general.

- To focus attention we begin with the simplest security of the pure discount bond.

- All economic agents must trade with the market and in line with the principles of conic finance the market serves as the passive counterparty for all financial transactions.

- The market is aware of a single risk neutral instantaneous spot rate process $r = (r(t), t \geq 0)$ at which funds may be transfered by the market through time.

- Suppose for simplicity that the underlying process for r is a one dimensional Markov process.

- Consider in this context the desire by an economic agent to buy from the market a unit face pure discount bond of unit maturity.

- If the market fixes the ask price at a, the market holds the random present value cash flow of

$$X(0,1) = a - e^{-\int_0^1 r(u)du}.$$

- The economic agent could hold the bond for unit time and then collect the unit face value.

- If the market prices this contract to acceptability using a convex set of test measures $\mathcal{M}$ then the ask price is given by

$$a = \sup_{Q \in \mathcal{M}} E^Q \left[e^{-\int_0^1 r(u)du} \right]$$

while the bid price is

$$b = \inf_{Q \in \mathcal{M}} E^Q \left[e^{-\int_0^1 r(u)du} \right],$$

and the mid quote or the reference two way price is the average of the bid and ask prices.

- Suppose now the economic agent wishes to have from the only market he or she must trade with, the opportunity to unwind this position at some earlier date and he or she wishes to see the terms at which this unwind my be possible.

- Essentially the economic agent asks the market for a schedule of bid and ask prices as functions of the prevailing spot rate at a frequency of $h = 1/N$. For $N = 4$ we have a quarterly schedule while $N = 12$ yields a monthly schedule.

- The market then has to first determine the bid and ask prices at time $1 - h$. At this time the present value of the risk X^a, X^b for an ask respectively bid price is

$$
\begin{aligned}
X^a(1 - h, 1) &= a - e^{-\int_{1-h}^{1} r(u)du} \\
X^b(1 - h, 1) &= e^{-\int_{1-h}^{1} r(u)du} - b
\end{aligned}
$$

- If the market uses the same cone of acceptability then these ask and bid prices are

$$a_{1-h}(r(1-h)) = \sup_{Q \in \mathcal{M}} E^Q \left[e^{-\int_{1-h}^{1} r(u)du} \right]$$

$$b_{1-h}(r(1-h)) = \inf_{Q \in \mathcal{M}} E^Q \left[e^{-\int_{1-h}^{1} r(u)du} \right].$$

- In principle this schedule may be computed.

Rolling back one more period

- We now consider the determination of the schedule at the next time step of $1 - 2h$.

- Now the market is ready to sell for $a(1 - h)$ at time $1 - h$ and we ask what price is the market willing to sell for at time $1 - 2h$.

- If it sells for a at time $1 - 2h$ we have the present value cash flow at time $1 - h$ of earning the interest and buying back at $1 - h$ for the ask price of $a_{1-h}(r(1 - h))$.

$$X^a(1-2h, 1-h) = a - a_{1-h}(r(1-h))e^{-\int_{1-2h}^{1-h} r(u)du}$$

- The corresponding bid cash flow is

$$X^b(1-2h, 1-h) = b_{1-h}(r(1-h))e^{-\int_{1-2h}^{1-h} r(u)du} - b$$

- It follows from making these risks acceptable that

$$a_{1-2h}(r(1-2h)) = \sup_{Q \in \mathcal{M}} E^Q \left[\begin{array}{c} a_{1-h}(r(1-h)) \times \\ e^{-\int_{1-2h}^{1-h} r(u)du} \end{array} \right]$$

$$b_{1-2h}(r(1-2h)) = \inf_{Q \in \mathcal{M}} E^Q \left[\begin{array}{c} b_{1-h}(r(1-h)) \times \\ e^{-\int_{1-2h}^{1-h} r(u)du} \end{array} \right]$$

- We thus get the ask and bid recursions on tenor h of

$$a_h(t-h) = \sup_{Q \in \mathcal{M}} \left(E^Q \left[e^{-\int_{t-h}^{t} r(u)du} a_h(t) \right] \right)$$

$$b_h(t-h) = \inf_{Q \in \mathcal{M}} \left(E^Q \left[e^{-\int_{t-h}^{t} r(u)du} b_h(t) \right] \right)$$

- The tenor specific discount curve is then given by the time zero mid quotes computed on each tenor h as

$$m_h(T) = \frac{a_h(0) + b_h(0)}{2}.$$

- The spreads between different tenors arise in these computations from liquidity considerations embedded in the cones of acceptable risks.

