Measuring Market Fear

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CONTENT

- Market Fear Components
  - Volatility
  - Liquidity
  - Herd-behavior
  - Counterparty Risk
- Volatility measuring by VIX
- Liquidity measuring by implied liquidity & conic finance
- Herd-behavior measuring by comonotonicity ratio
- Introducing the FIX – the market overall Fear Index

MARKET FEAR COMPONENTS

- There are a variety of market fear factors.
- We have market risk and nervousness. The higher the volatility the more market uncertainty there is and the wider swings in the market can occur.
- We have liquidity risk. The bid and ask spread widens in periods of high uncertainty.
- We have herd-behavior. In a systemic crisis, all assets move into the same direction. The more comonotone behavior we have the more assets move together and the more systemic risk there is.
- We have counterparty risk. In heavily distressed periods, counterparty risk is omnipresent. The failure of a counterparty could lead to a domino effect. Counterparty risk can be measured through Credit Default Swaps and other credit derivatives.

MARKET FEAR COMPONENTS

- The aim is to measure the market fear factors on the basis of market option data in a single intuitive number.
- The measure will be an overall market measure and hence will be based on vanilla index options and individual stock options.
- By making use of option data and not of historical data we have a forward looking measure indicating markets expectations for the near future.
- The classical example of using of option data is the measurement of market volatility by the VIX methodology.
- We will measure volatility, herd-behavior and liquidity in a similar manner and hence will be able of exactly decomposing the overall market fear into its components.

VIX

- The VIX index is often referred to as the fear index or fear gauge. It is a key measure of market expectations of near-term volatility conveyed by SP 500 stock index option prices.
- Since its introduction in 1993, the VIX has been considered by many to be a good barometer of investor sentiment and market volatility.
- It is a weighted blend of prices for a range of options on the SP500 index.
- The formula uses as inputs the current market prices for all out-of-the-money calls and puts for the front month and second month expirations.
- The goal is to estimate the implied volatility of the SP500 index over the next 30 days.

VIX

- The VIX calculation is very related to the implementation of a Variance Swap (cfr. work by P. Carr, D. Madan, A. Neuberger and others)
- On March 26, 2004, the first-ever trading in futures on the VIX Index began on CBOE Futures Exchange (CFE).
- As of February 24, 2006, it became possible to trade VIX options contracts.
- The VIX methodology has been applied on many other indices.
- On the January 5, 2011, CBOE announced to also VIX-ify individual stocks like (APPL, IBM, GS, GOOG, …).

VIX

- The magic VIX formula is :

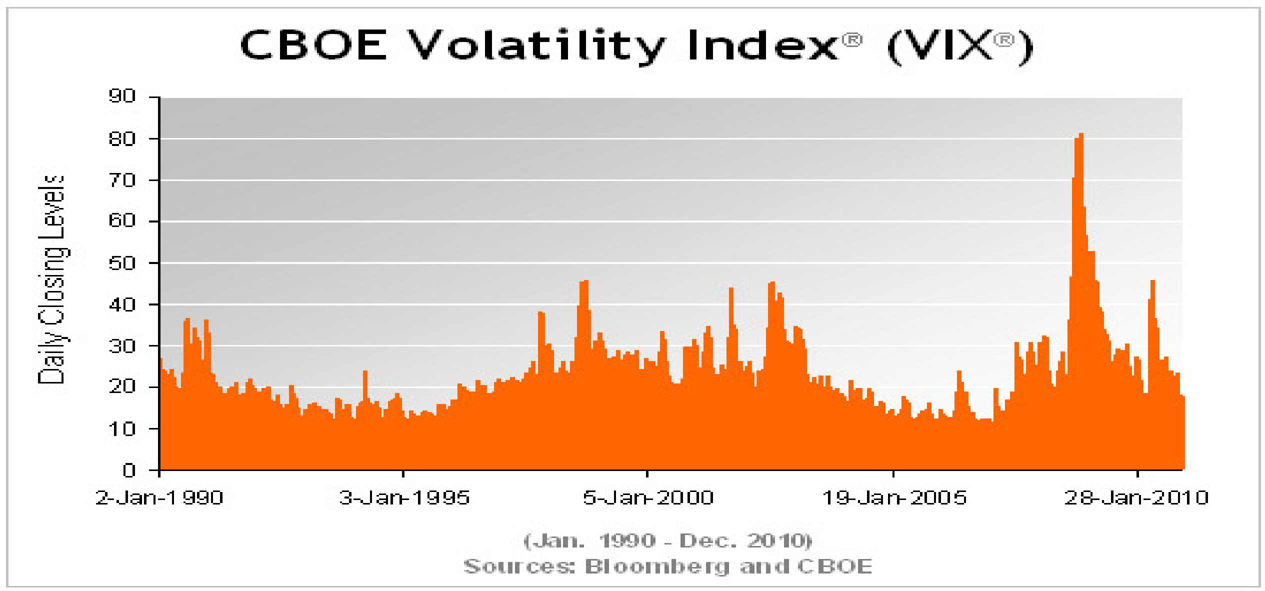
$$\sigma^2 = \frac{2}{T}\sum_i \frac{\Delta K_i}{K_i^2} e^{RT} Q(K_i) - \frac{1}{T}\left[\frac{F}{K_0} - 1\right]^2$$