- They are not credit related as we do not have any defaults but just a reluctance to take exposures.

- Observe that if we go back to the law of one price with a base risk neutral measure Q^0 we may rewrite the recursion as

$$
b_h(t - h) = E^{Q^0} \left[e^{-\int_{t-h}^t r(u)du} b_h(t) \right]
$$
$$
+ \inf_{Q \in \mathcal{M}_h} \left(E^Q \left[\begin{array}{c} e^{-\int_{t-h}^t r(u)du} b_h(t) \\ -E^{Q^0} \left[e^{-\int_{t-h}^t r(u)du} b_h(t) \right] \end{array} \right] \right)
$$

where we have the one step ahead expectation plus a risk charge based on the deviation. The risk charge is for exposure to deviations and could in principle be the same for two different tenors.

- However, the charge is for a risk exposure over an interval of length h and should be levied as a rate per unit time with the charge for h units of time being proportional to h. Hence the recursion employed for the tenor h is

$$b_h(t - h) = E^{Q^0}\left[e^{- \int_{t-h}^{t} r(u)du} b_h(t) \right]$$

$$+ h \inf_{Q \in \mathcal{M}} \left(E^Q \left[\begin{array}{c} e^{- \int_{t-h}^{t} r(u)du} b_h(t) \\ - E^{Q^0}\left[e^{- \int_{t-h}^{t} r(u)du} b_h(t) \right] \end{array} \right] \right)$$

$$a_h(t - h) = E^{Q^0}\left[e^{- \int_{t-h}^{t} r(u)du} a_h(t) \right]$$

$$+ h \sup_{Q \in \mathcal{M}} \left(E^Q \left[\begin{array}{c} e^{- \int_{t-h}^{t} r(u)du} a_h(t) \\ - E^{Q^0}\left[e^{- \int_{t-h}^{t} r(u)du} a_h(t) \right] \end{array} \right] \right)$$

- The resulting bid and ask price sequences are dynamically consistent nonlinear expectations operators associated with the solution of backward stochastic difference equations.

- We have presented them here without reference to this underlying framework.

Connections to NonLinear Expectations

- To establish this connection we first briefly review these concepts and the connection between them as they have been established in the literature.

- In the context of a discrete time finite state Markov chain with states e_i identified with the unit vectors of $\mathbb{R}^M$ for some large integer M, Cohen and Elliott (2010) have defined dynamically consistent translation invariant nonlinear expectation operators $\mathcal{E}(.|\mathcal{F}_t)$. The operators are defined on the family of subsets $\left\{ \mathbb{Q}_t \subset L^2(\mathcal{F}_T) \right\}$.

- For completeness we recall here this definition of an $\mathcal{F}_t-$consistent nonlinear expectation for$\{\mathbb{Q}_t\}$. This $\mathcal{F}_t-$consistent nonlinear expectation for$\{\mathbb{Q}_t\}$ is a system of operators

$$\mathcal{E}(.|\mathcal{F}_t) : L^2(\mathcal{F}_T) \to L^2\left(\mathcal{F}_t\right), \ 0 \leq t \leq T$$

satisfying the following properties:

- 1. For $Q, Q' \in \mathbb{Q}_t$, if $Q \geq Q'$ $\mathbb{P}-a.s.$ componentwise, then

$$\mathcal{E}(Q|\mathcal{F}_t) \geq \mathcal{E}(Q'|\mathcal{F}_t)$$

$\mathbb{P}-a.s.$ componentwise, with for each i,

$$e_i \mathcal{E}(Q|\mathcal{F}_t) = e_i \mathcal{E}(Q'|\mathcal{F}_t)$$

only if $e_i Q = e_i Q'$ $\mathbb{P}-a.s.$

- 2. $\mathcal{E}(Q|\mathcal{F}_t) = Q$ $\mathbb{P}-a.s.$ for any $\mathcal{F}_t-$measurable Q.