- VIX = σ x 100
- T is time to maturity
- F is forward index level
- K_i are strikes
- R is interest rate and
- Q(.) are mid prices

VIX

- The formula is applied to the front month (with T > 1 week) and the next month and is finally obtained by inter/extrapolation on the 30 days point:

$$\mathrm{VIX} = 100 \times \sqrt{\left\{ T_1\sigma_1^2 \left[\frac{N_{T_2} - N_{30}}{N_{T_2} - N_{T_1}} \right] + T_2\sigma_2^2 \left[\frac{N_{30} - N_{T_1}}{N_{T_2} - N_{T_1}} \right] \right\} \times \frac{N_{365}}{N_{30}}}$$



MEASURING LIQUIDITY

- How to measure and quantify in an isolated manner liquidity ?
- Bid-ask spread are a good indication but can be misleading.

- **Example:** Which European Call Option is the most liquid ?

EC1 on Stock1	EC2 on Stock2
Maturity = 1y	Maturity = 1y
Bid = 9 EUR	Bid = 9 EUR
Mid = 10 EUR	Mid = 10 EUR
Ask = 11 EUR	Ask = 11 EUR

A) EC1
B) EC2
C) Both
D) Can't say

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r=0%; q=0%	r=0%; q=0%
S1=100	S2=20
K=100	K=10
Vol=25.13%	Vol=1.0%
Bid = 9 EUR	Bid = 9 EUR
Mid = 10 EUR	Mid = 10 EUR
Ask = 11 EUR	Ask = 11 EUR

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Probability that Stock2 after one year will trade above 19.0 EUR is 0.9999997 (5 sigma event).
And hence option will "always" payout more than 9 EUR.

MEASURING LIQUIDITY

- It is very difficult to measure liquidity in an isolate manner.
- Bid and ask spreads can move around in a non-linear manner if spot, vol, or other market parameters move, without a change in liquidity.
- The concept of implied liquidity in a unique and fundamental founded way isolates and quantifies the liquidity risk in financial markets.
- This makes comparison over times, products and asset classes possible.
- The underlying fundamental theory is based on new concepts of the two-ways price theory of conic finance.
- These investigations open the door to stochastic liquidity modeling, liquidity derivatives and liquidity trading.

CONIC FINANCE

- We will make use of the minmaxvar distortion function:

$$\Phi(u;\lambda) = 1 - \left(1 - u^{\frac{1}{1+\lambda}}\right)^{1+\lambda}$$

- We use distorted expectation to calculate (bid and ask) prices.
- The distorted expectation of a random variable with distribution function *F(x)* is defined

$$de(X;\lambda) = \mathrm{E}^{\lambda}[X] = \int_{-\infty}^{+\infty} x \mathrm{d}\Phi(G(x);\lambda).$$

- The ask price of payoff *X* is determined as

$$ask(X) = -\exp(-rT)\mathrm{E}^{\lambda}[-X].$$

- The bid price of payoff X is determined as

$$bid(X) = \exp(-rT)\mathrm{E}^{\lambda}[X].$$

CONIC FINANCE

- These formulas are derived by noting that the cash-flow of selling X at its ask price and buying *X* at its bid price is acceptable in the relevant market .

- We say that a risk *X* is acceptable if

$$E_Q[X] \geq 0 \text{ for all measures } Q \text{ in a convex set } \mathcal{M}.$$

M is a set of test-measures under which cash-flows need to have positive expectation.

- Operational cones were defined by Cherney and Madan and depend solely on the distribution function *G(x)* of *X* and a distortion function. To have acceptability we need to have that the distorted expectation is positive:

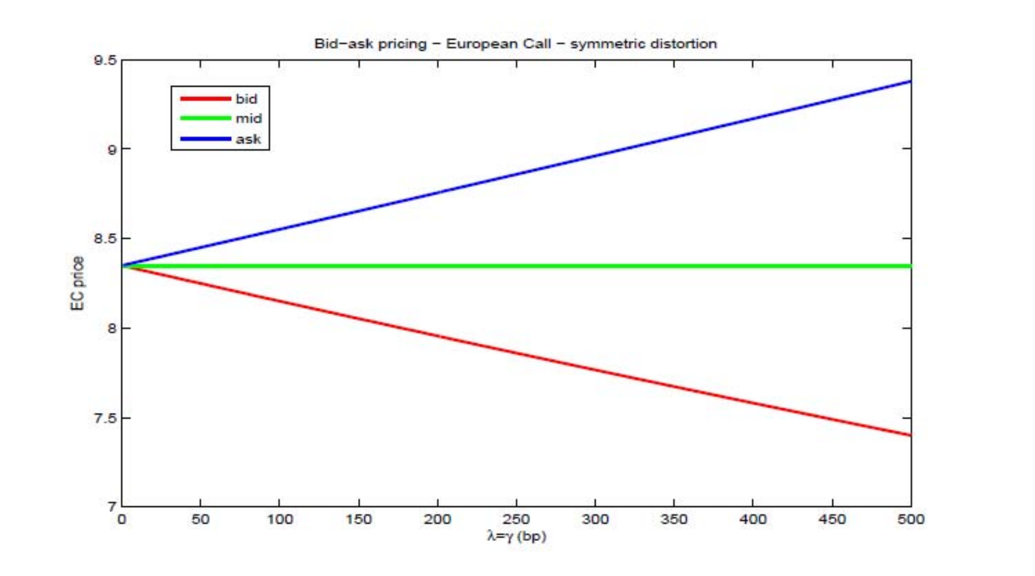
$$de(X;\lambda) = \mathrm{E}^{\lambda}[X] = \int_{-\infty}^{+\infty} x \mathrm{d}\Phi(G(x);\lambda).$$

CONIC FINANCE

$$\begin{aligned} bid(X) &= \exp(-rT)\int_0^{+\infty} x\,\mathrm{d}\Phi(G(x);\lambda), \\ ask(X) &= \exp(-rT)\int_{-\infty}^{0} (-x)\,\mathrm{d}\Phi(1-G(-x);\lambda). \end{aligned}$$

- For a EC (K,T), we have

$$G(x) = 1 - \mathrm{N}\left(\frac{\log(S_0/(K+x)) + (r-q-\sigma^2/2)T}{\sigma\sqrt{T}}\right), \qquad x \geq 0$$



IMPLIED LIQUIDITY

- We will call the parameter, fitting the bid-ask around the mid price, the **implied liquidity parameter**.

- Hence for the EC(K,T) with given market bid, *b*, and ask, *a*, prices, the implied liquidity parameter is the specific λ >0, such that:

$$a = -\exp(-rT)\mathrm{E}^{\lambda}[-(S_T - K)^+] \text{ and } b = \exp(-rT)\mathrm{E}^{\lambda}[(S_T - K)^+],$$

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Vol=25.13%	Vol=1.0%
Bid = 9 EUR	Bid = 9 EUR
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Ask = 11 EUR	Ask = 11 EUR
$\lambda = 626$ bp	$\lambda = 53769$ bp

A) EC1
B) EC2
C) Both
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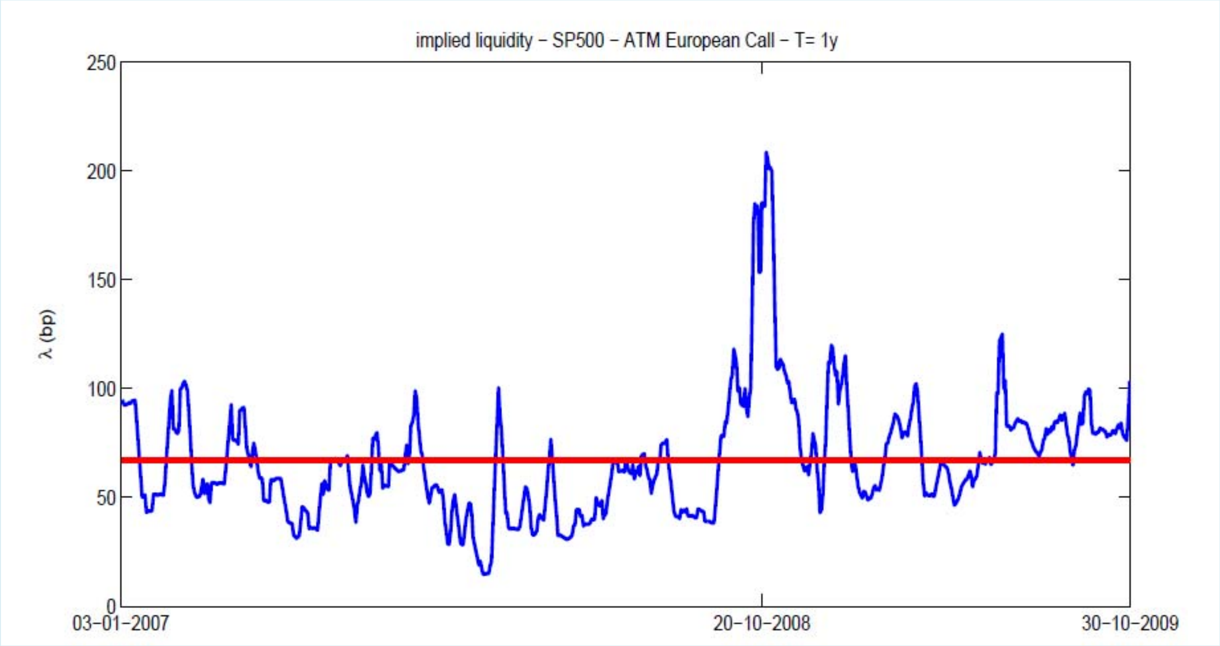
IMPLIED LIQUIDITY– EVOLUTION OVER TIME

- We clearly see that liquidity is non constant over time and exhibits a mean-reverting behavior.
- The long run average of the implied liquidity of the data set and over the period of the investigation this equals 67.11 bp.
- The highest value for the implied liquidity parameter was 283.1 bp on the 20th of October 2008. Around that day (and the week-end before) several European banks were rescued by government interventions.