- 3. $\mathcal{E}(\mathcal{E}(Q|\mathcal{F}_t)|\mathcal{F}_s) = \mathcal{E}(Q|\mathcal{F}_s)$ $\mathbb{P}-a.s.$ for any $s \leq t$

- 4. For any $A \in \mathcal{F}_t$, $\mathbf{1}_A \mathcal{E}(Q|\mathcal{F}_t) = \mathcal{E}(\mathbf{1}_A Q|\mathcal{F}_t)$ $\mathbb{P}-a.s.$

- Furthermore the system of operators is dynamically translation invariant if for any $Q \in L^2(\mathcal{F}_T)$ and any $q \in L^2(\mathcal{F}_t)$,

$$\mathcal{E}(Q + q|\mathcal{F}_t) = \mathcal{E}(Q|\mathcal{F}_t) + q.$$

Connection with BSDE

- Such dynamically consistent translation invariant non-linear expectations may be constructed from solutions of Backward Stochastic Difference Equations.

- These are equations to be solved simultaneously for processes Y, Z where Y_t is the nonlinear expectation and the pair (Y, Z) satisfy

$$Y_t - \sum_{t \leq u < T} F(\omega, u, Y_u, Z_u) + \sum_{t \leq u < T} Z_u M_{u+1} = Q$$

for a suitably chosen adapted map $F : \Omega \times \{0, \cdots, T\} \times \mathbb{R}^K \times \mathbb{R}^{K \times N} \to \mathbb{R}^K$ called the driver and for Q an $\mathbb{R}^K$ valued $\mathcal{F}_T$ measurable terminal random variable. We shall work in this paper generally with the case $K = 1$.

- For all t, (Y_t, Z_t) are $\mathcal{F}_t$ measurable.

- Furthermore for a translation invariant nonlinear expectation the driver F must be independent of Y and must satisfy the normalisation condition $F(\omega, t, Y_t, 0) = 0$.

- The drivers of the backward stochastic difference equations are the risk charges and for our ask and bid price sequences at tenor h we have drivers F^a, F^b where

$$
\begin{aligned}
F^a(\omega, u, Y_u, Z_u) &= h \sup_{Q \in \mathcal{M}} E^Q \left[Z_u M_{u+1} \right] \\
F^b(\omega, u, Y_u, Z_u) &= h \inf_{Q \in \mathcal{M}} E^Q \left[Z_u M_{u+1} \right],
\end{aligned}
$$

and the drivers are independent of Y.

- The process Z_t represents the residual risk in terms of a set of spanning martingale differences M_{u+1} and in our applications we solve for the nonlinear expectations Y_t without in general identifying either Z_t or the set of spanning martingale differences.

- We define risk charges directly for the risk defined for example as the zero mean random variable

$$e^{-\int_{t-h}^{t} r(u)du} a_h(t) - E^{Q^0}\left[e^{-\int_{t-h}^{t} r(u)du} a_h(t) \right].$$

- Leaving aside pure discount bonds we may consider for example a one year call option written on a forward or futures price $S(t)$ with zero risk neutral drift, unit maturity, strike K and payoff

$$(S(1) - K)^+.$$

Dynamically consistent forward bid and ask price sequences on the tenor h may be constructed as nonlinear expectations starting with

$$a(1) = b(1) = (S(1) - K)^+.$$

- Thereafter we apply the recursions

$$a_{t-h}(S(t-h)) = E^{Q^0}[a_t(S(t))]$$

$$+h \sup_{Q \in \mathcal{M}} \left(E^Q \left[\begin{array}{c} a_t(S(t)) \\ -E^{Q^0}[a_t(S(t))] \end{array} \right] \right)$$

$$b_{t-h}(S(t-h)) = E^{Q^0}[b_t(S(t))]$$

$$+h \inf_{Q \in \mathcal{M}} \left(E^Q \left[\begin{array}{c} b_t(S(t)) \\ -E^{Q^0}[b_t(S(t))] \end{array} \right] \right)$$

- Similar recursions apply to put options and other functions of the terminal stock price.

Path Dependent Claims

- For path dependent claims of an underlying Markov process with payoff on tenor h of

$$F\left((S(jh), 0 \leq j \leq J)\right) = V_J^a = V_J^b$$

- We determine the ask and bid value of the remaining uncertainty $V_j^a(S(jh))$, $V^b(S(jh))$ by the recursions

$$V_j^a(S(jh)) = E^{Q^0} \left[\begin{array}{c} F\left(S(kh), 0 \leq k \leq j+1\right) \\ -F\left(S(kh), 0 \leq k \leq j\right) \\ +V_{j+1}^a(S((j+1)h)) \end{array} \right]$$

$$+h \sup_{Q \in \mathcal{M}} E^Q \left[\begin{array}{c} F\left(S(kh), 0 \leq k \leq j+1\right) \\ -F\left(S(kh), 0 \leq k \leq j\right) \\ +V_{j+1}^a(S((j+1)h)) \\ -E^{Q^0} \left[\begin{array}{c} F\left(S(kh), 0 \leq k \leq j+1\right) \\ -F\left(S(kh), 0 \leq k \leq j\right) \\ +V_{j+1}^a(S((j+1)h)) \end{array} \right] \end{array} \right]$$

- The ask value of the claim is then

$$F\left(S(kh), 0 \le k \le j\right) + V_j^a(S(jh)).$$

- Similarly for the bid we have

$$V_j^b(S(jh)) = E^{Q^0} \left[\begin{array}{c} F\left(S(kh), 0 \le k \le j+1\right) \\ -F\left(S(kh), 0 \le k \le j\right) \\ +V_{j+1}^b(S((j+1)h)) \end{array} \right]$$

$$+h \inf_{Q \in \mathcal{M}} E^Q \left[\begin{array}{c} F\left(S(kh), 0 \le k \le j+1\right) \\ -F\left(S(kh), 0 \le k \le j\right) \\ +V_{j+1}^b(S((j+1)h)) \\ -E^{Q^0} \left[\begin{array}{c} F\left(S(kh), 0 \le k \le j+1\right) \\ -F\left(S(kh), 0 \le k \le j\right) \\ +V_{j+1}^b(S((j+1)h)) \end{array} \right] \end{array} \right]$$

and the bid value of the claim is

$$F\left(S(kh), 0 \le k \le j\right) + V_j^b(S(jh)).$$

- Tenor specific values may be constructed for a vast array of financial claims using the procedures developed for nonlinear expectations after the selection of an appropriate driver.

- The lower the tenor or the greater the frequency of quotations the more the liquidity that is being offered to economic agents.

- One might enquire into the nature of the limiting price associated with various drivers. These interesting questions are left for a considerable future research effort.

- For the moment we investigate the resulting tenor specific prices for bonds, stocks and options on stocks in a variety of contexts for a specific set of drivers based on distortions.

Drivers for nonlinear expectations based on distortions

- The driver for a translation invariant nonlinear expectation is basically a positive risk charge for the ask price and a positive risk shave for a bid price applied to a zero mean risk exposure to be held over an interim.

- We are then given as input the risk exposure ideally spanned by some martingale differences as $Z_u M_{u+1}$ or alternatively a zero mean random variable X with a distribution function $F(x)$.

- Cherny and Madan (2010) have constructed in the context of a static model law invariant bid and ask prices based on concave distortions.

- The bid and prices for a local exposure are then defined in terms of a concave distribution function $\Psi(u)$ defined on the unit interval as

$$
\begin{aligned}
b &= \int_{-\infty}^{\infty} x \, d\Psi(F(x)) \\
a &= -\int_{-\infty}^{\infty} x \, d\Psi\left(1 - F(-x)\right).
\end{aligned}
$$

- It is shown in Cherny and Madan (2010) that the set $\mathcal{M}$ of test measures seen as measure changes on the unit interval applied to $G(u) = F^{-1}(u)$ are all densities $Z(u)$ with respect to Lebesgue measure for which the antiderivative $H' = Z$ is distortion bounded, or $H \leq \Psi$.