HERD BEHAVIOR AND COMONOTONICITY

- Comonotonicity measures herd behavior.
- A random vector $Y = (Y_1, \ldots, Y_N)$ is comontonic if

$$Y =^d (F_{Y_1}^{[-1]}(U), \ldots, F_{Y_n}^{[-1]}(U)),$$

where *U* is a Uniform(0,1) random variable and

$$F_{Y_i}^{[-1]}(u) = \inf\{x \in \mathbb{R} | P(Y_i \leq x) = F_{Y_i}(x) \geq u\}.$$

- A comonotonic vector is driven by just one single factor.
- Given a vector $X = (X_1, \ldots, X_N)$ we call the comonotonic counter-part of *X* the vector

$$X^c = (X_1^c, \ldots, X_N^c) =^d (F_{X_1}^{[-1]}(U), \ldots, F_{X_n}^{[-1]}(U))$$

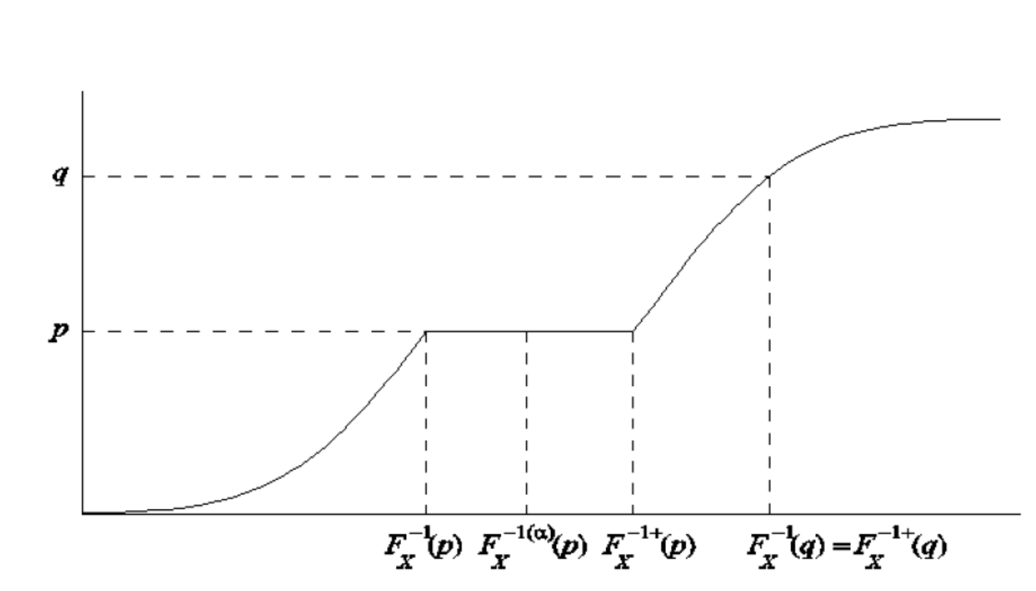
HERD BEHAVIOR AND COMONOTONICITY

- Inverse cdf fucntions:

$$F_{Y_i}^{[-1]}(u) = \inf\{x \in \mathbb{R} | P(Y_i \leq x) = F_{Y_i}(x) \geq u\}.$$

$$F_{Y_i}^{[-1+]}(u) = \sup\{x \in \mathbb{R} | P(Y_i \leq x) = F_{Y_i}(x) \leq u\}$$

$$F_{Y_i}^{[-1(\alpha)]}(u) = \alpha F_{Y_i}^{[-1]}(u) + (1-\alpha) F_{Y_i}^{[-1+]}(u)$$



HERD BEHAVIOR AND COMONOTONICITY

- Dow Jones, SP500 and any other indices are a weighted basket:

$$I(t) = \sum_{i=1}^{n} w_i S_i(t), \qquad t \geq 0$$

- Index Vanilla options can hence be seen as basket options:

$$(I(T) - K)^+ = \left(\sum_{i=1}^{n} w_i S_i(T) - K \right)^+$$

- We will denote by

$$I^c(T) = \sum_{i=1}^{n} w_i S_i^c(T).$$

where $S^c(T)$ is the comonontonic counterpart of

$$S(T) = (S_1(T), \ldots, S_n(T))$$

HERD BEHAVIOR AND COMONOTONICITY

- It is shown that

$$F_{I^c(T)}(x) = \sup\left\{p \in [0,1] | \sum_{i=1}^{n} w_i F^{[-1]}_{S_i(T)}(p) \leq x\right\}$$

and

$$F^{[-1(\alpha)]}_{I^c(T)}(p) = \sum_{i=1}^{n} w_i F^{[-1(\alpha)]}_{S_i(T)}(p)$$

and

$$E[(I^c(T) - K)^+] = \sum_{i=1}^{n} w_i E\left[\left(S_i(T) - F^{[-1(\alpha)]}_{S_i(T)}\left(F_{I^c(T)}(K)\right)\right)^+\right]$$

where $\alpha \in [0,1]$ is such that $F^{[-1(\alpha)]}_{I^c(T)}\left(F_{I^c(T)}(K)\right) = K$

HERD BEHAVIOR AND COMONOTONICITY

- The expression

$$E[(I^c(T)-K)^+] = \sum_{i=1}^{n} w_i E\left[\left(S_i(T) - F_{S_i(T)}^{[-1(\alpha)]}\left(F_{I^c(T)}(K)\right)\right)^+\right]$$

  basically tells us that the price of a vanilla call under the comotonic setting $C^c(K,T)$quals a weighted sum of calls $C_i(K_i,T)$ on the components.

- The weights and maturity are the same; the strikes vary.

$$K_i^* = F_{S_i(T)}^{[-1(\alpha)]}\left(F_{I^c(T)}(K)\right)$$

- In order to determine these we need to have the cdf of all the components and of the comonotonic index. The later was given on previous slide.

- It is well know that the cdf of the stocks can be extracted out option info:

$$F_{S_i(T)}(x) = 1 + \exp(rT)\frac{\partial C_i(x+,T)}{\partial K}$$

HERD BEHAVIOR AND COMONOTONICITY

- Summarizing we have

$$C(K,T) \leq C^c(K,T) = \sum_{i=1}^{n} w_i C_i(K_i^*,T).$$

- We have derived an upper bound for the vanilla index options in terms of the component options.

- Moreover, we know that under a comonotonic setting the index options prices are coinciding with the upper bound.

- Therefore the gap $\sum_{i=1}^{n} w_i C_i(K_i^*,T) - C(K,T)$ between the true market price and the upper bound is a good indicator of how far one is away from the comonotonic situation.