- We consider in the rest of the paper drivers based on the distortion $minmaxvar$.

- In this case

$$
\begin{aligned}
F^b(Z_u M_{u+1}) &= \int_{-\infty}^{\infty} x\, d\Psi^\gamma(\Theta(x)) \\
F^a(Z_u M_{u+1}) &= -\int_{-\infty}^{\infty} x\, d\Psi^\gamma(1 - \Theta(-x)) \\
\Theta(x) &= \Pr(Z_u M_{u+1} \le x).
\end{aligned}
$$

- The distortion $\Psi^\gamma(u)$ is given by

$$
\Psi^\gamma(u) = 1 - \left(1 - u^{\frac{1}{1+\gamma}}\right)^{1+\gamma}.
$$

- Importantly it was shown in Carr, Madan and Vicente Alvarez (2010) that for such distortions in general that the mid quote lies above the risk neutral expectation if a claim has large exposures at quantiles above the median and low exposures below the median.

- The opposite is the case for large exposures at the lower quantiles and low ones above.

- The quantile exposure is measured by the sensitivity or derivative of the inverse of the distribution function.

Tenor Specific Discount Curves for the CIR spot rate model

- The construction of tenor specific discount curves require access to the probability law of random variables of the form

$$X^a(t, t+h) = e^{-\int_t^{t+h} r(u)du} a_{t+h}(r(t+h)).$$

- Hence one needs access to the joint law of the forward spot rate and the integral over the interim.

- This is available for the Cox, Ingersoll, Ross spot rate process defined by the stochastic differential equation

$$dr = \kappa(\theta - r)dt + \lambda\sqrt{r}dW$$

where κ is the rate of mean reversion, θ is the long term equilibrium interest rate and λ is the spot rate volatility parameter.

- The Laplace transform of the forward spot rate given the current rate is available in closed form and an application of an inverse Laplace transform along the lines of Abate and Whitt gives us access to the distribution function. The forward spot rate may then be simulated by the inverse uniform method.

- The Laplace transform of the integral given the initial spot rate and the final spot rate is also available in closed form (Pitman and Yor (1982)) and once again an inverse Laplace transform allows us to draw from the density of the integral given the rates at the two ends.

- In this way we may simulate readings on $X^a(t, t+h)$ and $X^b(t, t+h)$. Working backwards from a one year maturity for the first step we just need the law of the integral.

- Thereafter we first simulate $r(t + h)$ we then interpolate from stored values of bid and ask prices at the later time step the value for $a_{t+h}(r(t+h)), b_{t+h}(r(t+h))$. Then we draw from the distribution of the integral given the rates at the two ends to do the discounting and construct a single reading on X^a or X^b.

- We are then in a position to perform the recursion at different tenors back to time zero.

Estimating CIR

- For a sample of parameter values to work with we employ the joint characteristic function for the rate and its integral

$$\phi_r(u, v) = E\left[\exp\left(iur(t) + iv\int_0^t r(s)ds\right)\right]$$

and determine the risk neutral pure discount bond prices as

$$P(0, t) = \phi_r(0, i).$$

- This model for bond prices was to the OIS discount curve for data on December 15 2010.

- The estimated parameters were

$$
\begin{aligned}
\kappa &= 0.3712 \\
\theta &= 0.0477 \\
\lambda &= 0.0599 \\
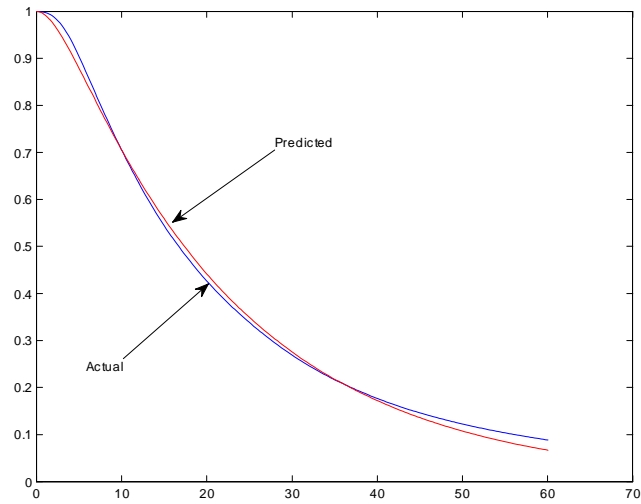r_0 &= 0.00004.
\end{aligned}
$$