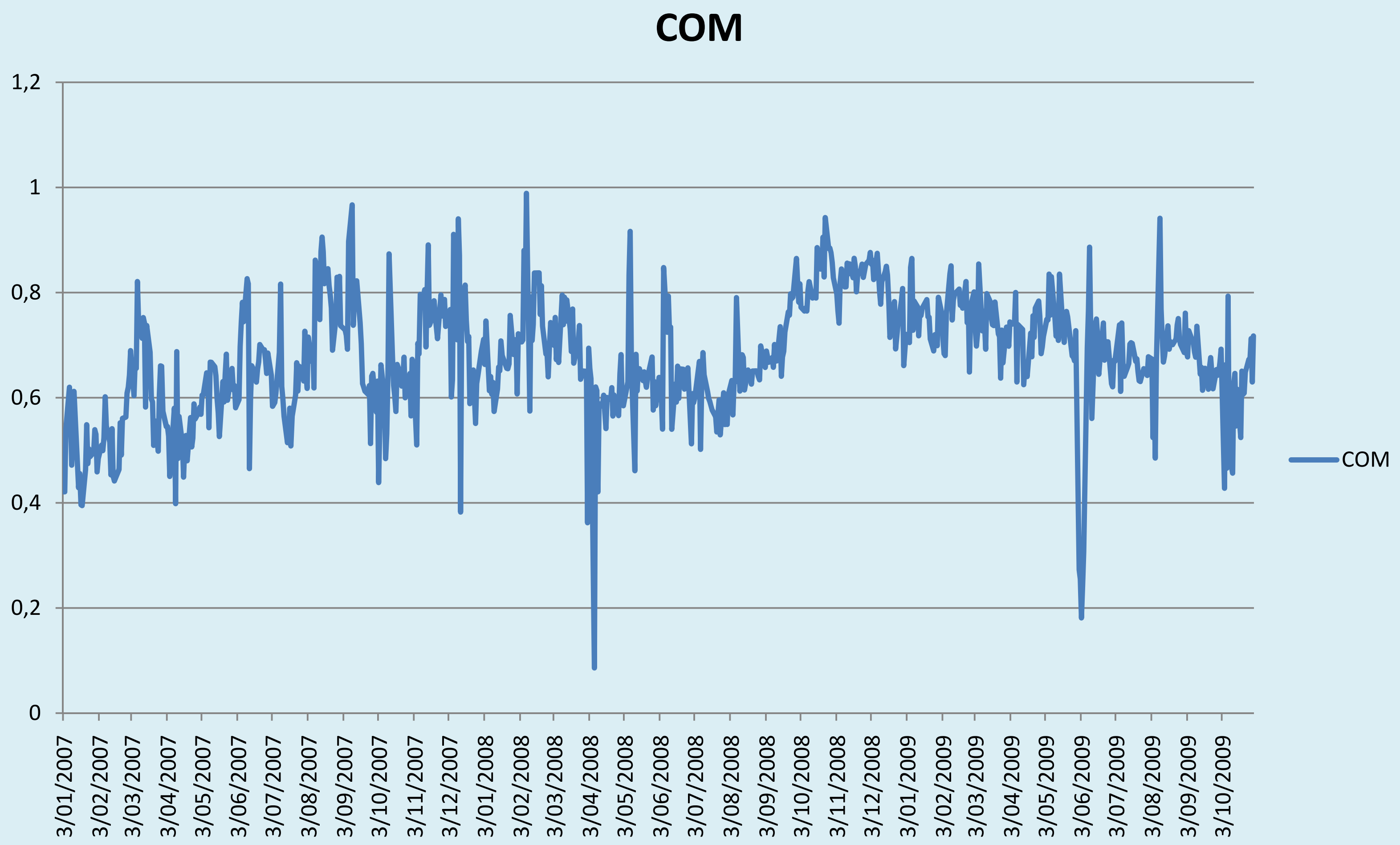
HERD BEHAVIOR AND COMONOTONICITY

- We call the quantity

$$\frac{C(K,T)}{C^c(K,T)}$$

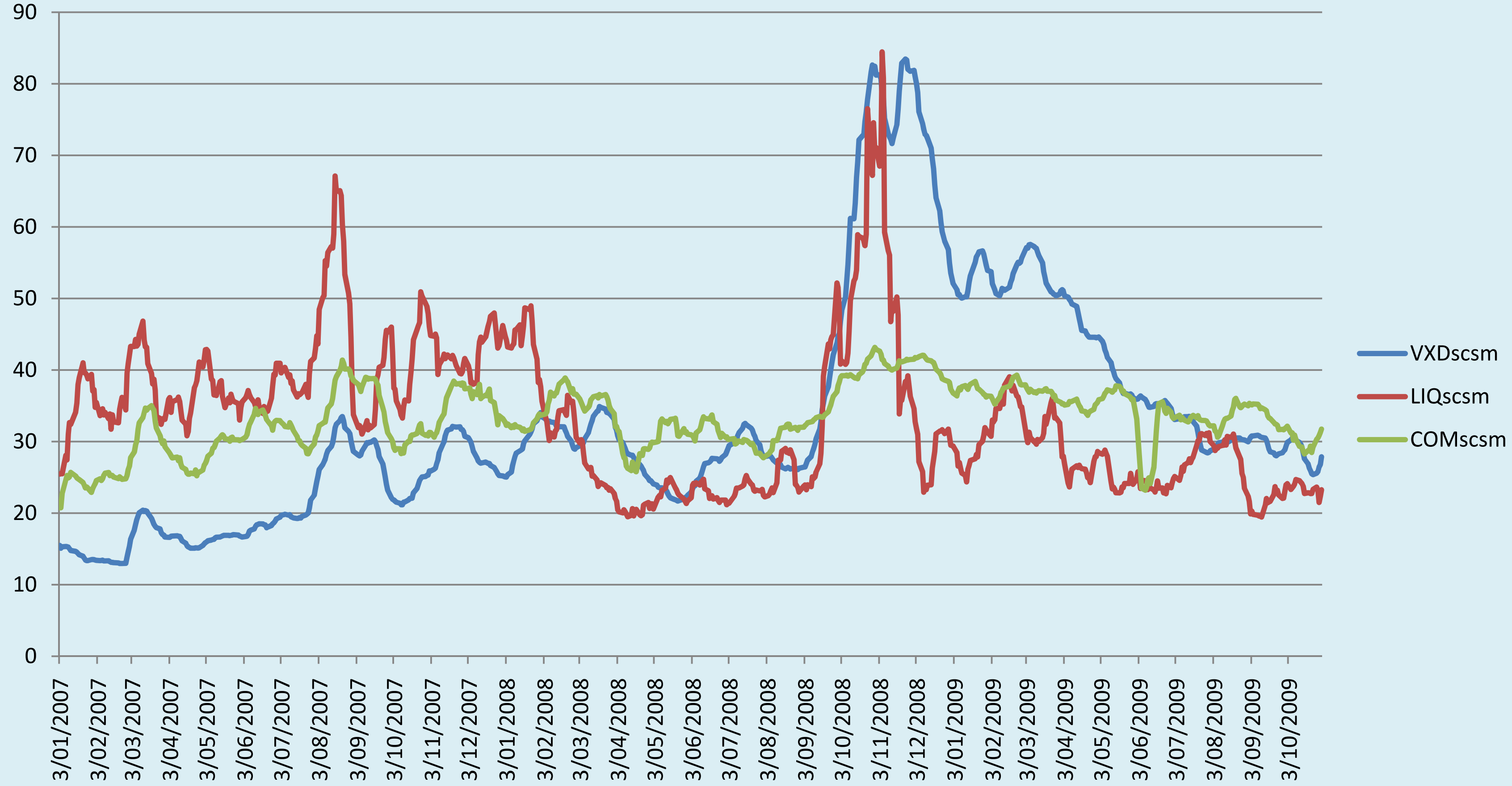
  the comonotonicity ratio.
- The closer this number is to 1, the closer we are to the comonotonic situation.
- If the ratio equals 1, we hence have perfect herd behavior.
- In conclusion, the above gives us a way to compute how much herd behavior there is on the basis of option surfaces.
- Furthermore, the gap between fully comontonic and the current market situation can be monetized via a long-short position in options.