Figure 3: Actual and CIR model predicted OIS discount bond prices for maturities up to 60 years.

- A graph of the actual and fitted bond price curves are presented in Figure (3)

- The recursions for bid and ask prices were performed using the $minmaxvar$ distortion at a stress level of 0.75, for all the local risk charges.

- Figure (4) presents discount five year bond prices at time 0 for 3, 6 and 12 month tenors as a function of the initial spot rate that we let vary to levels reached at the first time point of 3, 6 and 12 months.

- An increase in the price of pure discount bonds associated with the shorter tenor is observed in the model in line with market data for such tenor specific discount curves.

- The theory for tenor specific pricing presented in this paper is capable of generating tenor specific discount curves of the form observed in markets post the crisis of 2008.
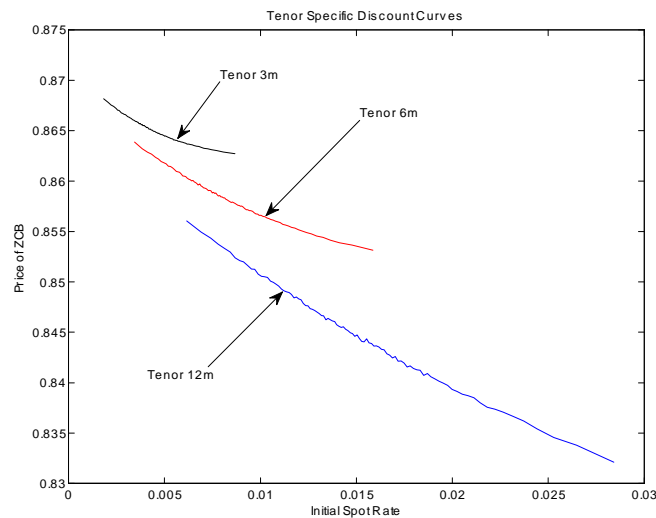
Figure 4: Tenor Specific Discount curves generated from mid quotes of dynamic sequences of bid and ask prices constructed at the tenors of 3, 6, and 12 months in black, red and blue respectively.

Tenor Specific Forward Stock Prices

- Consider tenor specific pricing for an underlying risk neutral process that is forward price martingale.

- Our first example is that of geometric Brownian motion. The risk neutral process here is

$$S(t) = S(0) \exp\left(\sigma W(t) - \frac{\sigma^2 t}{2}\right)$$

for a Brownian motion $W(t)$ and we take the initial stock price $S(0)$ to be 100.

- Economic agents trading with the market do not have access to this risk neutral process that represents the underlying risk priced by the market.

- Consider first the forward prices for delivery of stock in one year for a variety of volatilities and quoting tenors.

- We take for the two way price of the market, the mid quote constructed using the distortion $minmaxvar$ at the stress level 0.75.

- Table 1 presents the resulting midquotes.

TABLE 1
Mid Quotes under GBM

Tenor	Volatility		
	.2	.3	.4
12m	101.8420	103.7851	106.9213
6m	101.1343	102.4458	104.5791
3m	100.4782	101.4591	102.5282
1m	100.1019	100.0572	100.2699

- It is clear that for the geometric Brownian motion model the single time step mid quote is above the risk neutral value and furthermore as one enhances liquidity by decreasing the tenor the prices fall towards the risk neutral value.

- The positive skewness of the lognormal distribution lifts the supremum and results in a mid quote above the risk neutral value.

- This effect is dampened for the shorter tenors.