COMOTONICITY RATIO

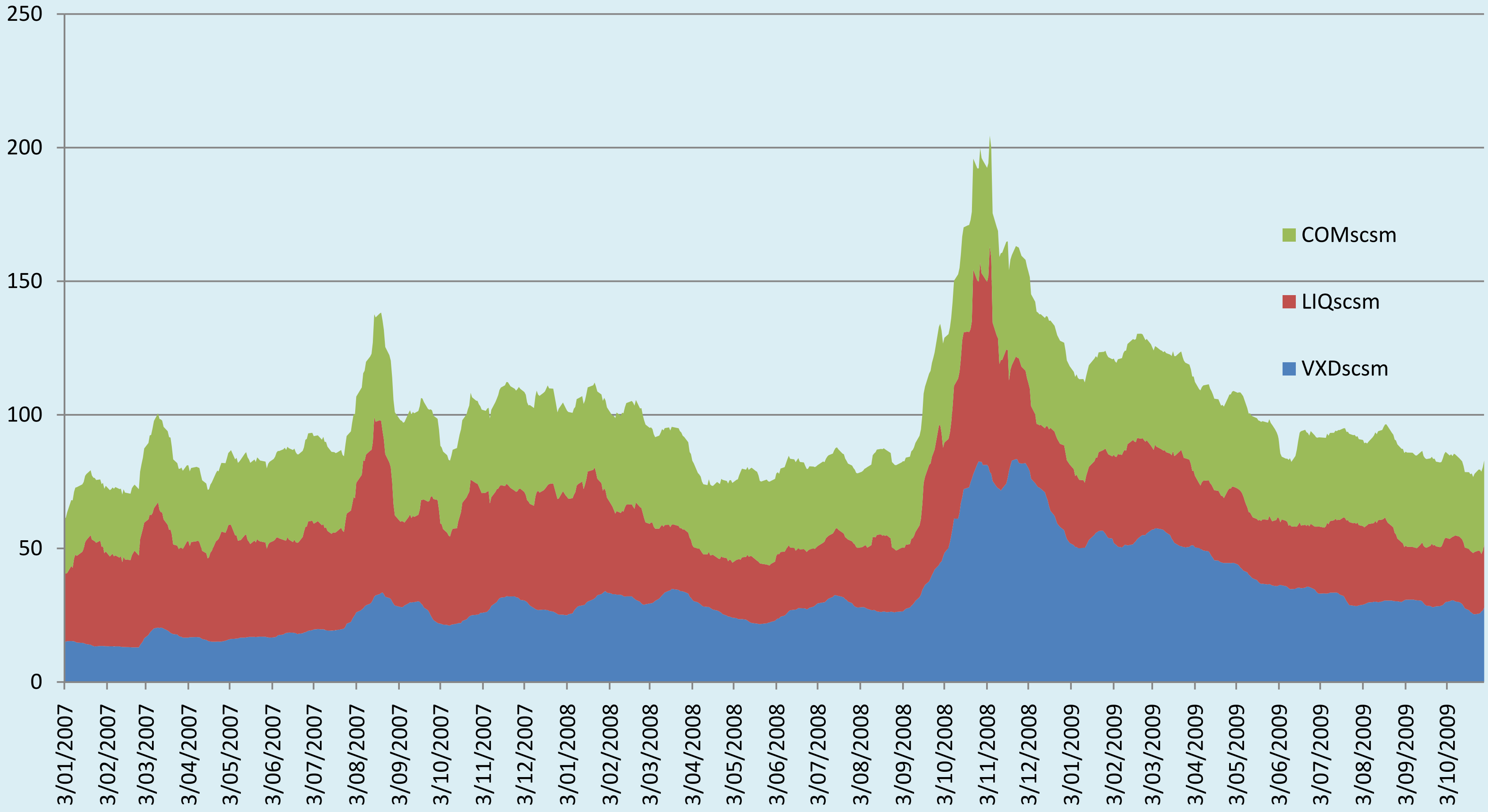


THE MARKET FEAR COMPONENTS

We smooth and rescale the 3 fear components :