- Actual risk neutral stock price distributions have a considerable left skewness as reflected in the implied volatility smiles. It is therefore instructive to investigate mid quotes in tenor specific pricing for models that fit the smile. For this we turn next to the variance gamma model.

- For a set of stylized parameter values we fix σ the volatility of the Brownian motion at .2 as a control on volatility.

- We then take some moderate and high values for skewness and excess kurtosis via setting θ at $-.3, -.6$ and setting ν at $.5, 1.5$.

- For these four settings we report in Table 2 on the midquotes for a quarterly tenor on a one year forward quote.

  TABLE 2
  Mid Quotes under VG

σ	ν	θ	midquote
.2	.5	$-.3$	97.7905
.2	.5	$-.6$	97.0130
.2	1.5	$-.3$	96.0998
.2	1.5	$-.6$	95.2685

- It may be observed that in all these cases the mid quote is below the risk neutral value.

- Preliminary numerical investigations confirm that as we increase liquidity we do get a convergence to the risk neutral value and hence it appears that an increase in liquidity raises the two way price quote on stocks for two price markets.

Tenor Specific Option Prices

- We now report on the mid quote and the risk neutral value of out of the money options and loan type contracts for an underlying geometric Brownian motion with a 30% volatility and the four VG processes considered in section 6.

- The out of the money options are a put struck at 80 and a call struck at 120 with an annual maturity.

- The loan or risky debt type contract pays the minimum of 1.25 times the stock price and a 100 dollars. Loss is then taken for stock prices below 80.

- In each case the risk neutral value and the mid quote are reported at each of two tenors, quarterly and monthly.

- The results are in tables 3 and 4, one for the quarterly tenor and the other for the monthly tenor.

- The loan mid quotes are below risk neutral values and rise as the tenor comes down.

- The opposite is the case with out-of-the-money options reflecting the expected convergence to risk neutral values.

TABLE 3
Tenor Specific Options, Tenor 3m

Model	Risky Debt		80 Put		120 Call	
	RNV	MQ	RNV	MQ	RNV	MQ
GBM	95.78	94.18	3.40	4.68	5.14	7.77
VG1	95.88	93.20	3.33	5.48	2.44	3.48
VG2	91.20	87.46	7.02	10.01	6.95	7.92
VG3	93.42	89.62	5.31	8.35	2.46	3.20
VG4	87.30	82.39	10.29	14.22	8.93	9.08

TABLE 4

Tenor Specific Options, Tenor 1m

	Risky Debt		80 Put		120 Call	
Model	RNV	MQ	RNV	MQ	RNV	MQ
GBM	95.80	95.19	3.37	3.86	4.88	5.83
VG1	96.02	94.44	3.19	4.45	2.42	2.75
VG2	91.40	88.94	6.84	8.81	6.88	6.71
VG3	93.64	91.31	5.11	6.98	2.43	2.63
VG4	87.39	84.07	10.03	12.68	8.87	8.31

Conclusion

- Fixed income markets now construct pure discount curves based on a variety of tenors for rolling over funds between time points.

- This gives rise to tenor specific prices for zero coupon bonds and raises the issue of the possibility of tenor specific pricing for all financial contracts.

- It is recognized that the law of two prices, bid and ask, as constructed in theory of conic finance set out in Cherny and Madan (2010), yields prices that are nonlinear functions of the random variables being priced.

- Dynamically consistent sequences of such prices are then related to the theory of nonlinear expectations and its connections with solutions to backward stochastic difference equations.

- The drivers for the stochastic difference equations are related to concave distortions that implement risk charges for the local risk specific to the tenor.

- This theory is applied at a variety of tenors to generate such tenor specific bid and ask prices for discount bonds, stocks, and options on stocks.

- It is observed that such tenor specific prices given by the mid quotes of bid and ask converge to the risk neutral price as the tenor is decreased.

- The greater liquidity of lower tenors may lead to an increase or decrease in prices depending on whether the lower liquidity of a higher tenor has a mid quote above or below the risk neutral value.

- Generally for contracts with a large upside and a bounded downside the prices fall with liquidity while the opposite is the case for contracts subject to a large downside and a bounded upside.