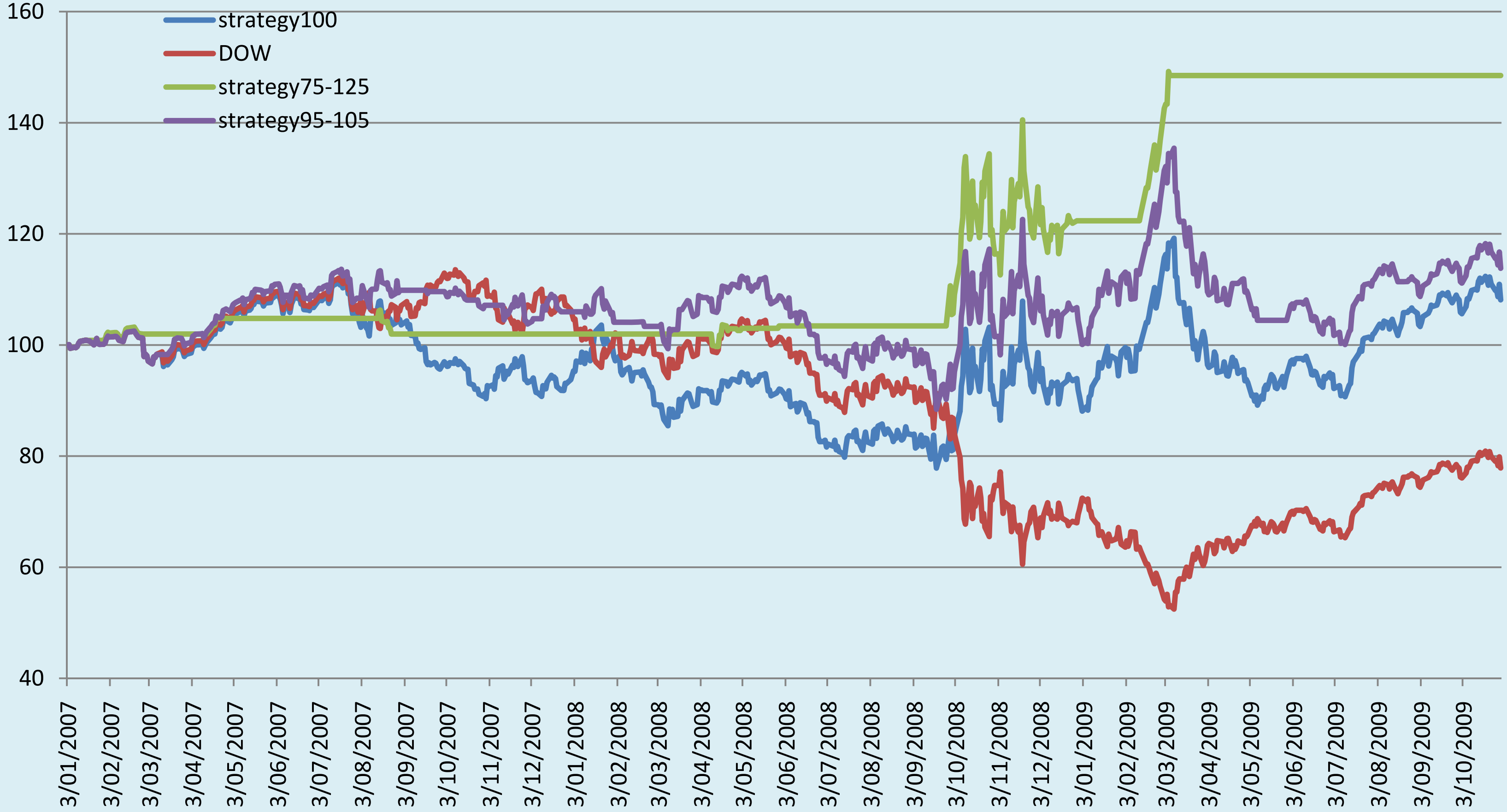


WE PROUDLY PRESENT YOU : THE FIX



100 is base value; a value above 100 reflects a more than average stress situation; a value below 100 is a less than average stress situation

TRADING STRATEGIES



DOW : long DJI

Strategy100 : short DJI if FIX >100; long DJI if FIX < 100

Strategy75-125 : short DJI if FIX >125; long DJI if FIX < 75

Strategy95-105 : short DJI if FIX >105; long DJI if FIX < 95

CONCLUSION

- There are a variety of market fear factors.
- We have market risk and nervousness. The higher the volatility the more market uncertainty there is and the wider swings in the market can occur.
- We have liquidity risk. The bid and ask spread widens in periods of high uncertainty.
- We have herd-behavior. In a systemic crises, all assets move into the same direction. The more comonotonic behavior we have the more assets move together and the higher the systemic risk there is.
- The aim is to measure the market fear factors on the basis of market option data in a single intuitive number.
- We have presented the FIX as an overall market measure. The calculations are solely based on vanilla index options and individual stock options.

CONCLUSION

"'Be careful'! All you can tell me is 'be careful'?"

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